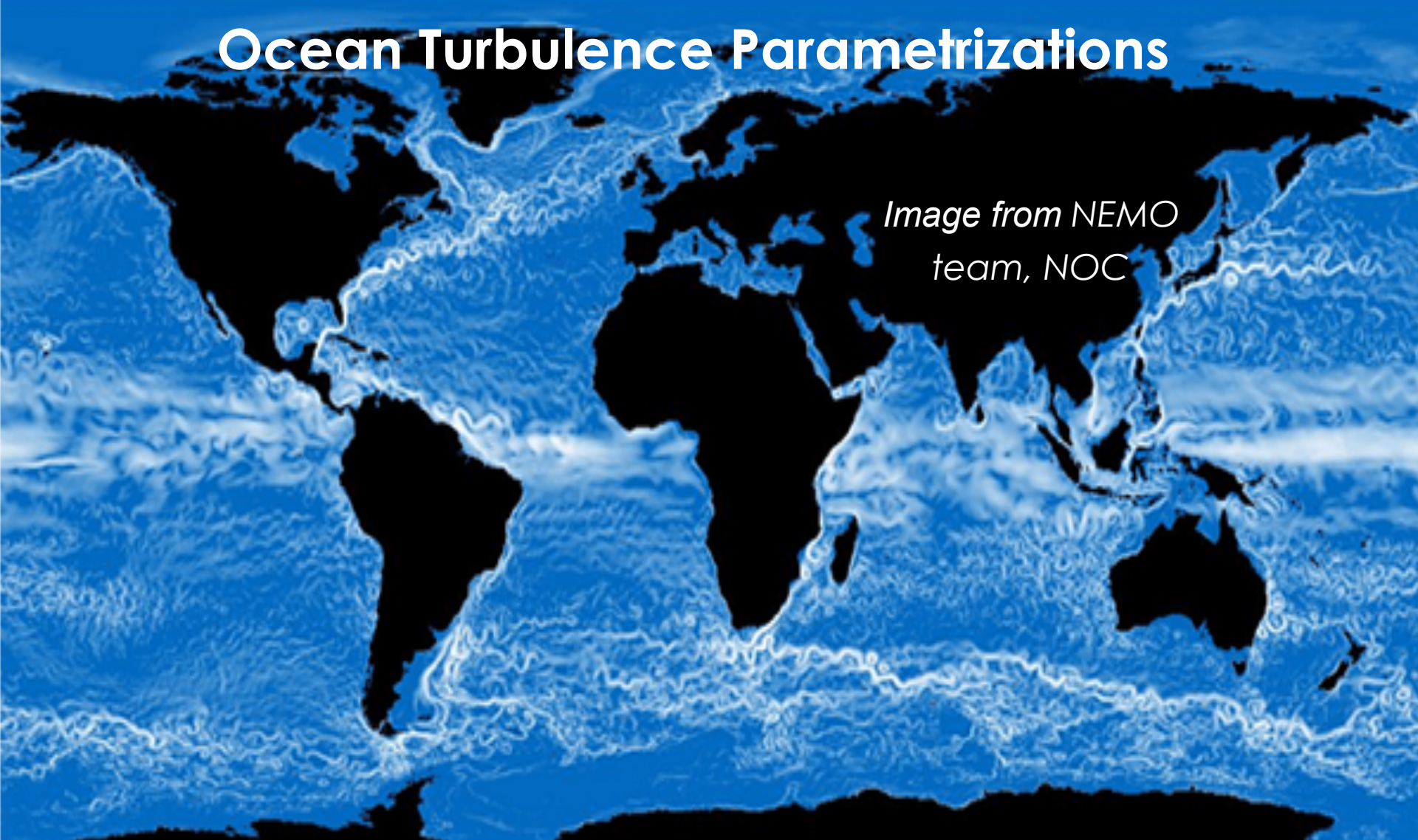


# Ocean Turbulence Parametrizations

*Image from NEMO  
team, NOC*



**Laure Zanna**

U. of Oxford, Dept of Physics  
on sabbatical @ Princeton/GFDL

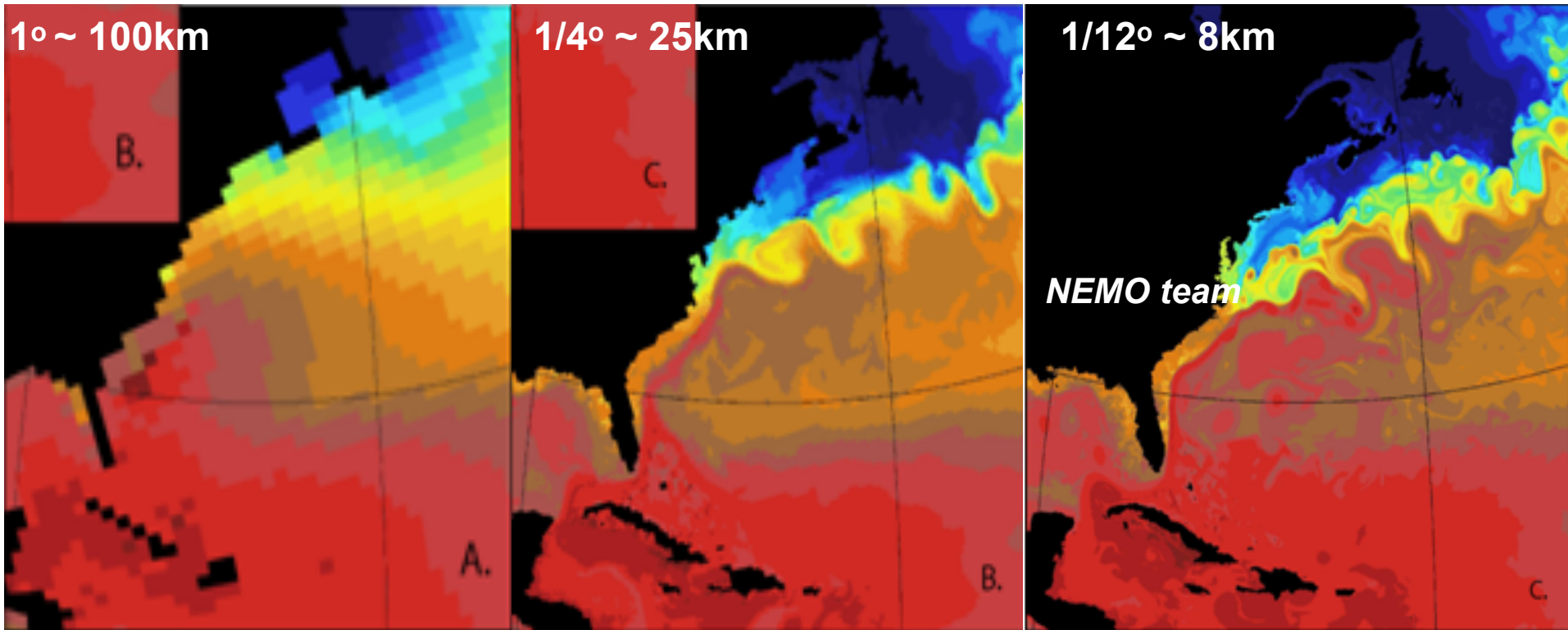
# Outline (with an ocean turbulence focus)

---

- The sub-grid parametrization problem
- The traditional approach & its caveats
- Our (& others) complementary approach: focus on sub-grid turbulence forcing, rather than one process due to the lack of scale separation
  - 1) coarse-graining from models + new mathematically- and physically-constrained relationships
  - 2) imperfect “observations” & machine learning (Neural Networks)
- ▶ Some results with implementations, caveats & ways forward

# Ocean Models & Resolution

Resolution



Timestep

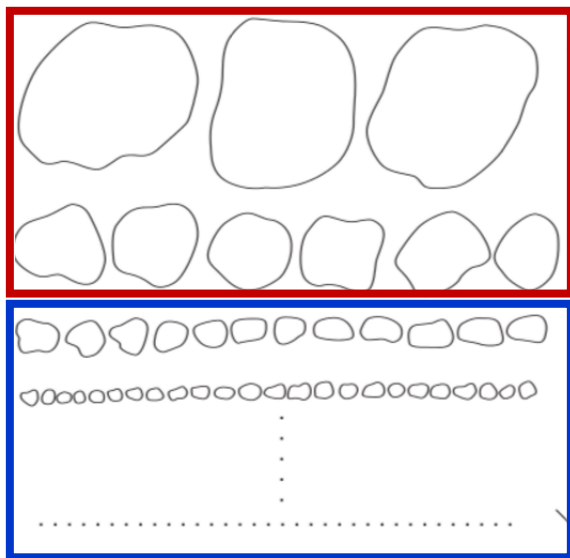
Computational cost

# Aim: Turbulence Closure in Numerical Simulations

- Governing Equations, e.g. zonal/x-direction:

$$\frac{\partial u}{\partial t} + \mathbf{u} \cdot \nabla_h \mathbf{u} - f v = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{F_x}{\rho_0}$$

- Reynolds-Averaged Equation of motions:



(-) = slow/large-scale fluctuations  
> grid-box size

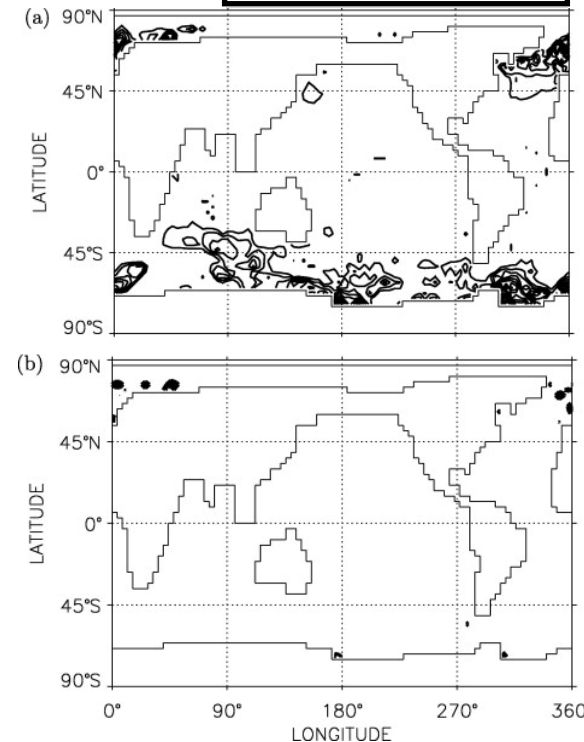
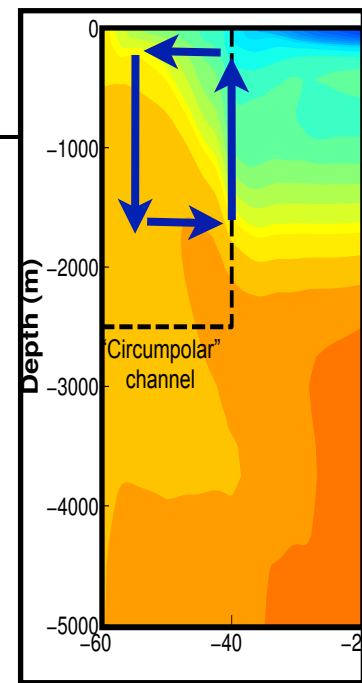
( )' = fast/small-scale (eddy) fluctuations  
< grid-box size

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} - f \bar{v} = -\frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x} + \frac{\bar{F}_x}{\rho_0} - \underbrace{\frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{u'v'}}{\partial y}}$$

- **Turbulence Closure Problem:** how to mimic the eddy/Reynolds stress which represent the effects of small/fast fluctuations on large-scale

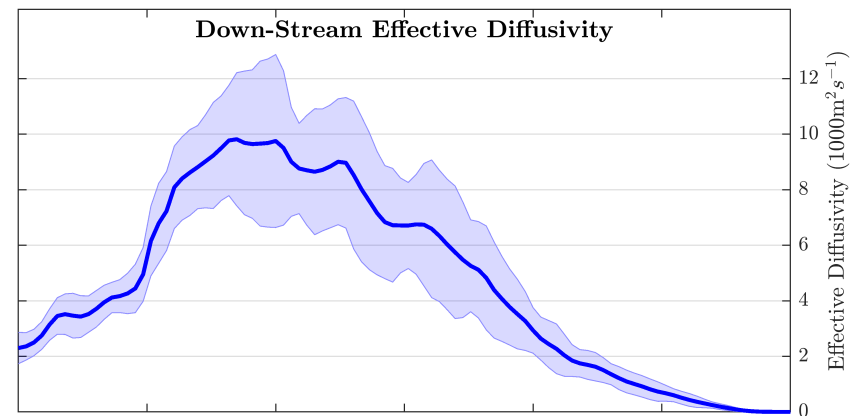
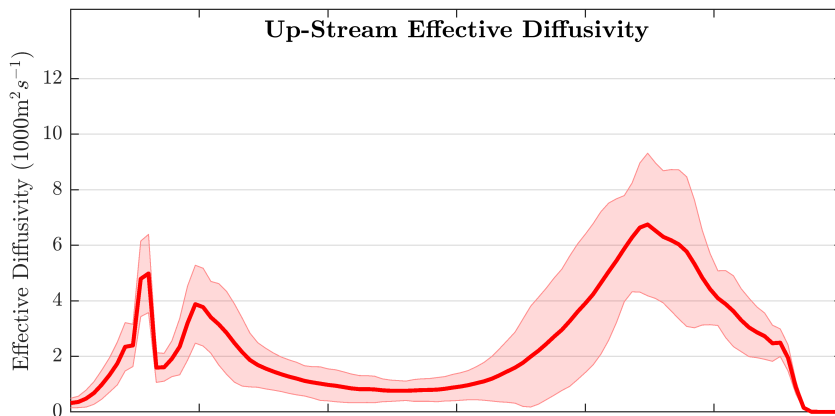
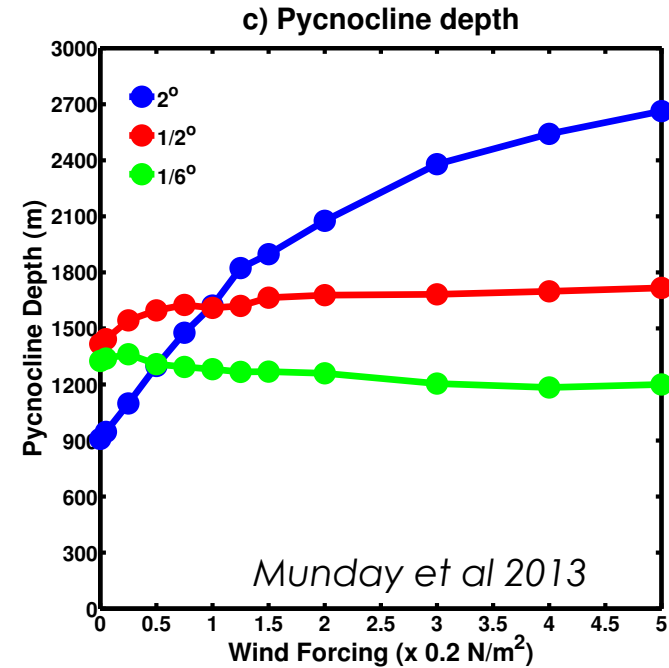
# Traditional Approach to Parametrization

- **One process:** baroclinic mesoscale eddies (*Gent-McWilliams, 1990, Gent et al 1995*)
  - **One effect** associated with the process: baroclinic instability with conversion of APE into KE & flattening of isopycnal
  - **One functional form:** depends on the gradient of buoyancy
  - **One parameter** = positive definite & spatially-dependent
- ➔ *Large improvements to large-scale circulation, especially Southern Ocean & density distribution, stratification, eliminated spurious convection*



# Caveats: Missing processes & Unknown Parameters

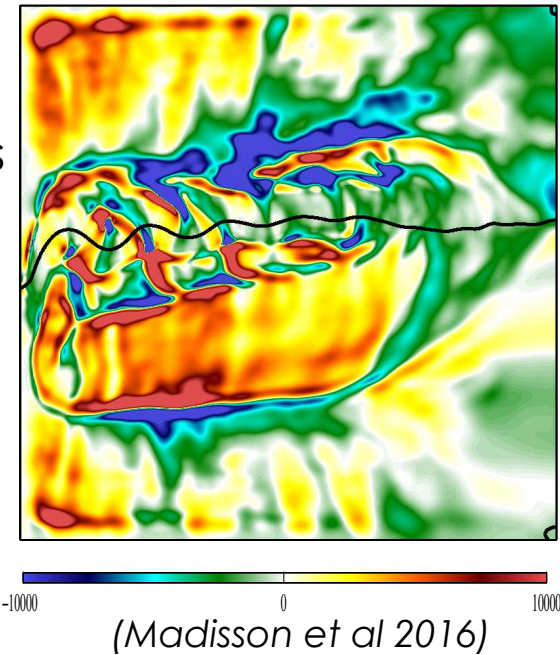
- Imperfect parametrizations b/c:
  - Missing effects (e.g., sensitivity/response to forcing)
  - Unknown parameters which are often not observable (or physical)
  - Missing Interaction with other processes & different scales



Bolton, Abernathy & Zanna, In Prep

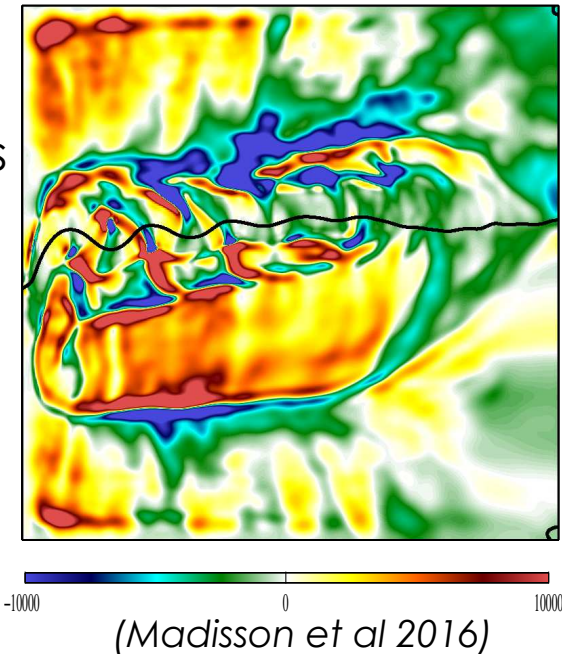
# Improving on Current sub-grid parametrizations

- **Constraining parameters:** using observations or model output with data-assimilations; caveat: assumes a perfect parametrization & enough information
- **Additional parametrizations:** to include more effects or to fix problems introduced by other sub-grid scales - e.g., energy backscatter (*Jansen et al 2015*)

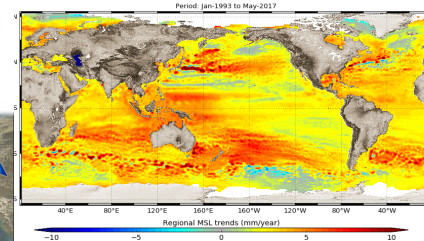
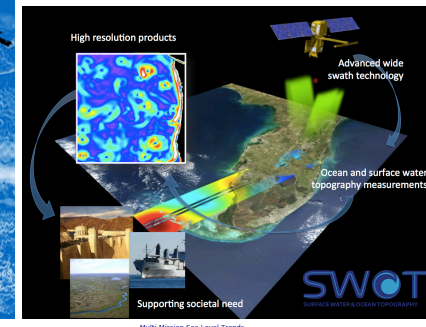


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- **Focus on the total sub-grid tendencies:** Extract/diagnose the entire sub-grid tendency & its statistics from models or observations → deduce something about the missing physics & its effect



# Using high-resolution simulations & Coarse-Graining

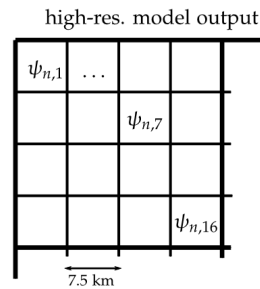
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- Coarse-graining + find relationship between terms - deterministic or stochastic - which relate to the resolved physics
- Key aspects missing: quasi-2D turbulence, jet sharpening, enhanced shear
- Diagnostics in an idealized baroclinic 3 layers potential vorticity (PV) model

# Using high-resolution simulations & Coarse-Graining

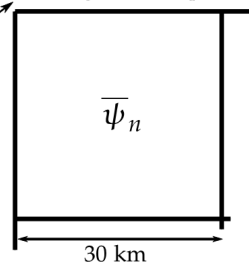
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**Eddy Resolving  
(7.5km)**

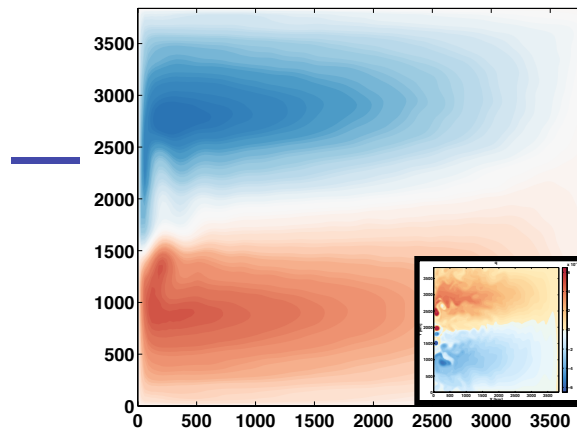
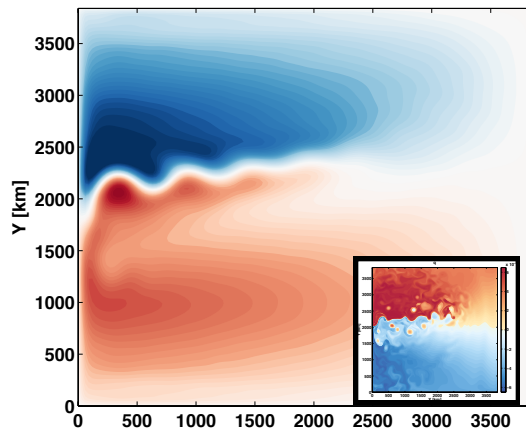


$$\bar{\psi}_n = \frac{1}{16} \sum_{i=1}^{16} \psi_{n,i}$$

coarse-grained output



**Eddy Permitting  
(30km)**



- Eddy Forcing:

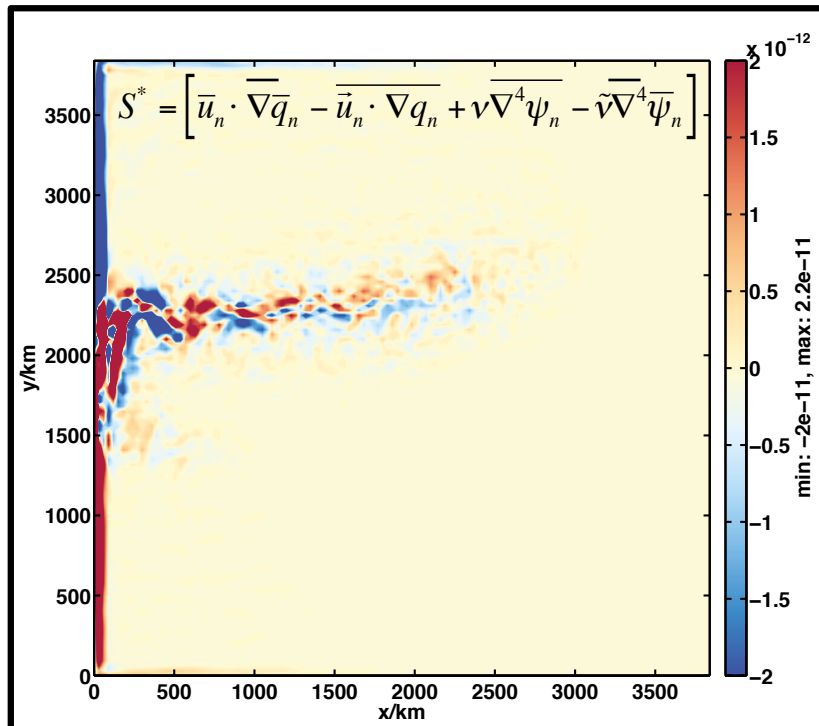
$$S^* = \bar{\nabla} \cdot (\bar{\mathbf{u}}\bar{q}) - \overline{\nabla \cdot (\mathbf{u}q)} + \nu \bar{\nabla}^4 \bar{\psi} - \tilde{\nu} \bar{\nabla}^4 \bar{\psi}.$$

LowRes — High-Res

# Using high-resolution simulations & Educated Guess

- Eddy Forcing = a visco-elastic stress x parameter depending on resolution
- depends on the local rate of shear (with some memory) & the deformation of the parcel
- Conserves PV & momentum
- Holds under a very wide range of Reynolds

Diagnosed Turbulence Forcing from  
coarse-grained models

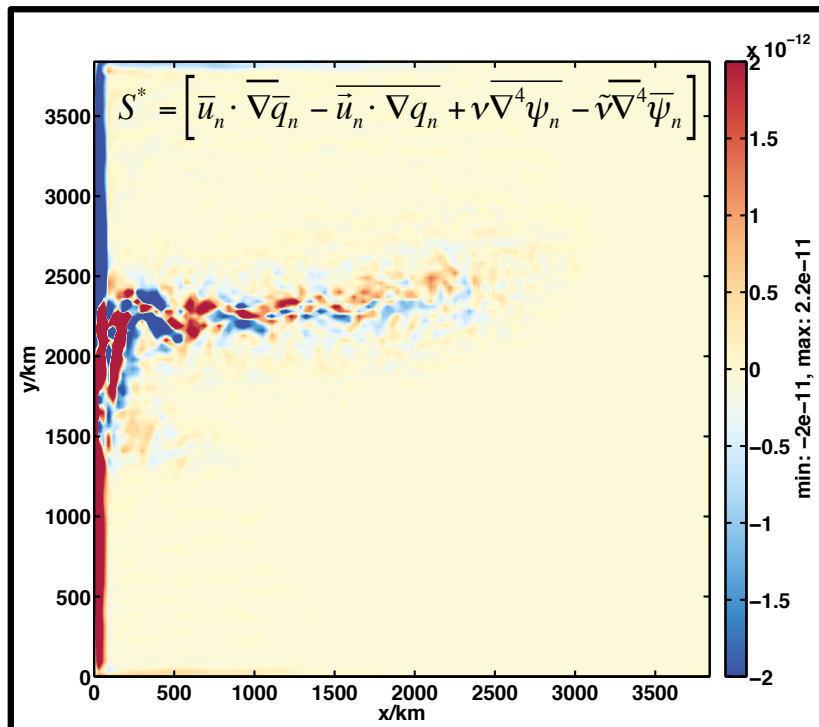


Mana & Zanna, 2014

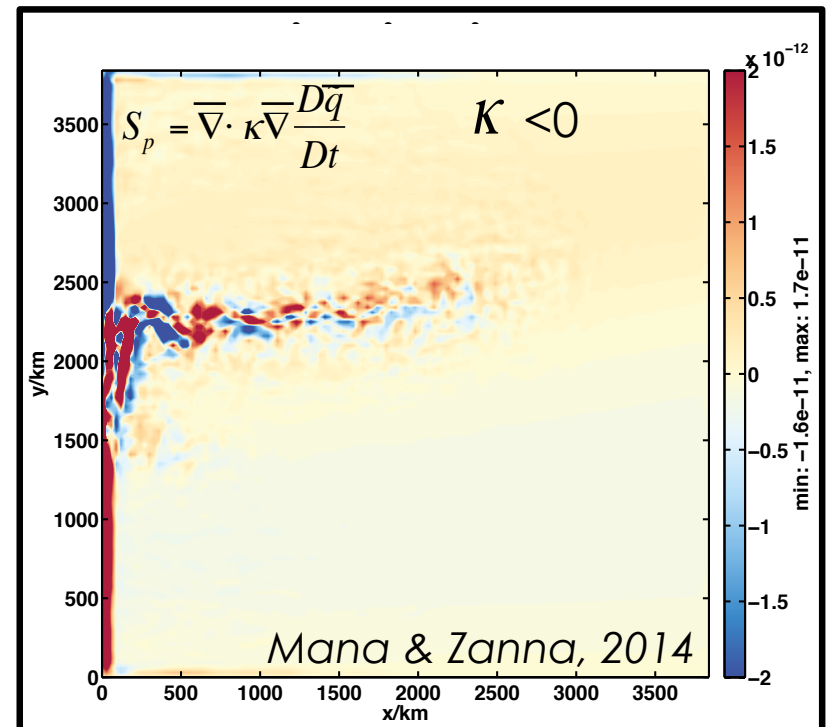
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Diagnosed Turbulence Forcing from coarse-grained models

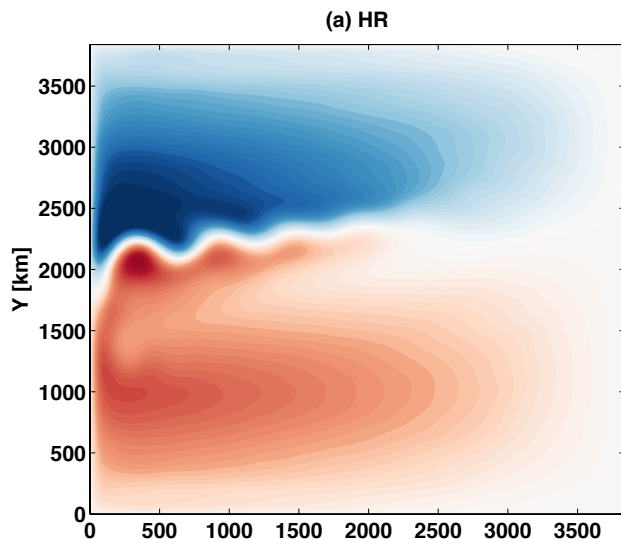


Non-Newtonian Parametrization based on coarse-grained resolved scales

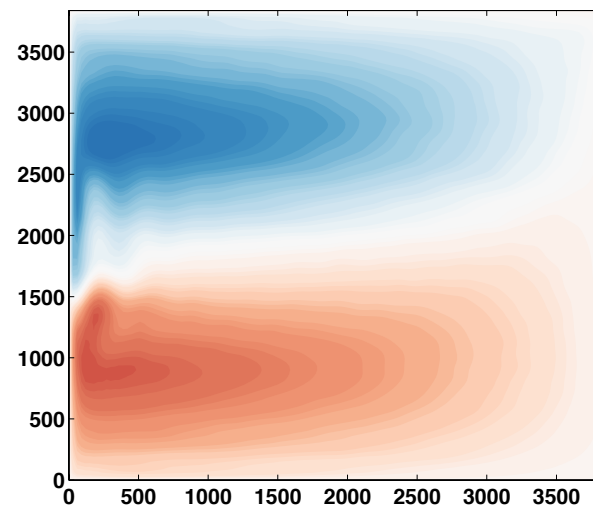


# Implementation into QG: Steady-State Streamfunction

**7.5 km  
(truth)**

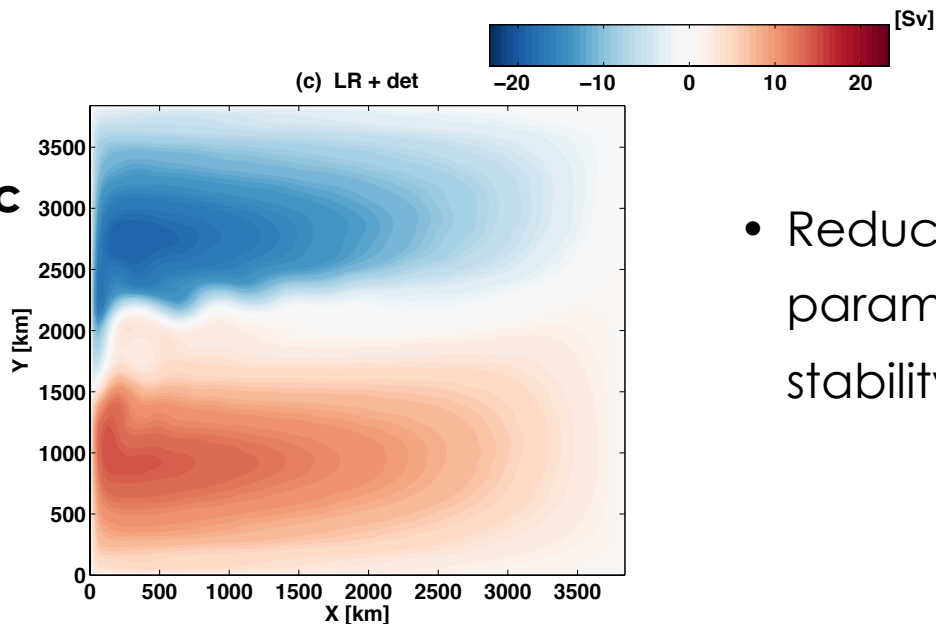


(b) LR



**30 km  
(no  
param)**

(c) LR + det

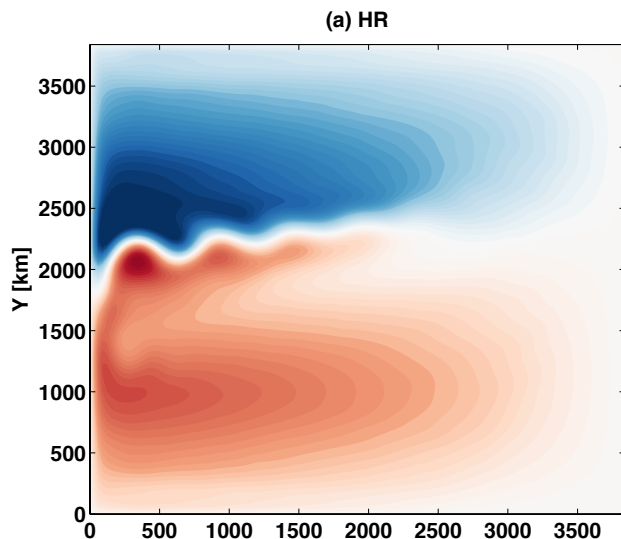


**30 km +  
deterministic**

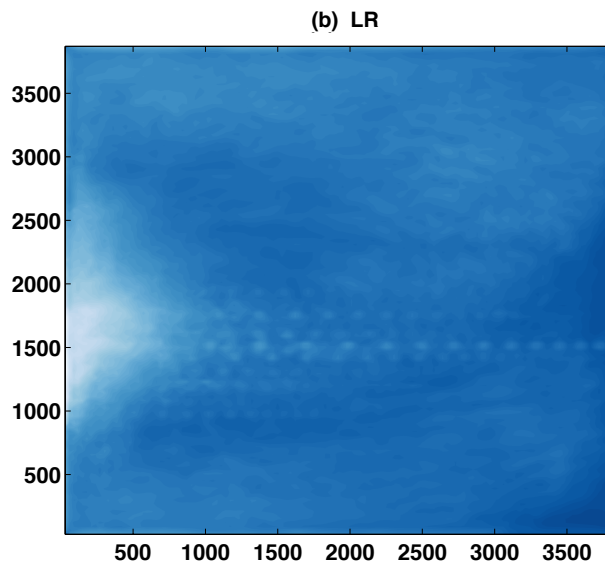
- Reduced amplitude of parameter for numerical stability

# Implementation into QG: Steady-State Streamfunction

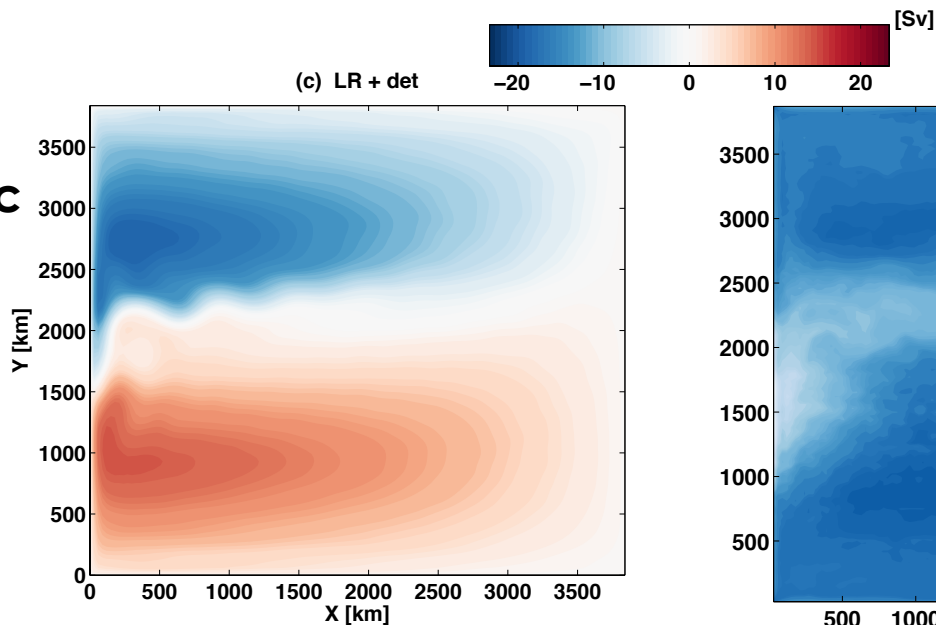
**7.5 km  
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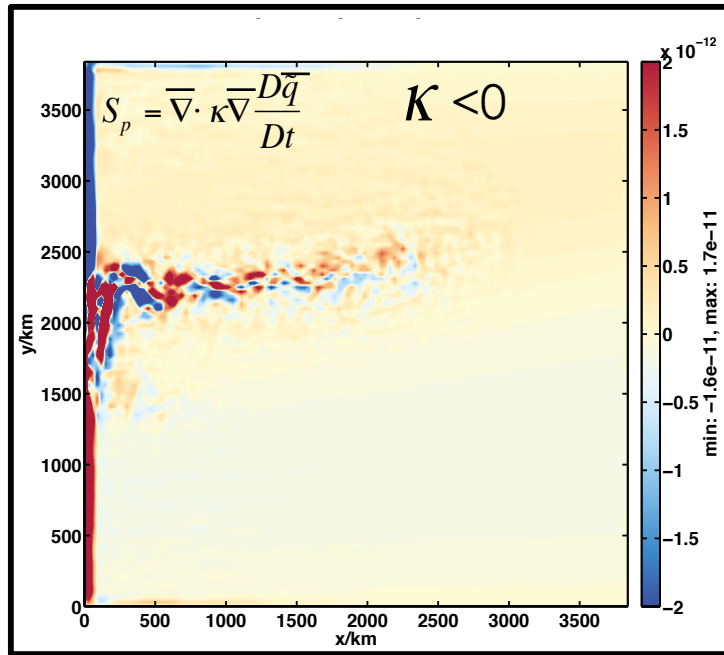
**30 km  
(no  
param)**



**30 km +  
deterministic**

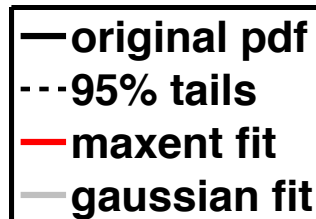


# Using high-resolution simulations & Educated Guess



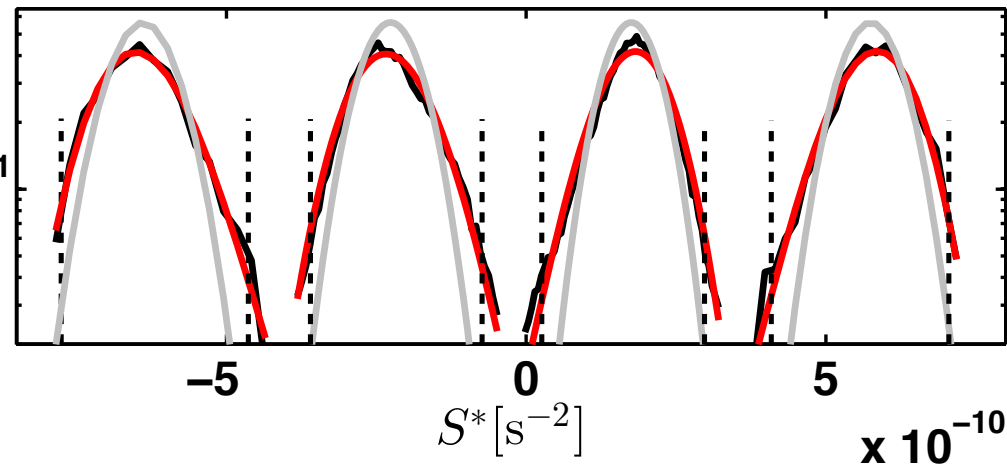
- Turbulence forcing is not fully deterministic
- Moments depends resolution, surface forcing, stratification

Conditional PDFs of eddy forcing on visco-elastic stress



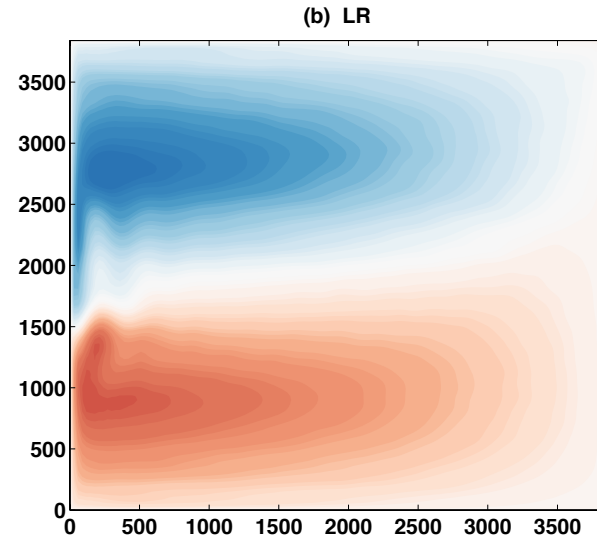
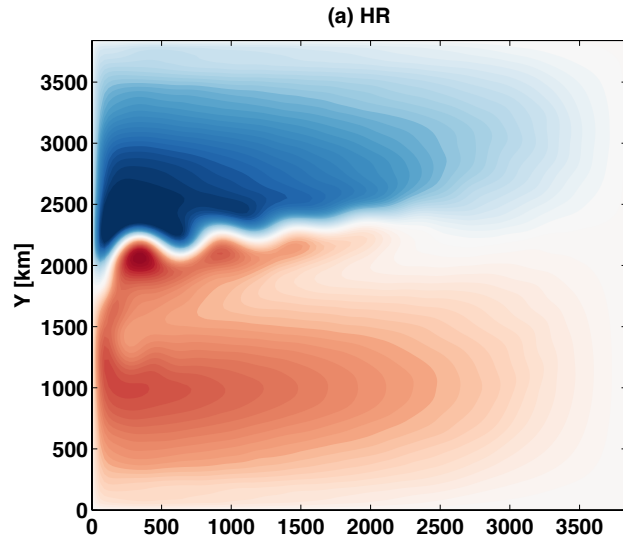
prob. density [freq/s<sup>-2</sup>]

MaxEnt fit with four moments

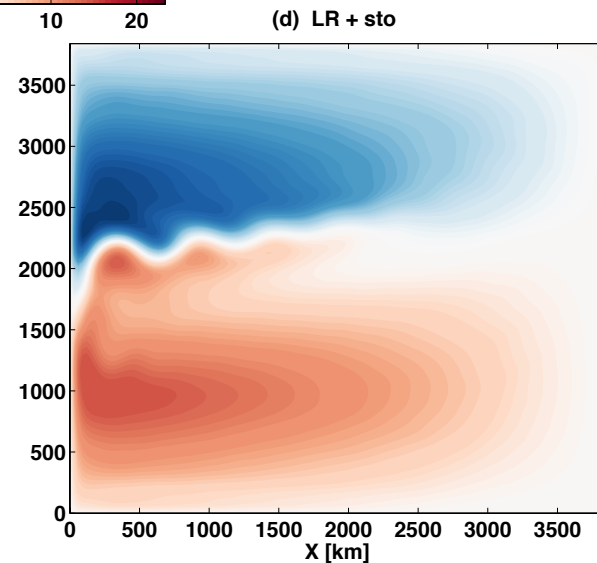
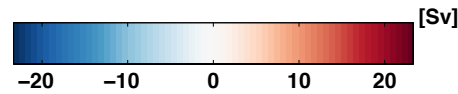
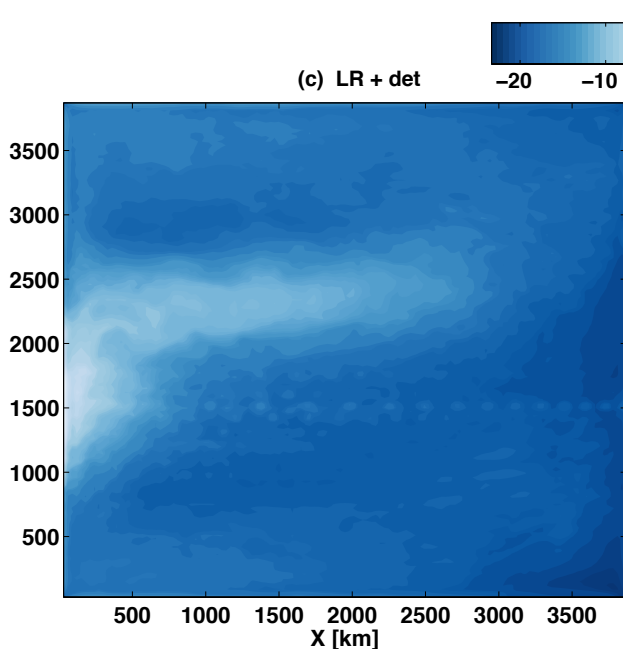


# Implementation into QG: Steady-State Streamfunction

**7.5 km  
(truth)**



**30 km  
(no  
param)**



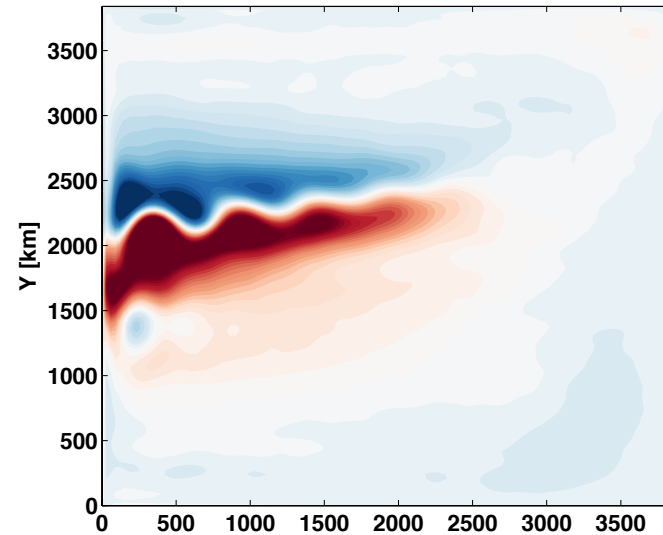
**30 km +  
stochastic**

# Using high-resolution models: Caveats & Ways Forward

---

- **Difficulties transplanting ideas** from idealised models into complex ocean/climate models
- ongoing implementation:
  - with Alistair Adcroft (Princeton, GFDL) in MOM6
  - with Scott Bachman (NCAR) in MITgcm
- Assumes the high-res model is the “truth”

MOM6: HR- parametrised run



# Using high-resolution models: Caveats & Ways Forward

---

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ongoing implementation:

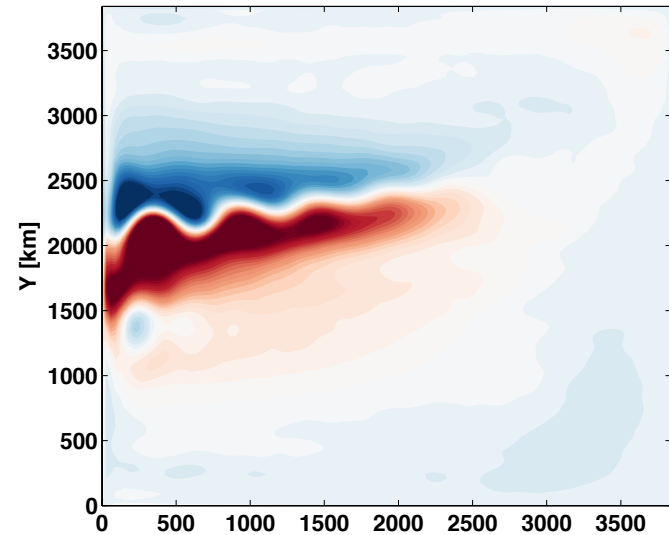
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- **Need for some good (human) guesses** based on physics for the relationships between sub-grid closure & large scale - but could be done more efficiently via an optimization?

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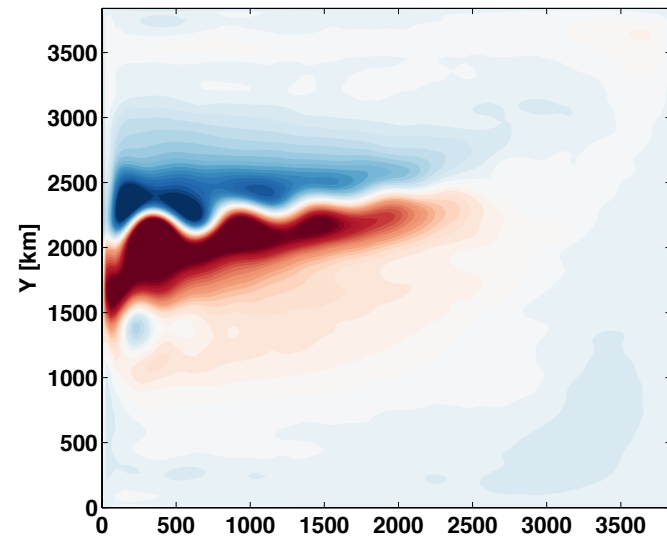
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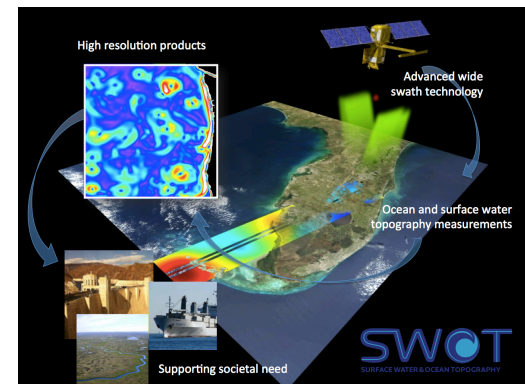
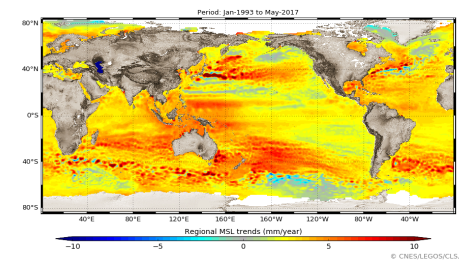
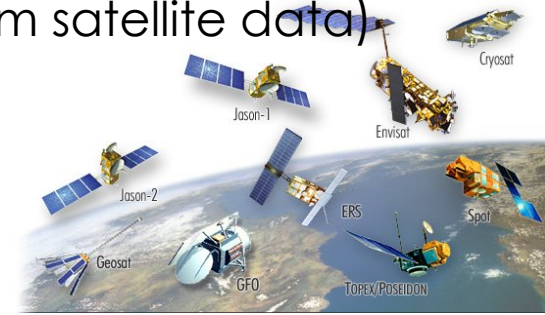
MOM6: HR- parametrised run



- **Need for some good (human) guesses** based on physics for the relationships between sub-grid closure & large scale - but could be done more efficiently via an optimization?
- **Disconnect between diagnostic & implementation:** numerical instabilities, unbounded solutions - impose constraints for numerical stability (e.g., 2nd-order deterministic or stochastic closures)?

# Using observations - Learning the Unseen but felt

- Similar idea using imperfect/smoothed//filtered observations
- Tool: same model data, Gaussian filtered
- Goal: Prediction of sub-grid momentum forcing when Neural Network is trained on smooth streamfunction (equivalent to SSH from satellite data)

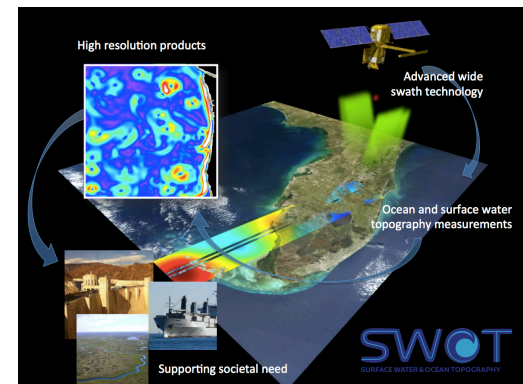
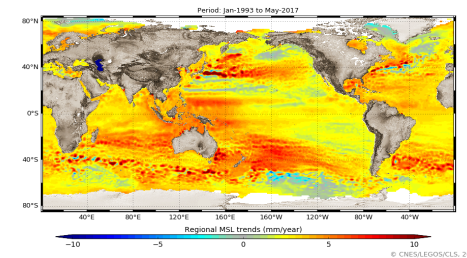
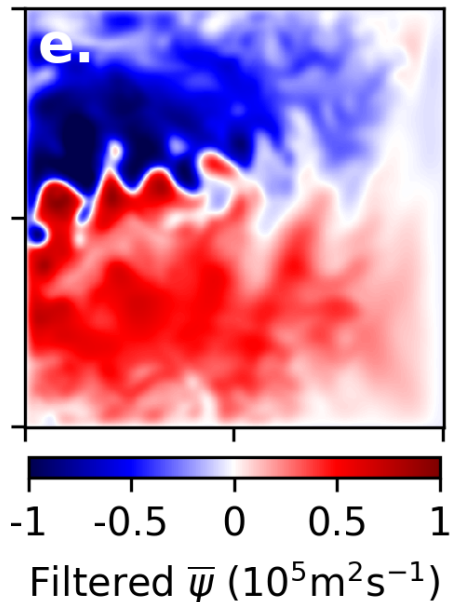


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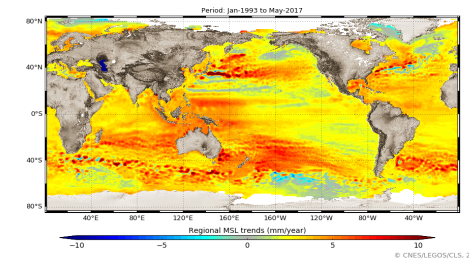


Snapshot  $\bar{\psi}$



# Using observations - Learning the Unseen but felt

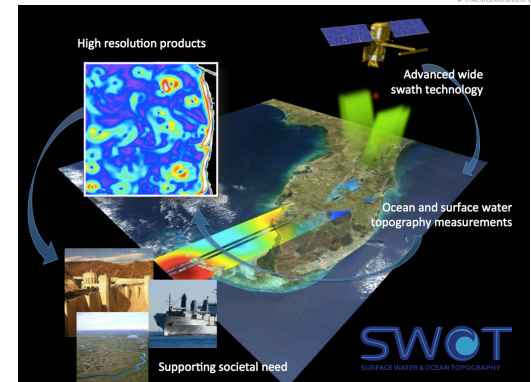
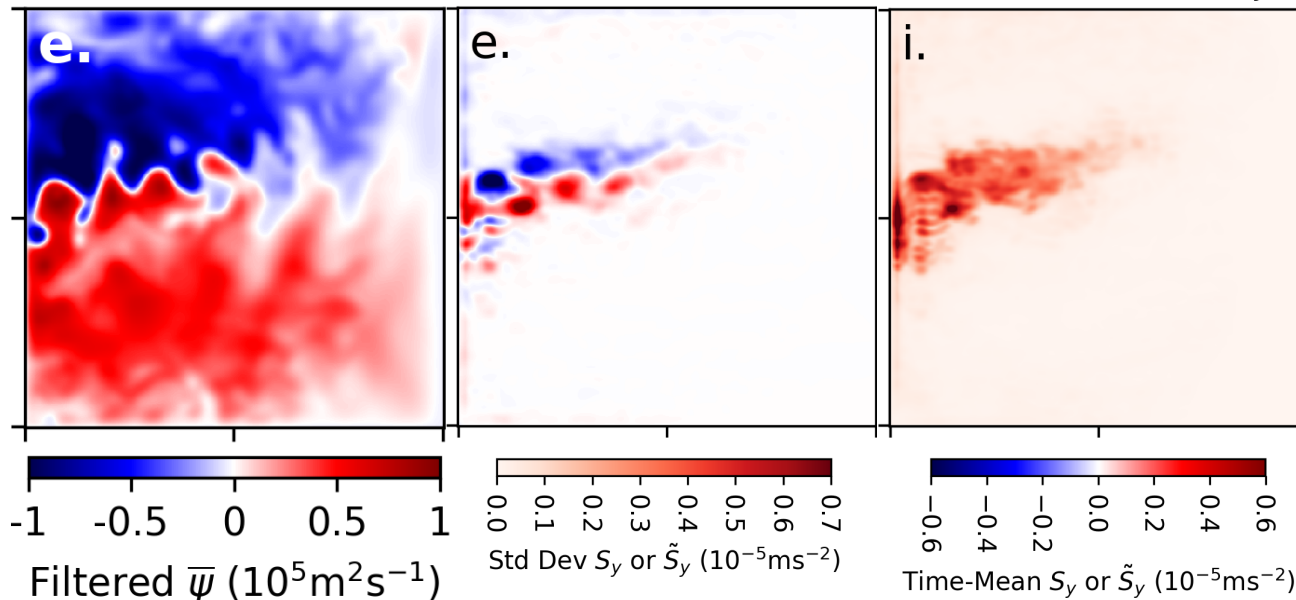
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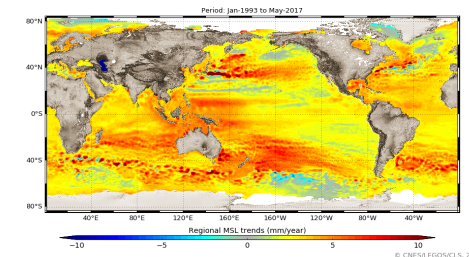
Mean of True  $S_y$

Std Dev of True  $S_y$



# Using observations - Learning the Unseen but felt

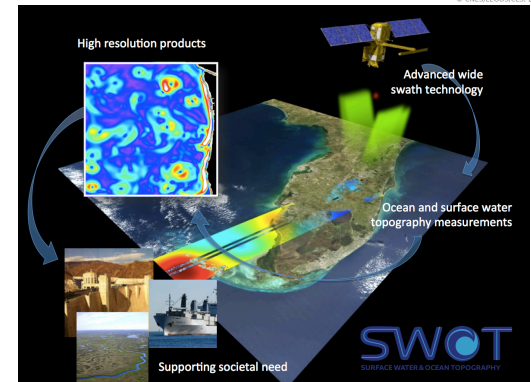
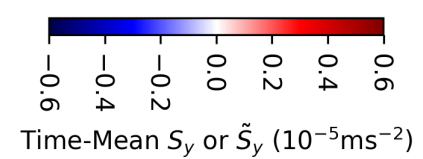
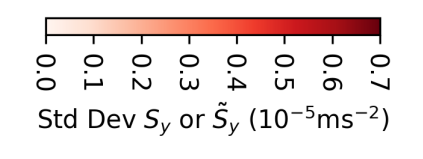
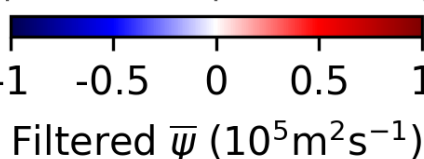
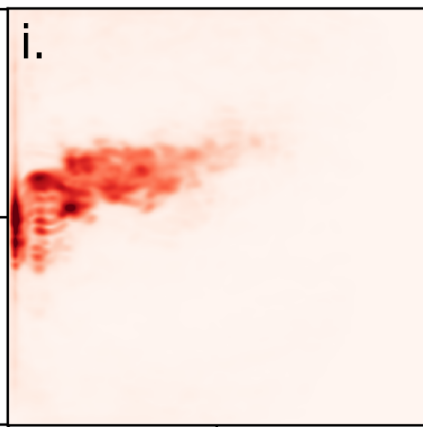
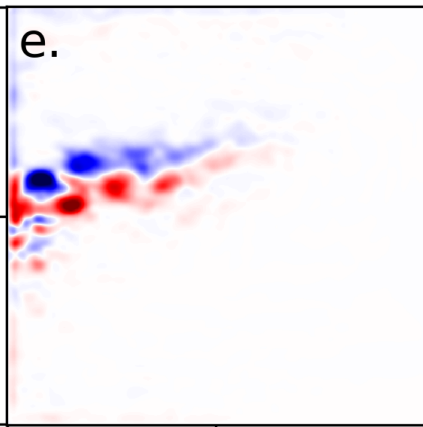
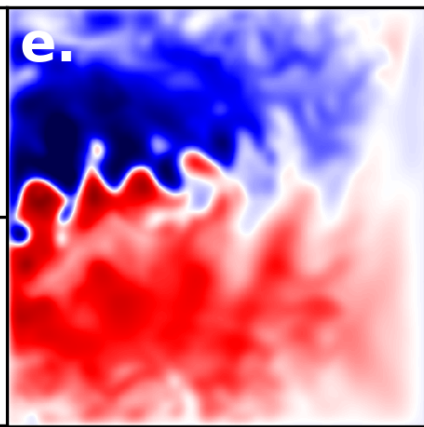
- Similar idea using imperfect/smoothed//filtered observations
- Tool: same model data, Gaussian filtered
- Goal: Prediction of sub-grid momentum forcing when Neural Network is trained on smooth streamfunction (equivalent to SSH from satellite data)
- Next step: use the NN as a way to design & implement a parametrization



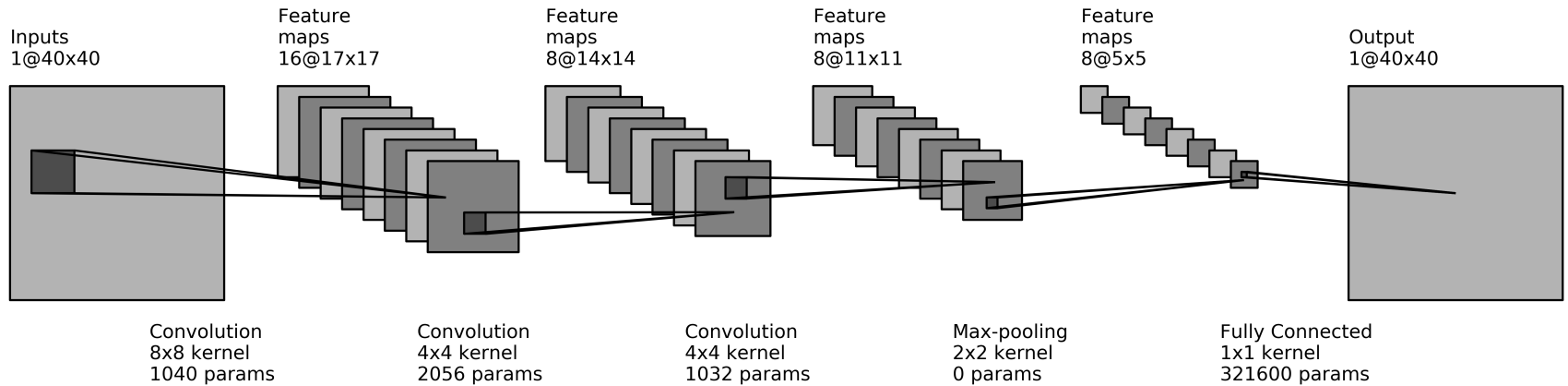
Snapshot  $\bar{\psi}$

Mean of True  $S_y$

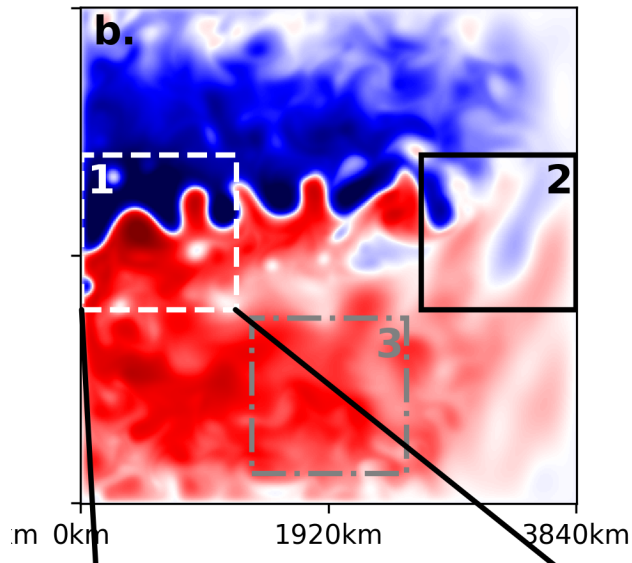
Std Dev of True  $S_y$



# Neural Networks Architecture

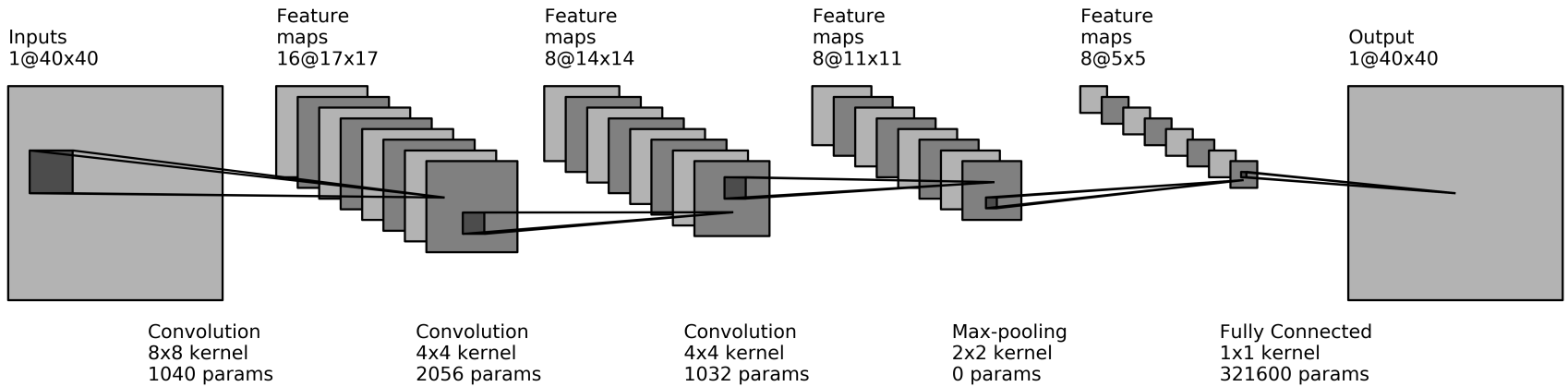


Snapshot of High-Res (7.5km)  $\psi$

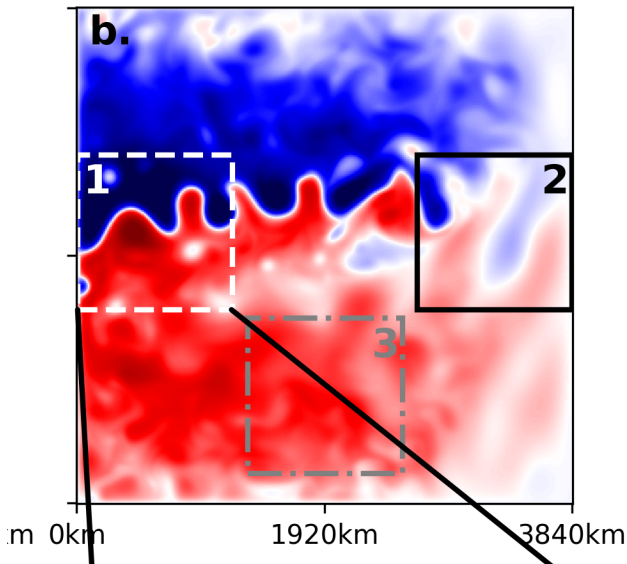


- 9 yrs for training
- 1 yrs for prediction

# Neural Networks Architecture

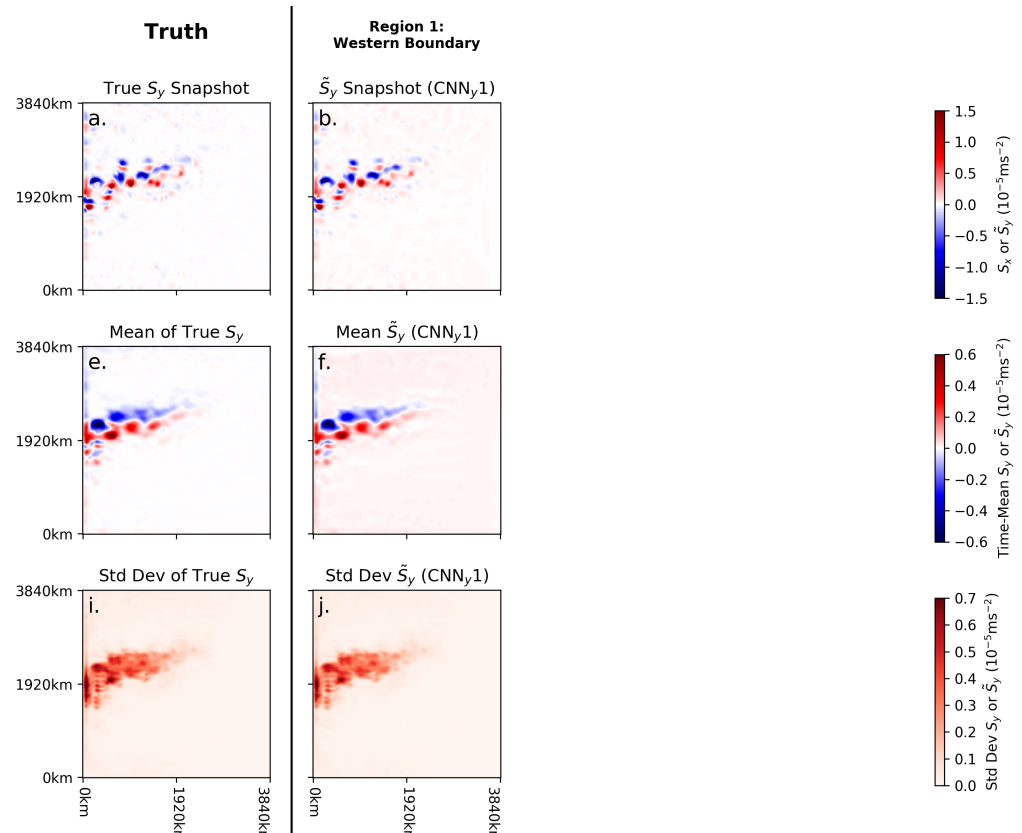


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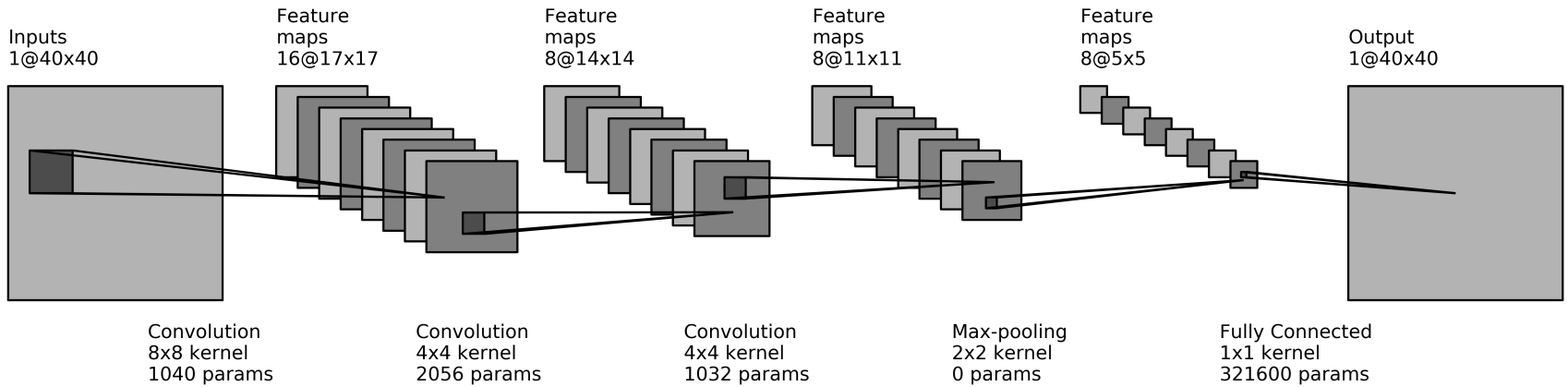


- Performance based on the region used for training

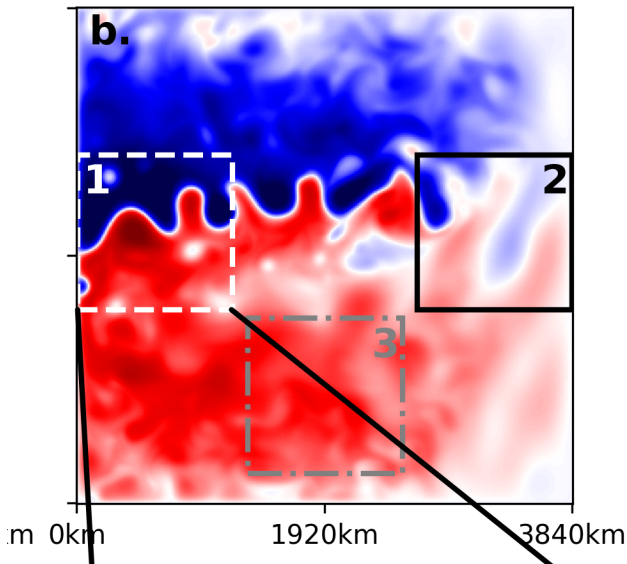
*Bolton & Zanna, In Prep*



# Neural Networks Architecture

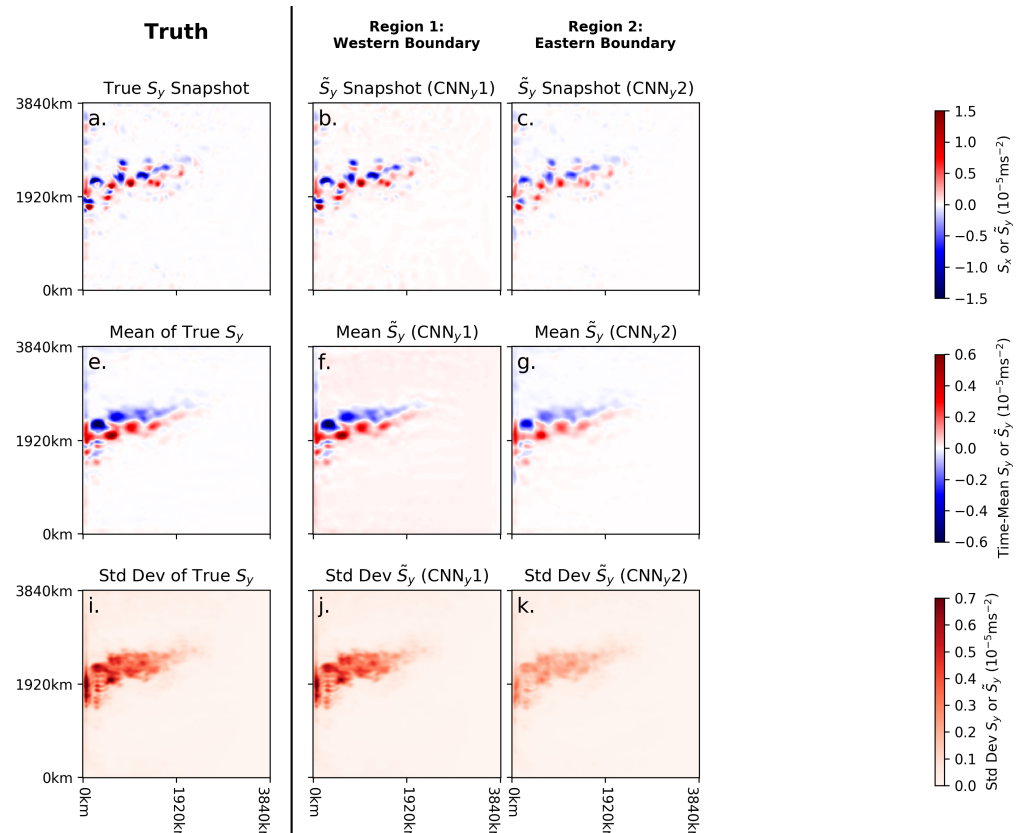


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Performance based on the region used for training

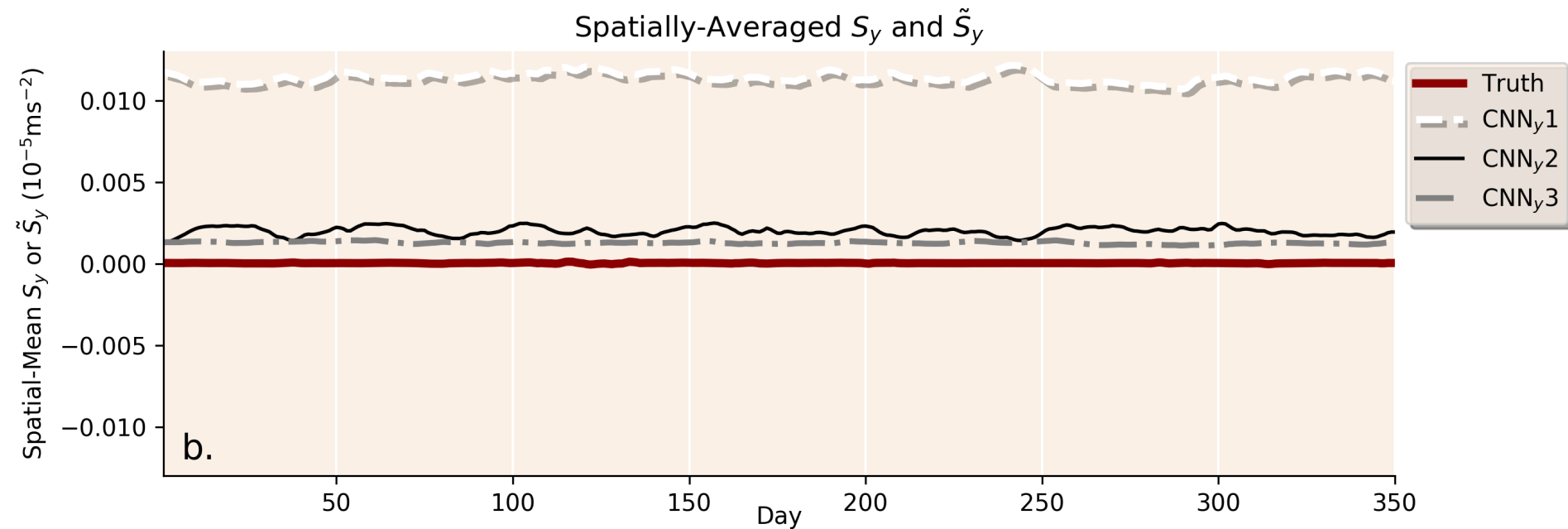
*Bolton & Zanna, In Prep*





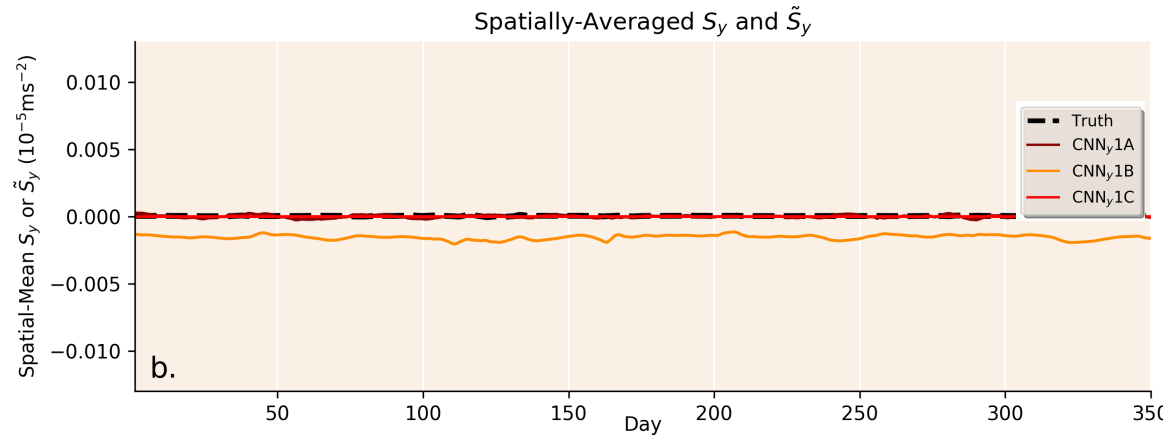
# Using observations - Caveats

- Quality/relevance of the training + length
- No physics - except for the physics implicit into the input function
- Numerical instabilities generated when implemented into a prognostic model (in progress)



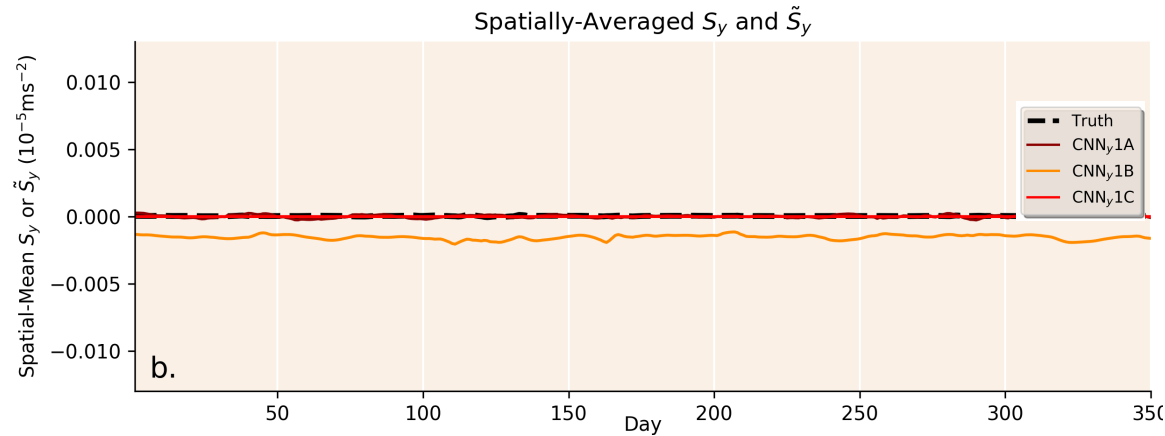
# Using Observations - Next Steps

- Minimisation: better targets or constraints (based on the application)

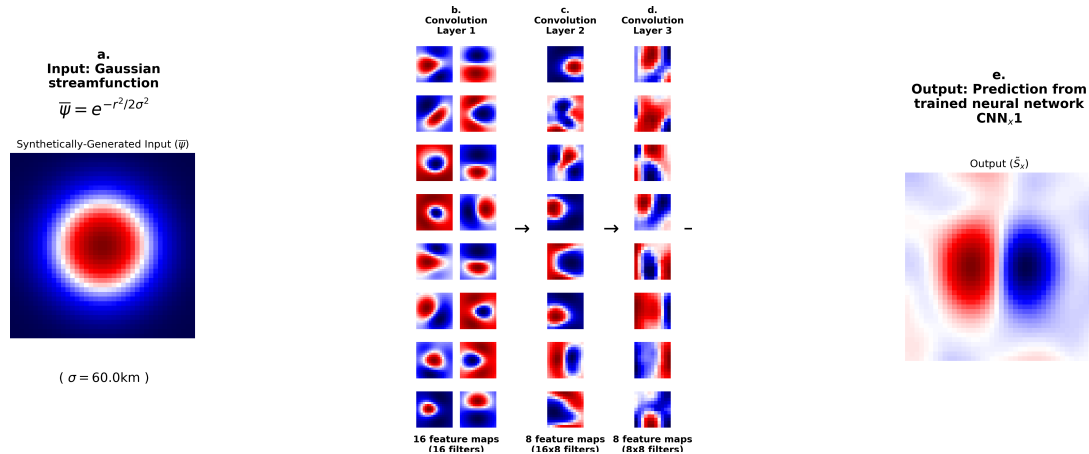
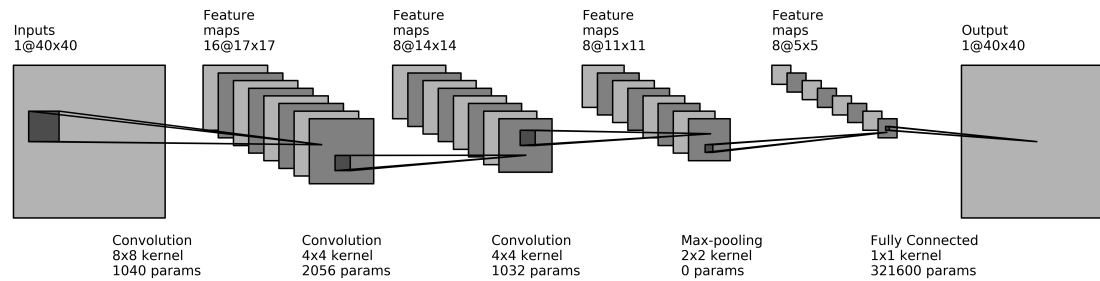


# Using Observations - Next Steps

- Minimisation: better targets or constraints (based on the application)



- Learning from the Networks' Architecture

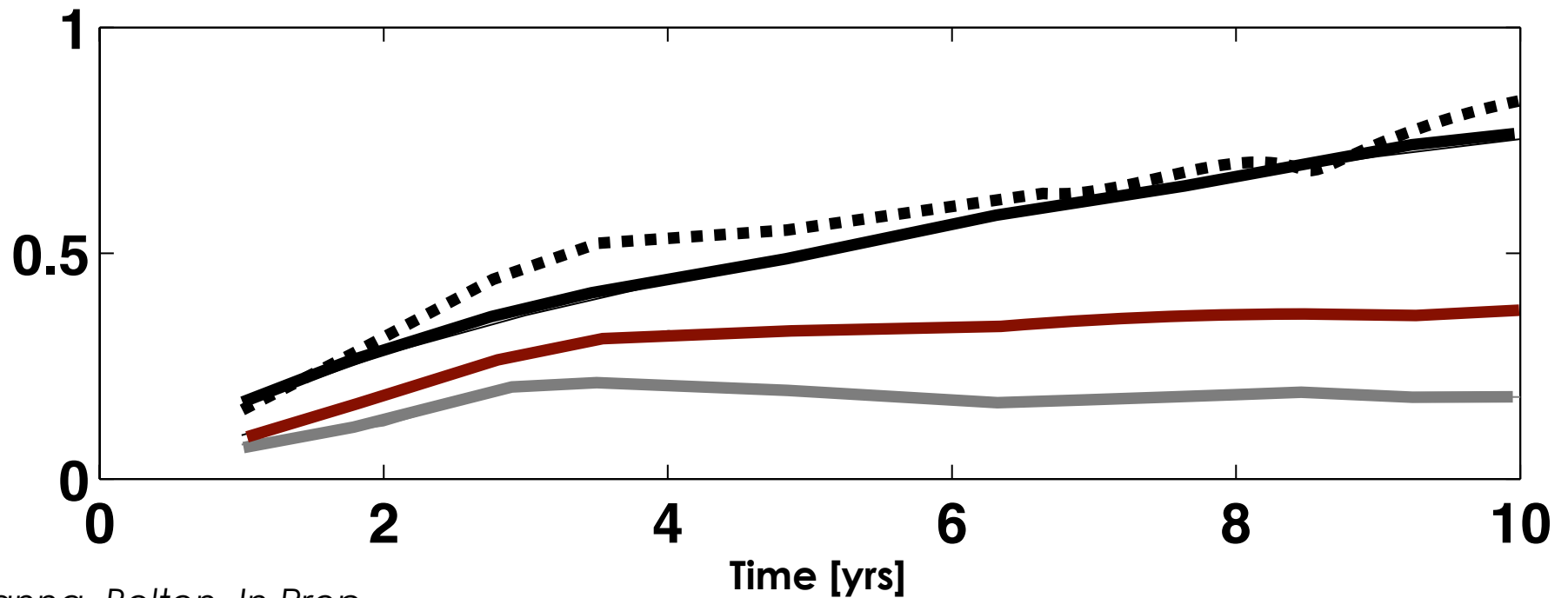


# Error in Predictions in QG

---

- Low-res without parametrization (black)
- Low-res + NN (dashed)
- Low-res + non-Newtonian (red)
- Low-res + NN + non-Newtonian parametrization (grey)

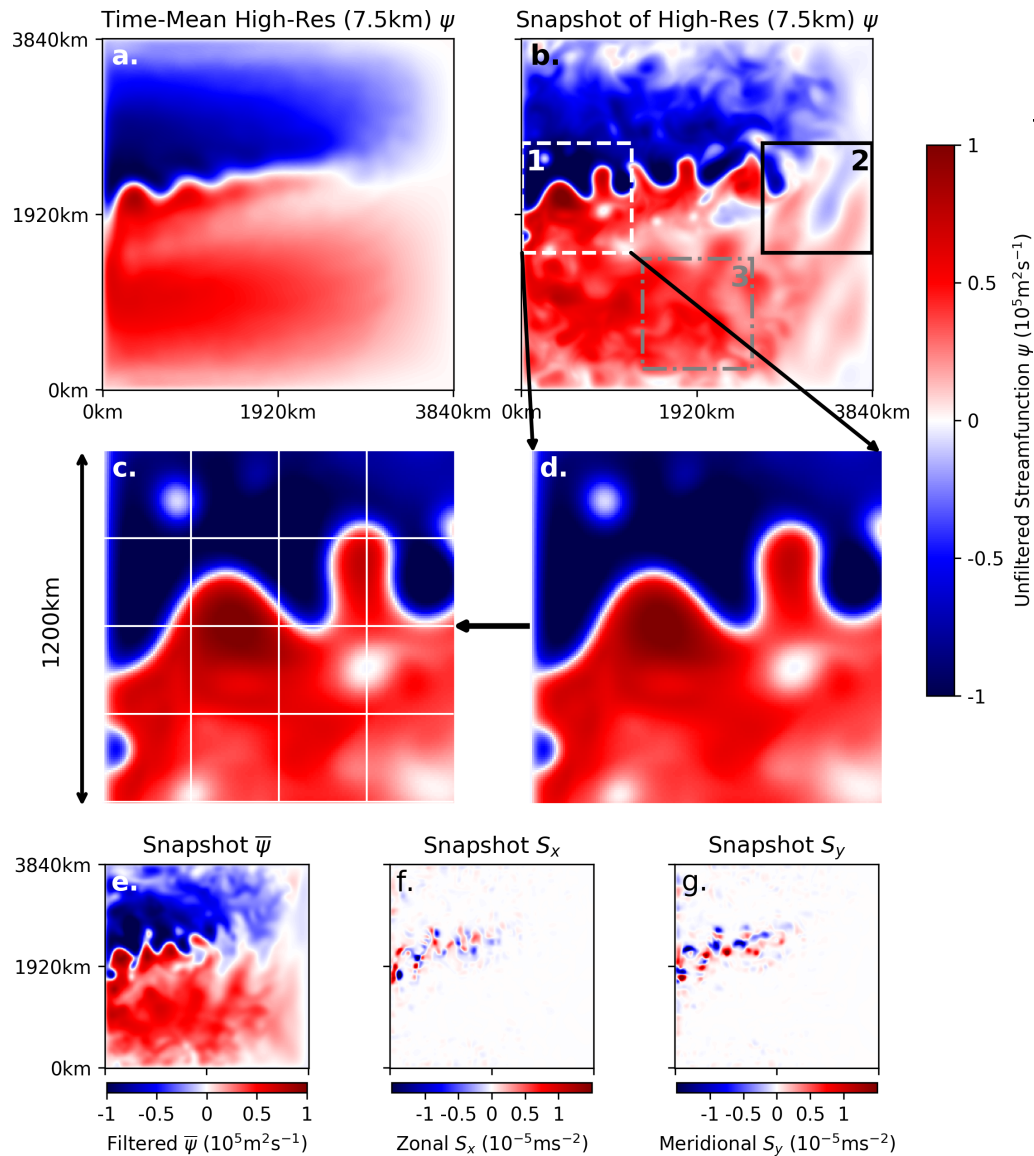
Root Mean Square Error as a function of time



# Summary

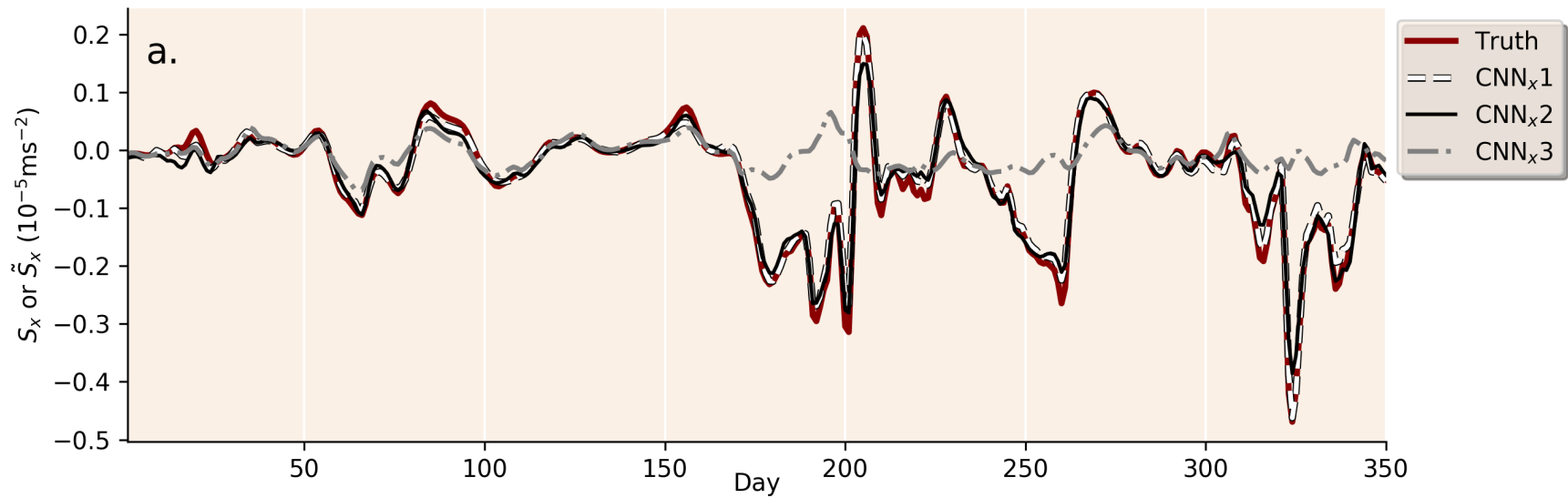
---

- Complementary approaches to developing traditional parametrizations
- Need for good model + observational data
- Constant need for a combination of physics + human + machines
- Unconventional approaches can still teach us something about the physics of processes & their representation in models

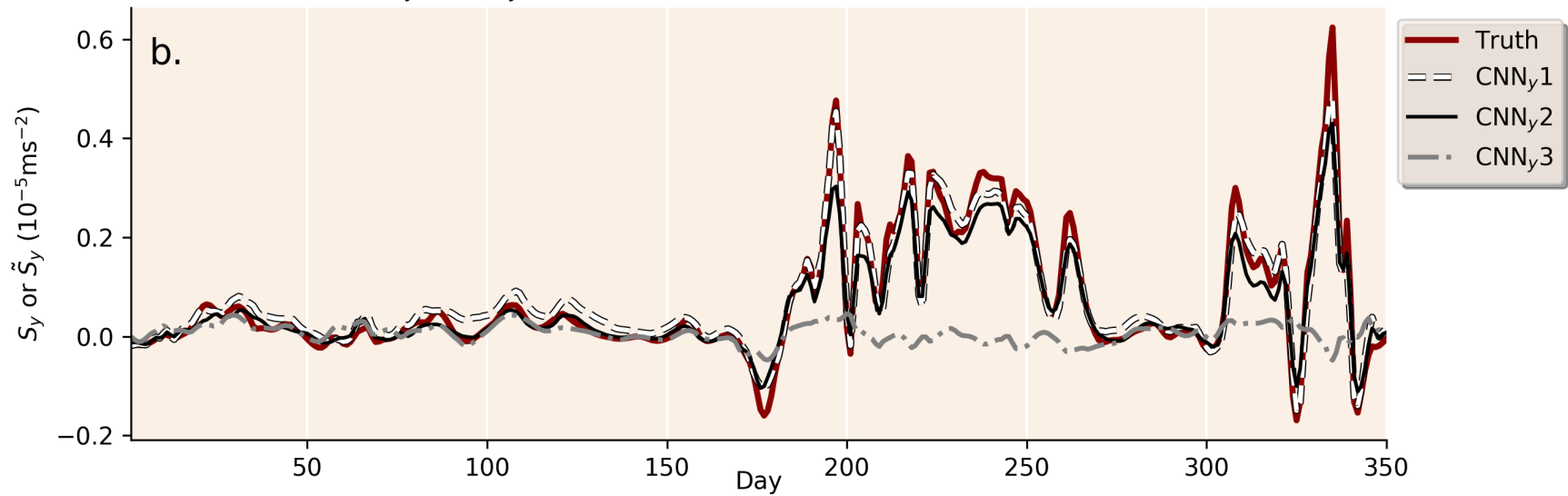


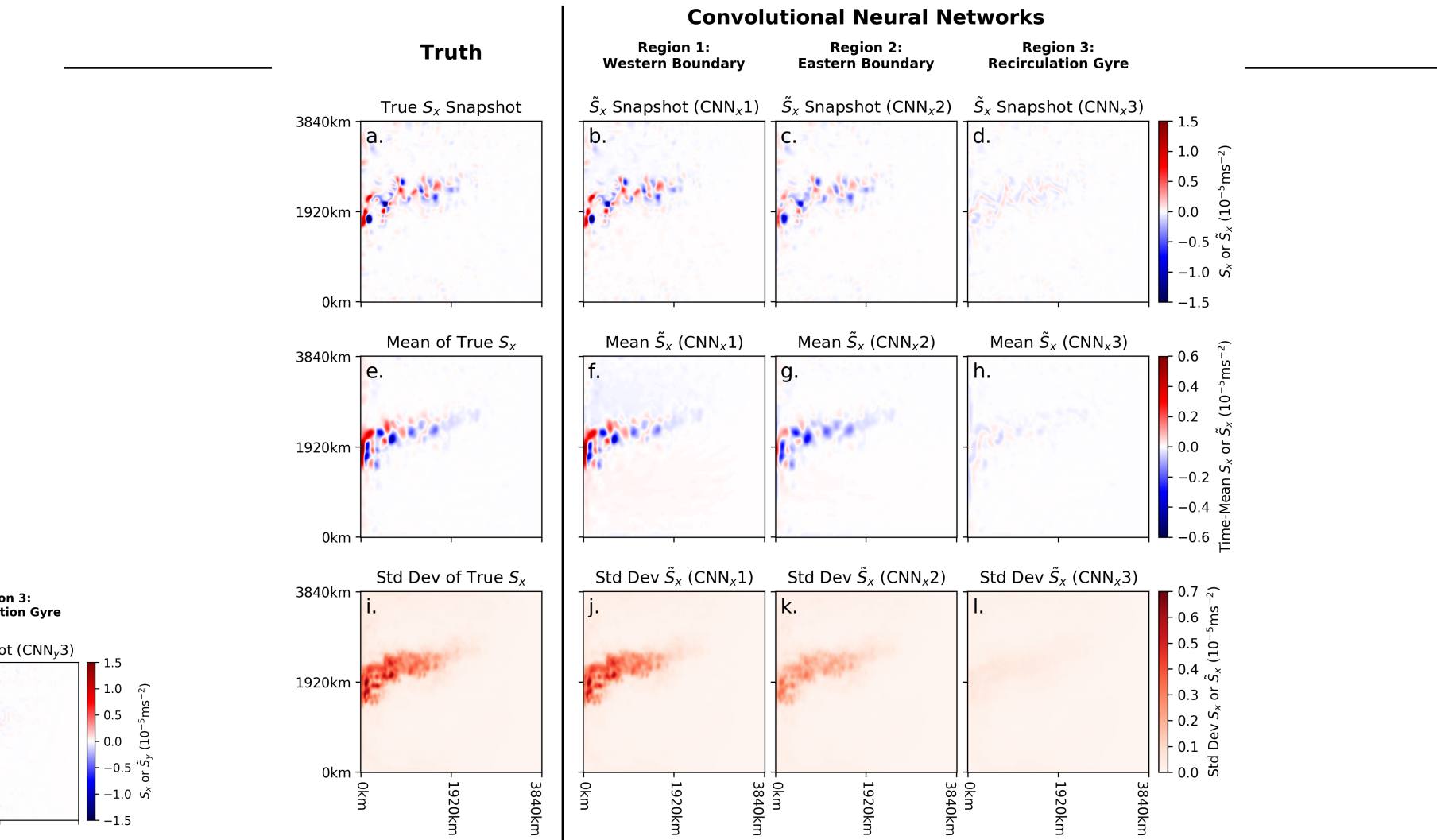
**Figure 1.** The time-mean and a snapshot of the upper-layer (un-filtered) streamfunction  $\psi$  are shown in (a) and (b) respectively. Training region 1 (whited-dashed), training region 2 (black-solid), and training region 3 (grey-dot-dashed), are illustrated in (b); data extracted from each region is used to train separate neural networks. A close-up of training region 1 is shown in (d), with the sixteen  $40 \times 40$  sub-regions illustrated in (c). Each training region has length 1200km, with each  $40 \times 40$  sub-region of length 300 km.

$S_x$  and  $\tilde{S}_x$  Time-Series (at  $x = 1920\text{km}$ ,  $y = 2400\text{km}$ )

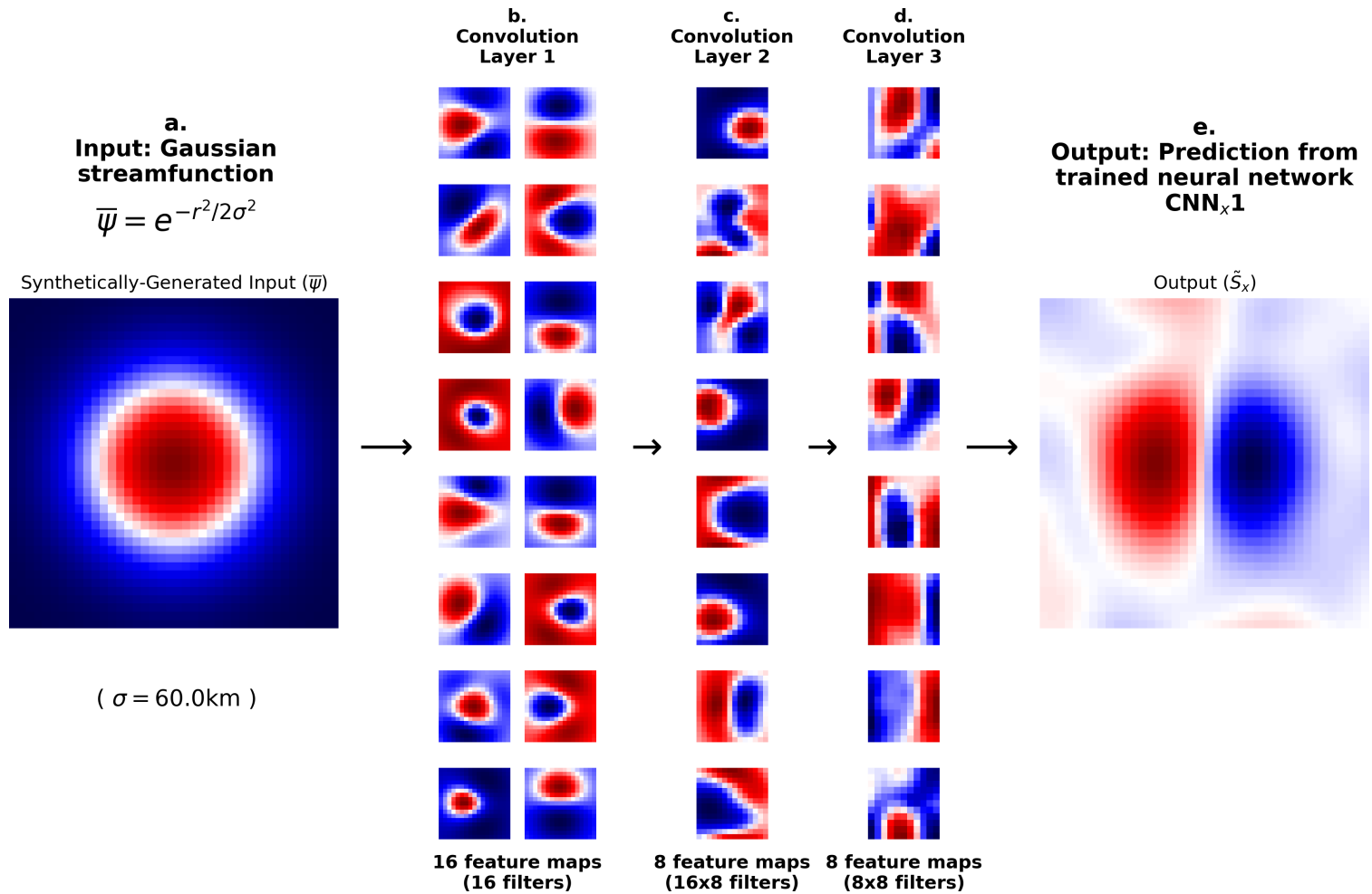


$S_y$  and  $\tilde{S}_y$  Time-Series (at  $x = 1920\text{km}$ ,  $y = 2400\text{km}$ )



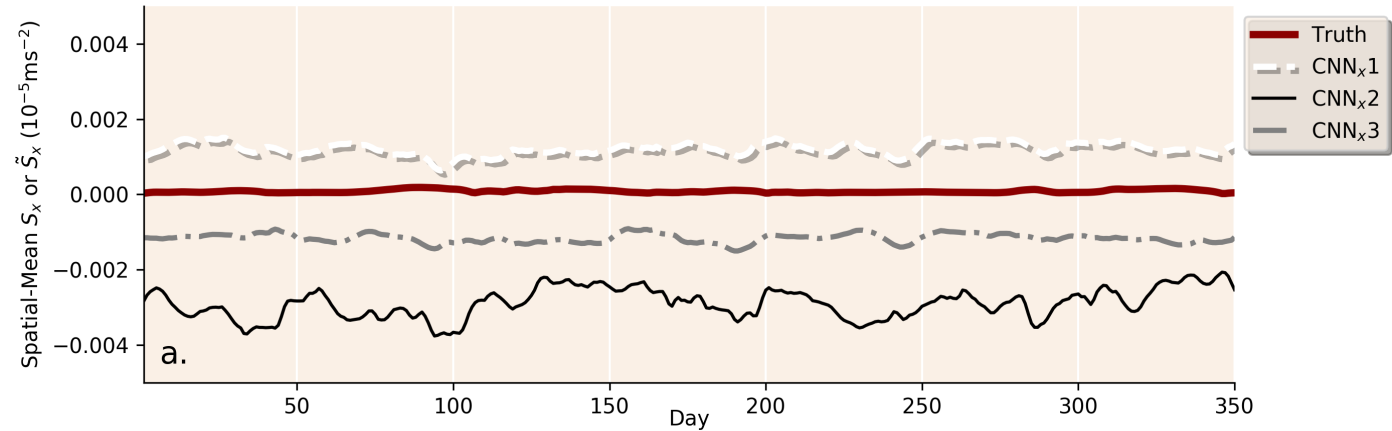


**Figure 4.** Comparisons of the true zonal component of the sub-filter momentum forcing  $S_x$ , with the neural networks trained using data from three different regions. The top, middle, and bottom rows compare a snapshot, the time-mean, and the standard deviation, respectively. The first column contains the diagnostics using the true zonal sub-filter momentum forcing  $S_x$ , while columns two, three, and four use predictions  $\tilde{S}_x$  from the neural networks  $\text{CNN}_{x1}$ ,  $\text{CNN}_{x2}$ , and  $\text{CNN}_{x3}$ , respectively. All diagnostics were calculated using the validation data.

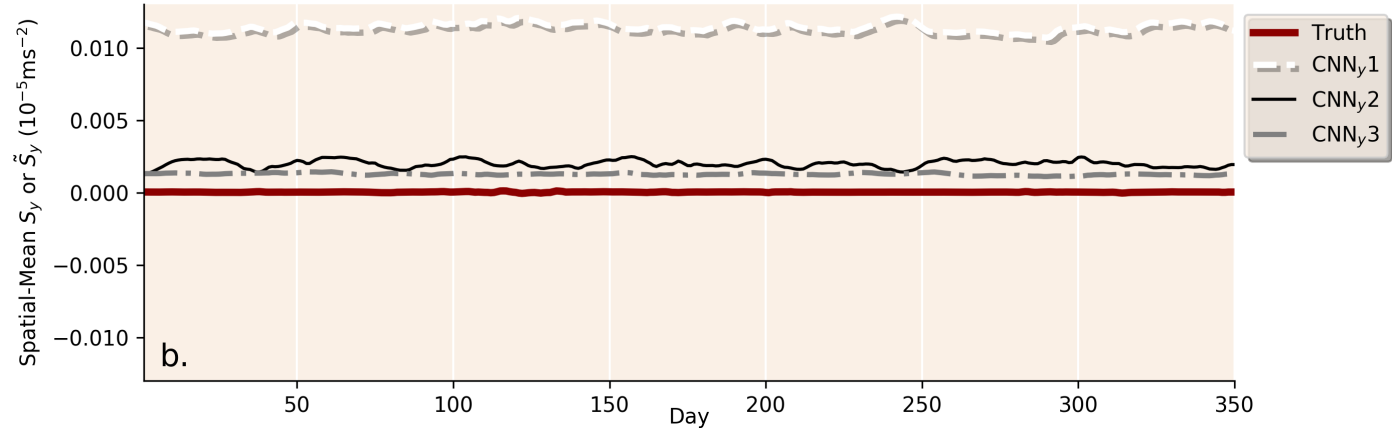


**Figure 10.** The feature maps at each stage of the neural network  $\text{CNN}_x1$ , for a synthetically-generated input streamfunction. Panel (a) shows the synthetically-generated input, where a radially-symmetric two-dimensional Gaussian function is used; the standard deviation is set to 60 km, in order to approximately match the length-scale of an eddy. Panels (b), (c), and (d) show the resulting feature maps of convolution layers 1, 2, and 3 respectively; there is a feature map

Spatially-Averaged  $S_x$  and  $\tilde{S}_x$

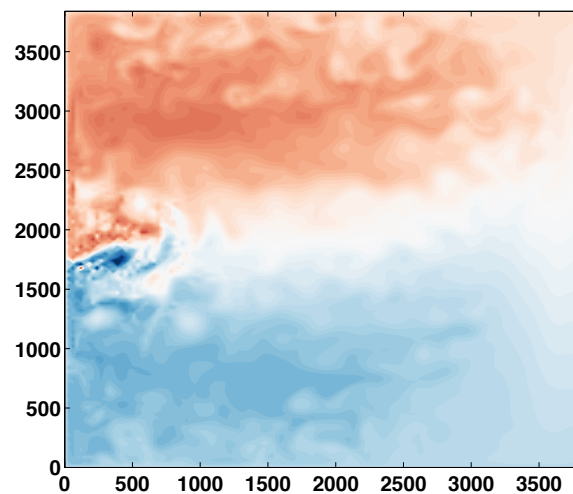
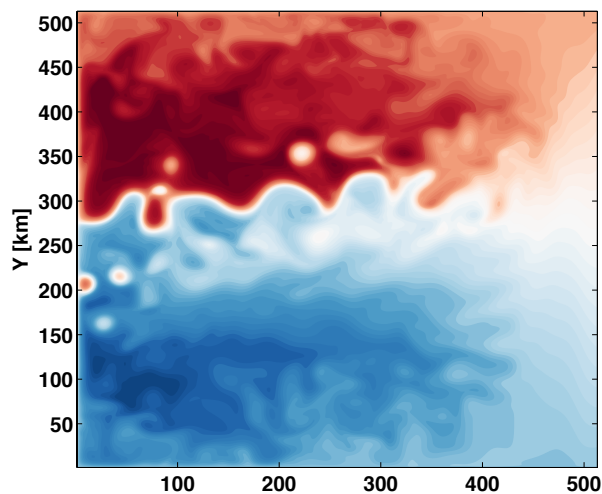
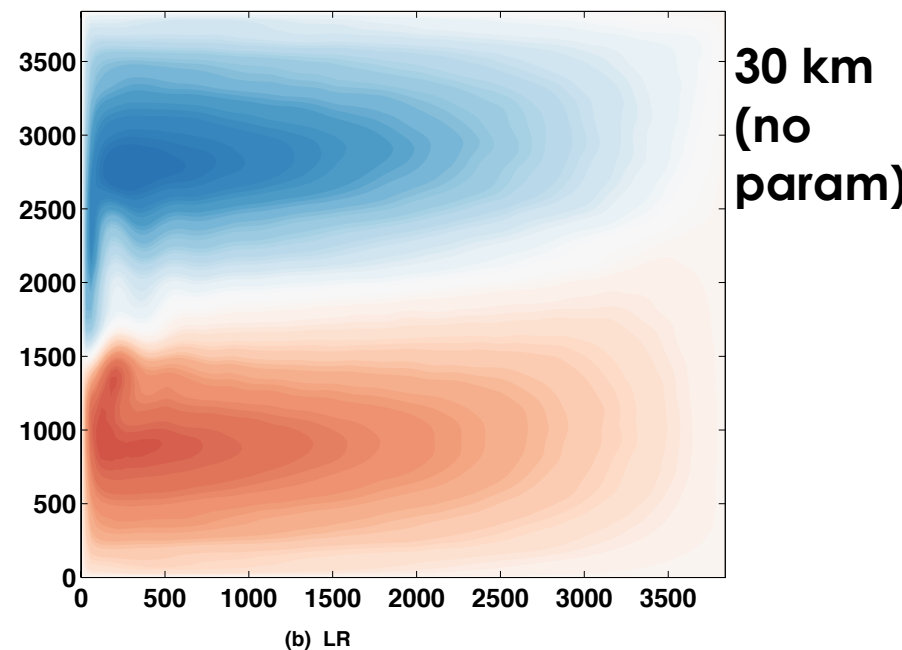
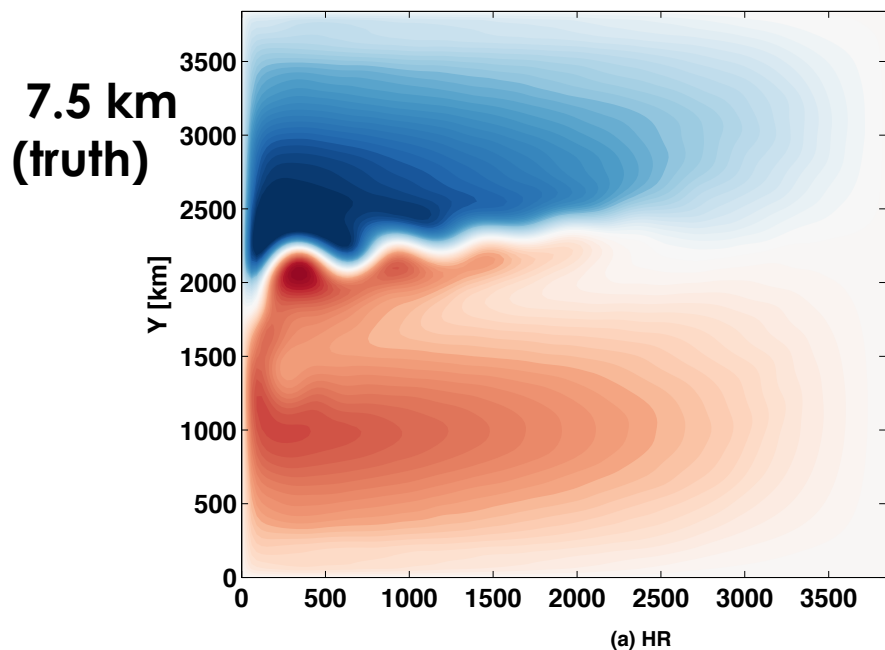


Spatially-Averaged  $S_y$  and  $\tilde{S}_y$



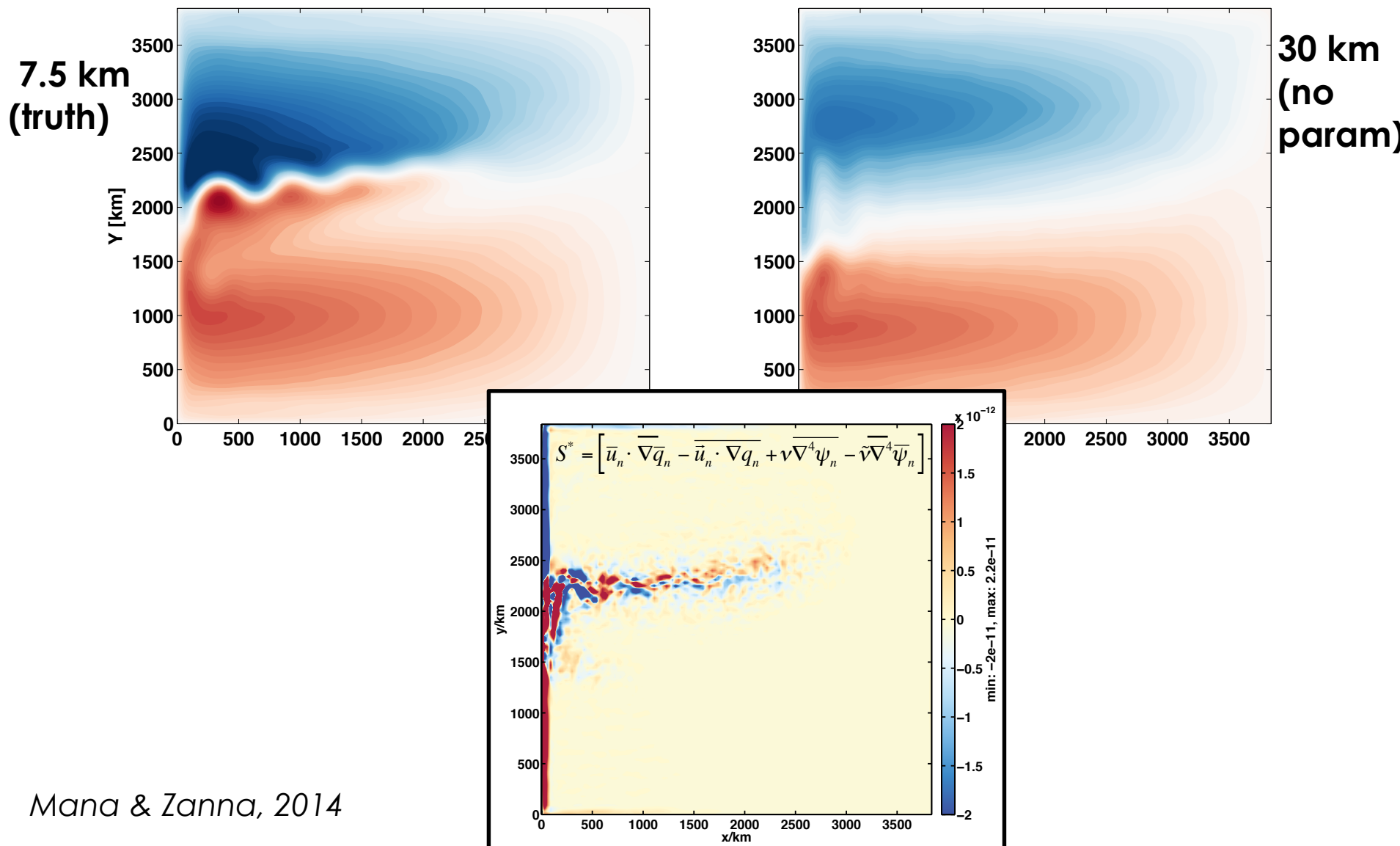
# Rethinking Subgrid Eddy Parametrizations

- Key aspects missing: energy backscatter from quasi-2D turbulence, jet sharpening, enhanced shear

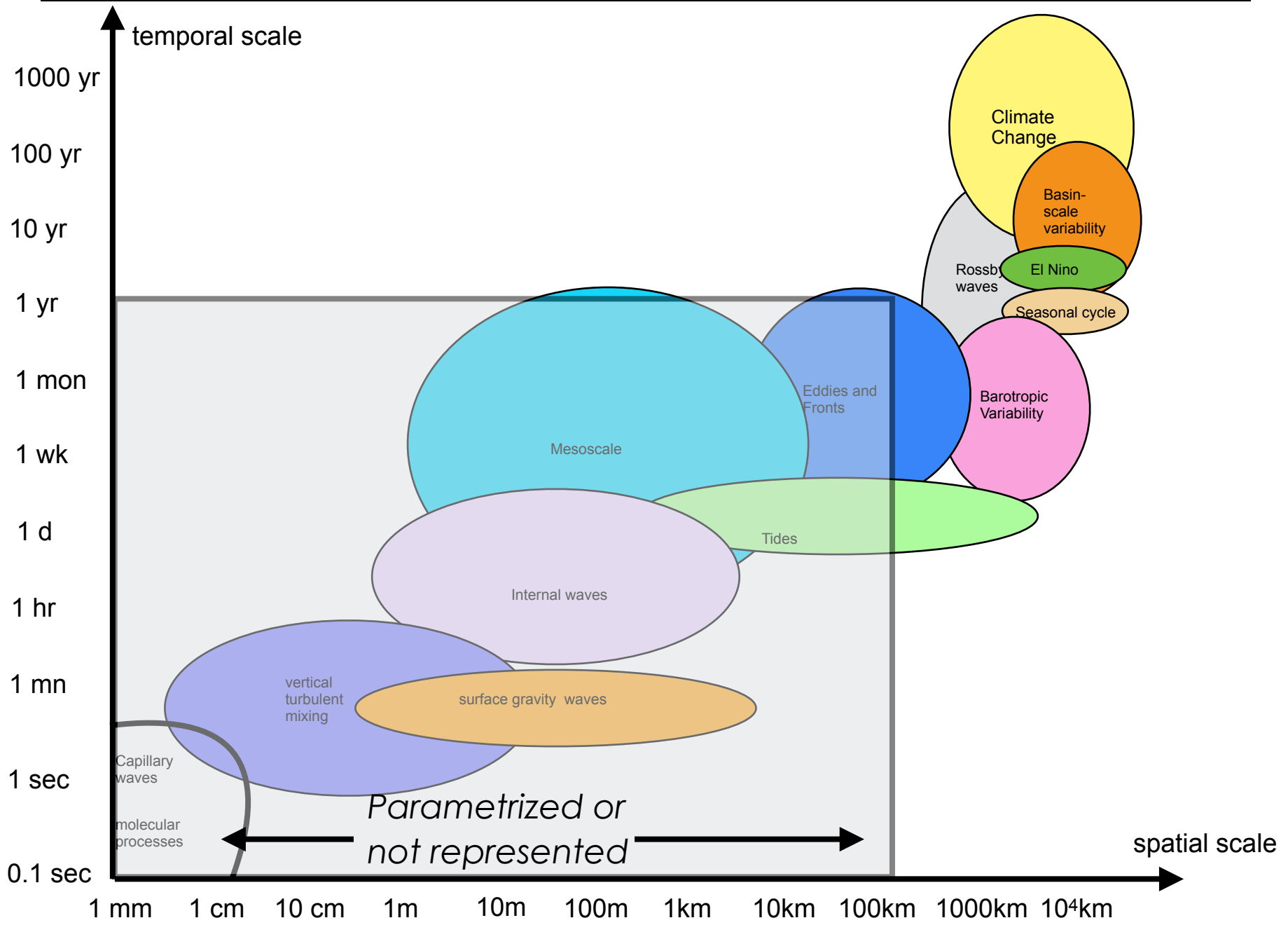


# Rethinking Subgrid Eddy Parametrizations

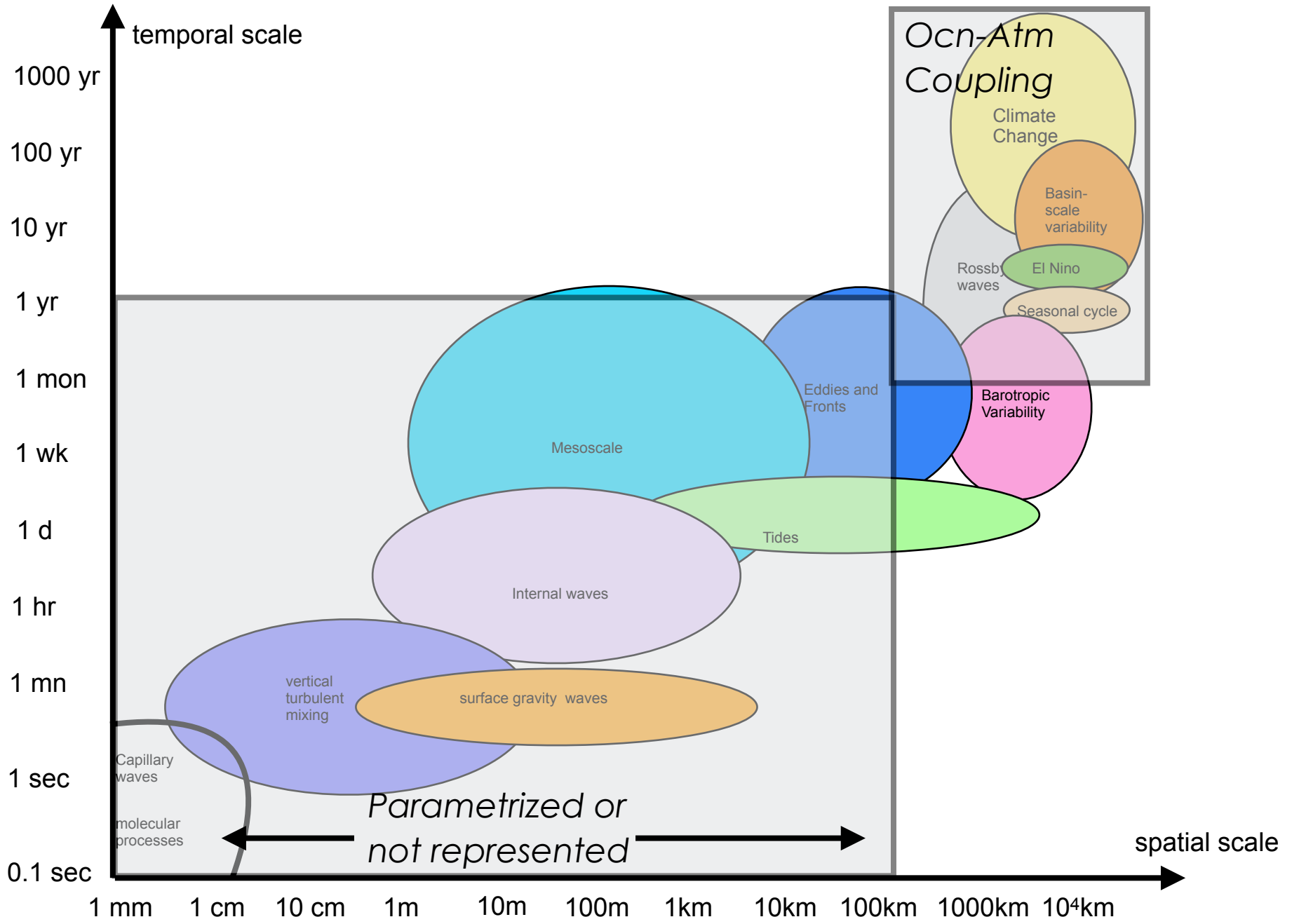
- Key aspects missing: energy backscatter from quasi-2D turbulence, jet sharpening, enhanced shear



# Ocean Processes & Subgrid-scale parametrization

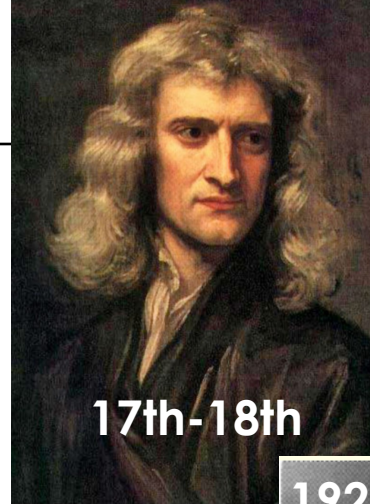


# Ocean Processes: Temporal vs. Spatial Scales

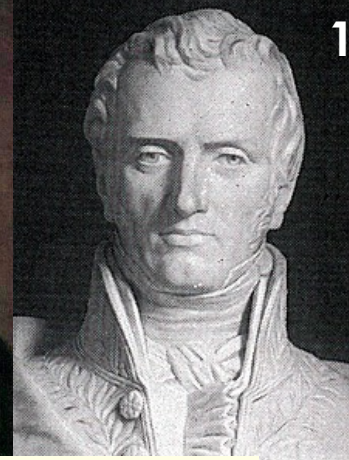


# Outline

- Ocean/Climate physics & models



17th-18th

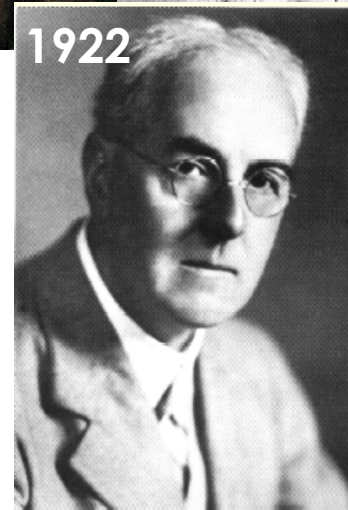


19th



- Fundamental Problems in Ocean/Climate, uncertainty and modelling:

- Regional projections of Sea Level Rise
- Uptake of heat, carbon, oxygen

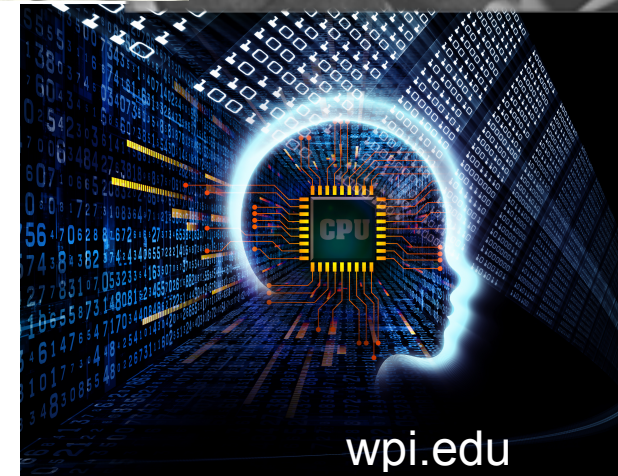


1922



1969

- New avenues for ocean modelling: combining advances in machine learning together with our knowledge of physics and its modelling



# Our governing equations

---

- **Newton's laws** (Navier-Stokes)

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v} + \mathbf{F} + \mathbf{F}_c \rightarrow \text{Coriolis}$$

wind, gravity...  
↓

- **Thermodynamics** - heat & salt for a stratified fluid (tracer equations),

$$\frac{\partial C}{\partial t} + \mathbf{v} \cdot \nabla C = \kappa \nabla^2 C + m_C$$

- **Conservation of mass/** continuity for incompressible fluid

$$\nabla \cdot \mathbf{v} = 0$$

- **Equation of state** for density (we don't know what it is really)

$$\rho = \rho(T, S, p) \approx \rho_0(1 - \alpha T + \beta S)$$

# Multi-scale Interaction

Energy Source

Wind + Buoyancy Work

Energy Transfer

Large-scale circulation + stratification  
1000-10000km

Instability

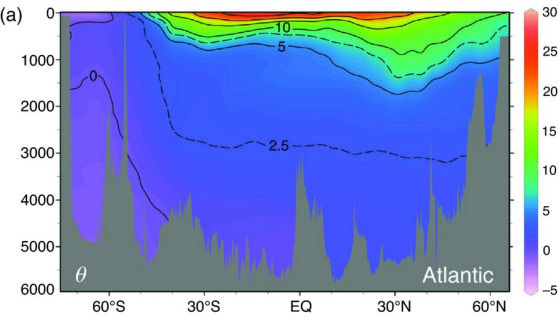
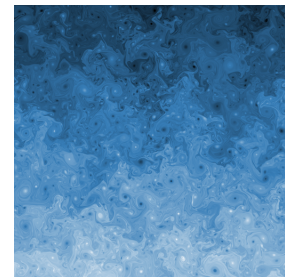
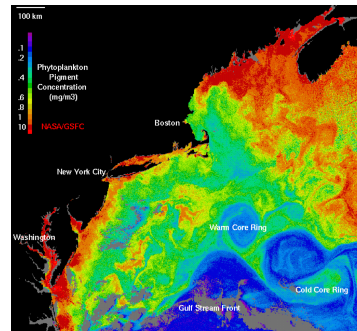
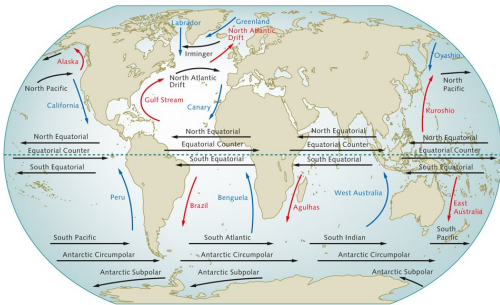
**Inverse Cascade**

Mesoscale Eddies  
10km-100km

**Inverse Cascade**

Energy Sink  
Viscous Dissipation  
1mm-1cm

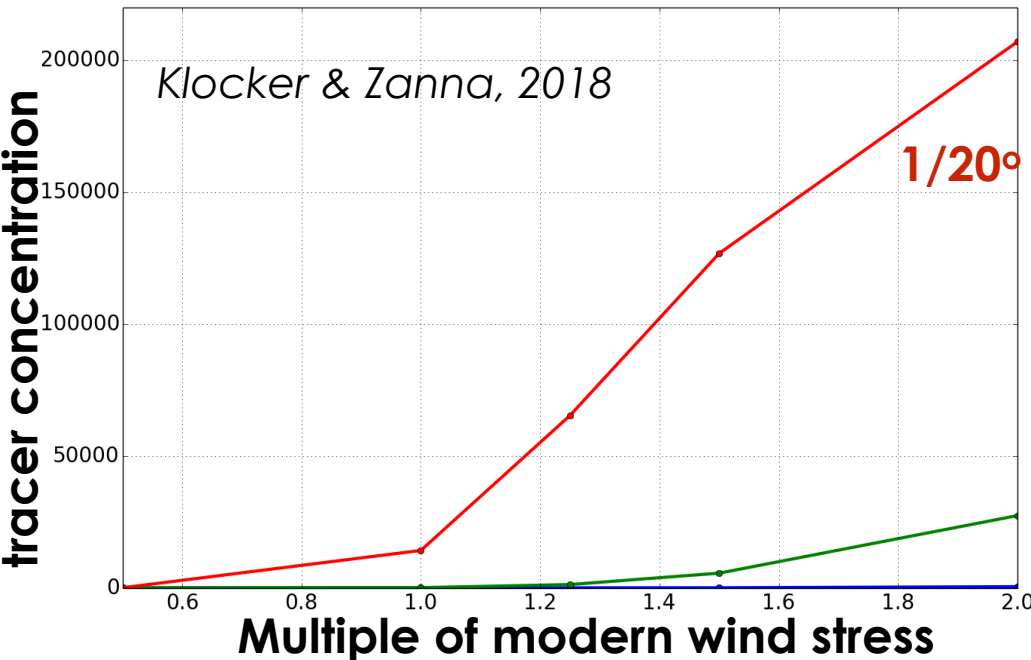
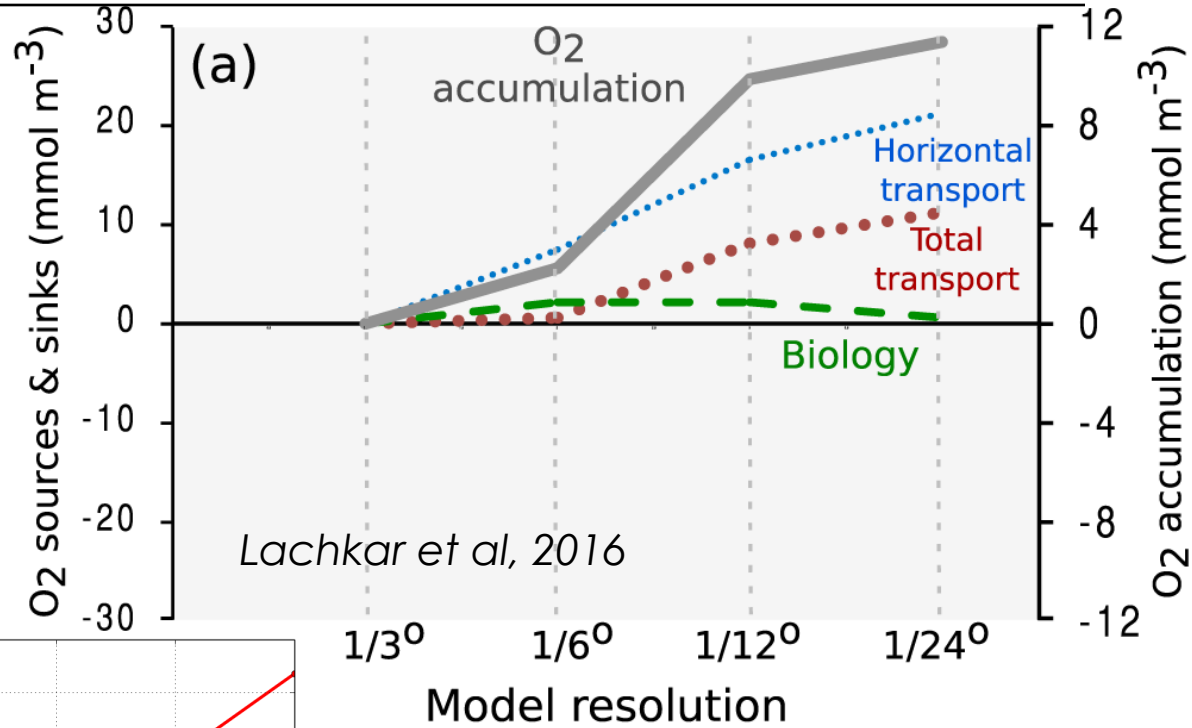
Sub-mesoscale, filaments ...  
<10km



**Many of these interactions are poorly represented in models, understood in nature, and their impact on climate (physics, biology, chemistry) is unknown**

# How much heat, carbon, oxygen are taken up?

- “Resolution” sets transport, accumulation of tracer

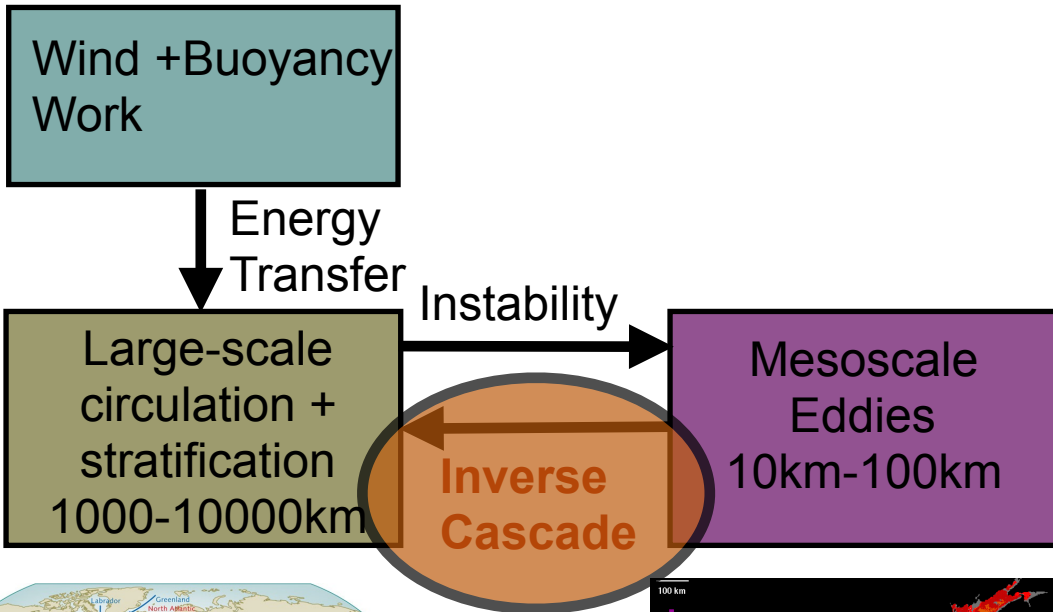


and its sensitivity to forcing  
(important for climate change)

1/12°  
1°

# How to improve estimates of tracer uptake?

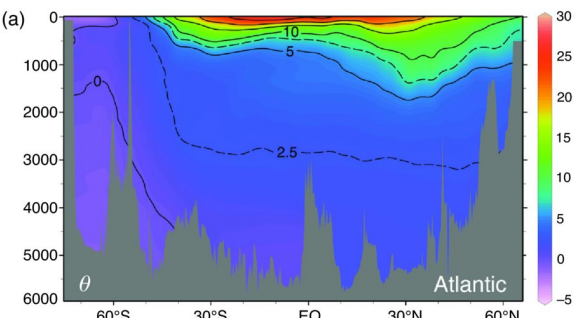
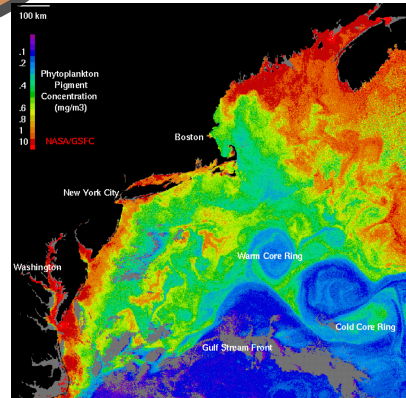
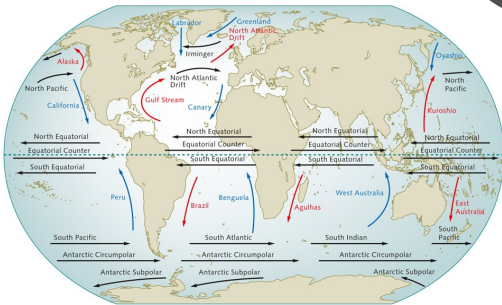
- or include missing mesoscale effects (10-100km)



Energy Sink

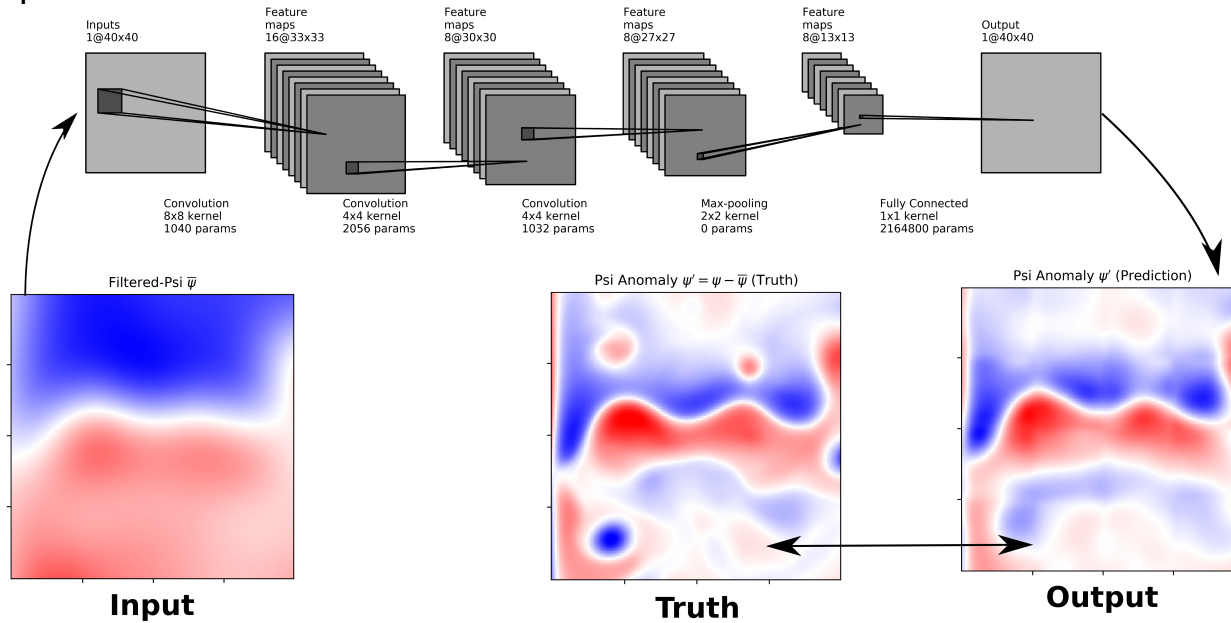
Viscous Dissipation  
1mm-1cm

Sub-mesoscale filaments  
<10km



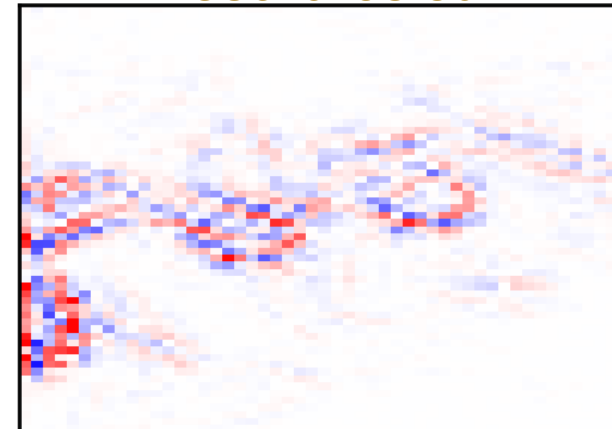
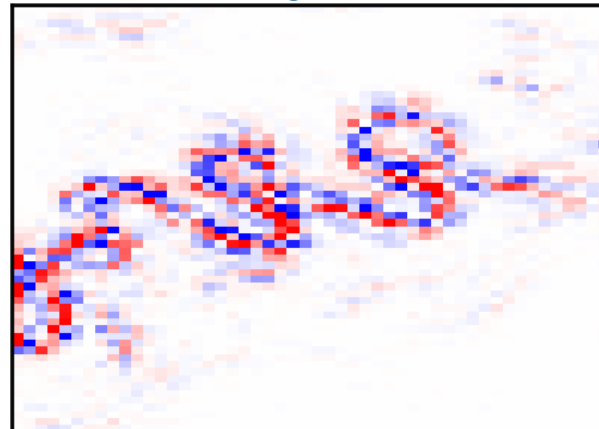
# Neural Network to find Missing Forcing

- Can neural network help determining the missing forcing or optimise the parameters ?



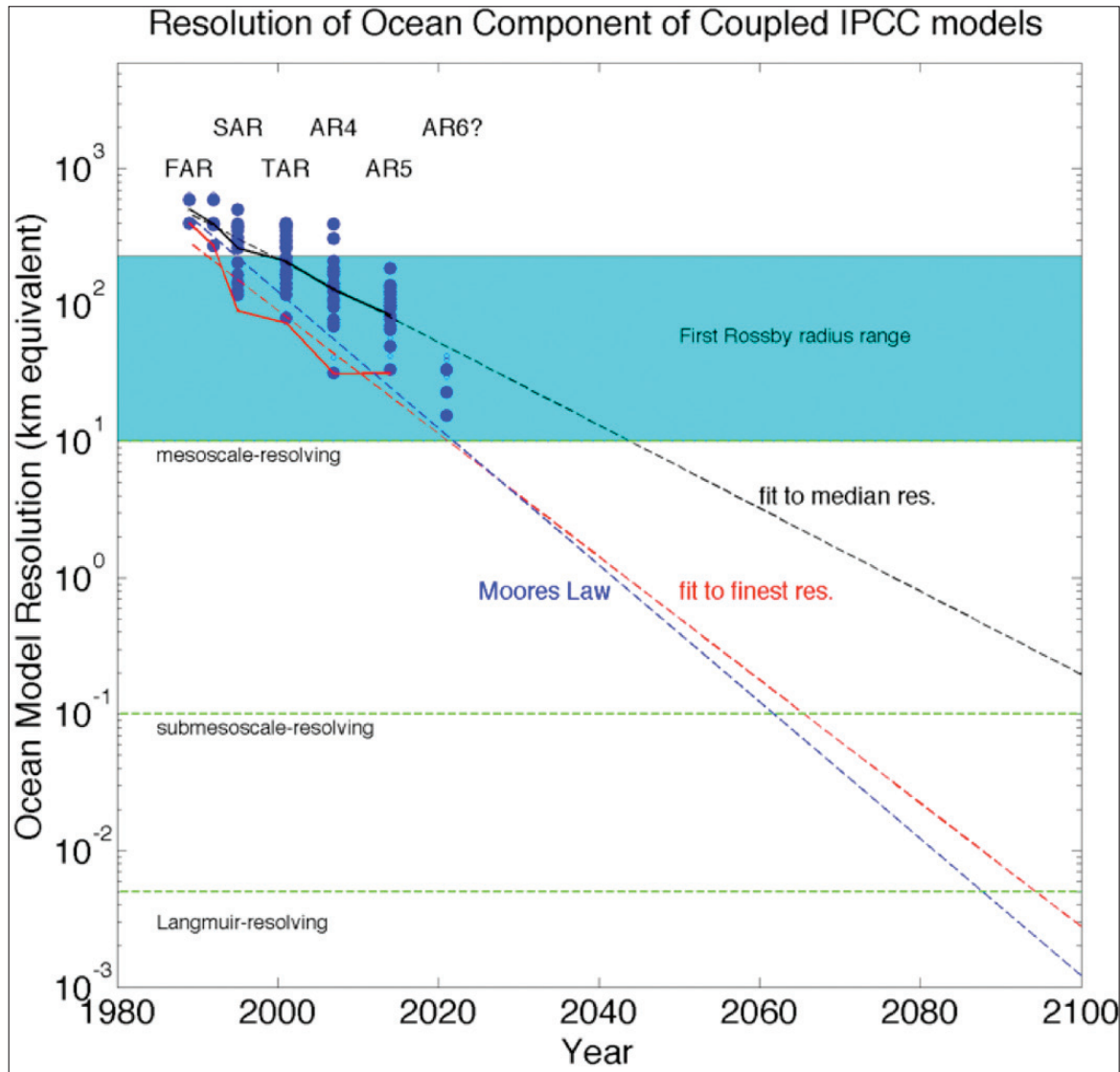
Truth

Reconstructed



# Summary & New Avenues

- Limited computational resources for ocean-climate models



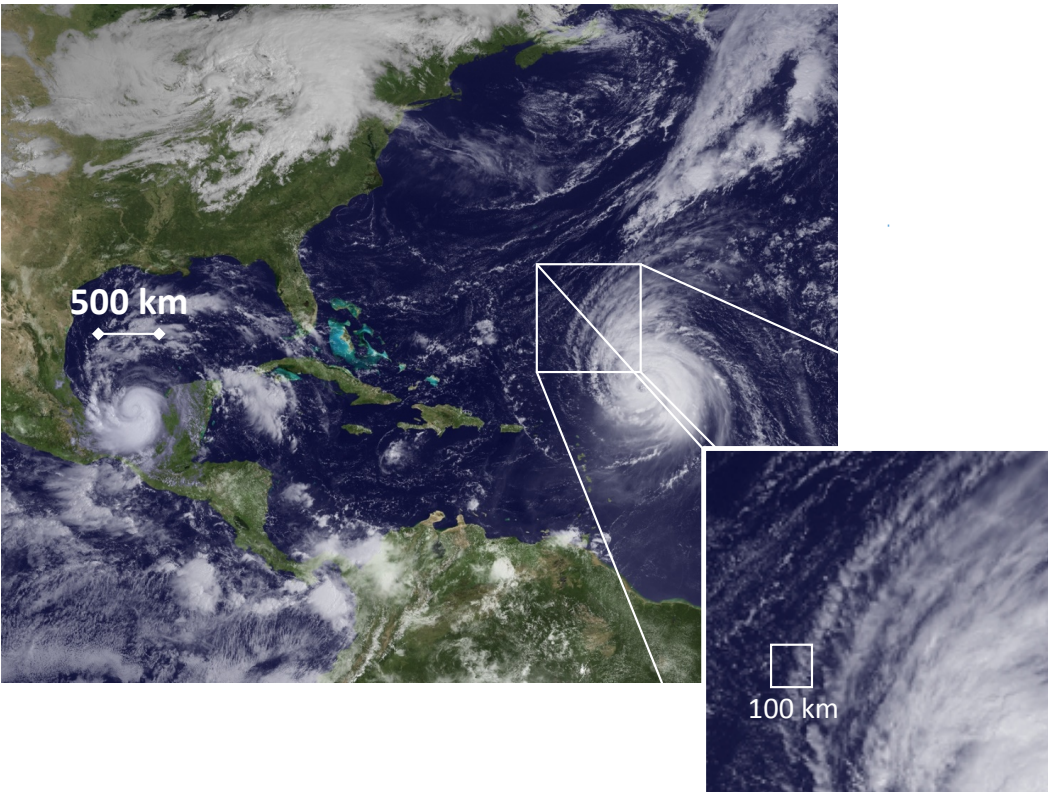
*Fox-Kemper et al 2014*

# Summary & New Avenues

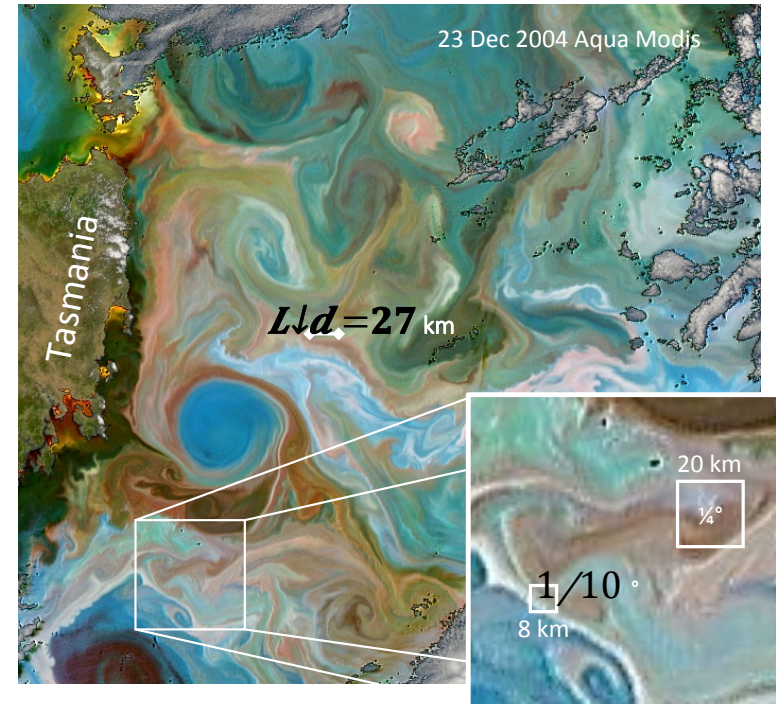
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- Limited computational resources for ocean-climate models
- Scales to resolve in the ocean smaller than in the atmosphere

Clouds visualized from FV3 simulation (NOAA)

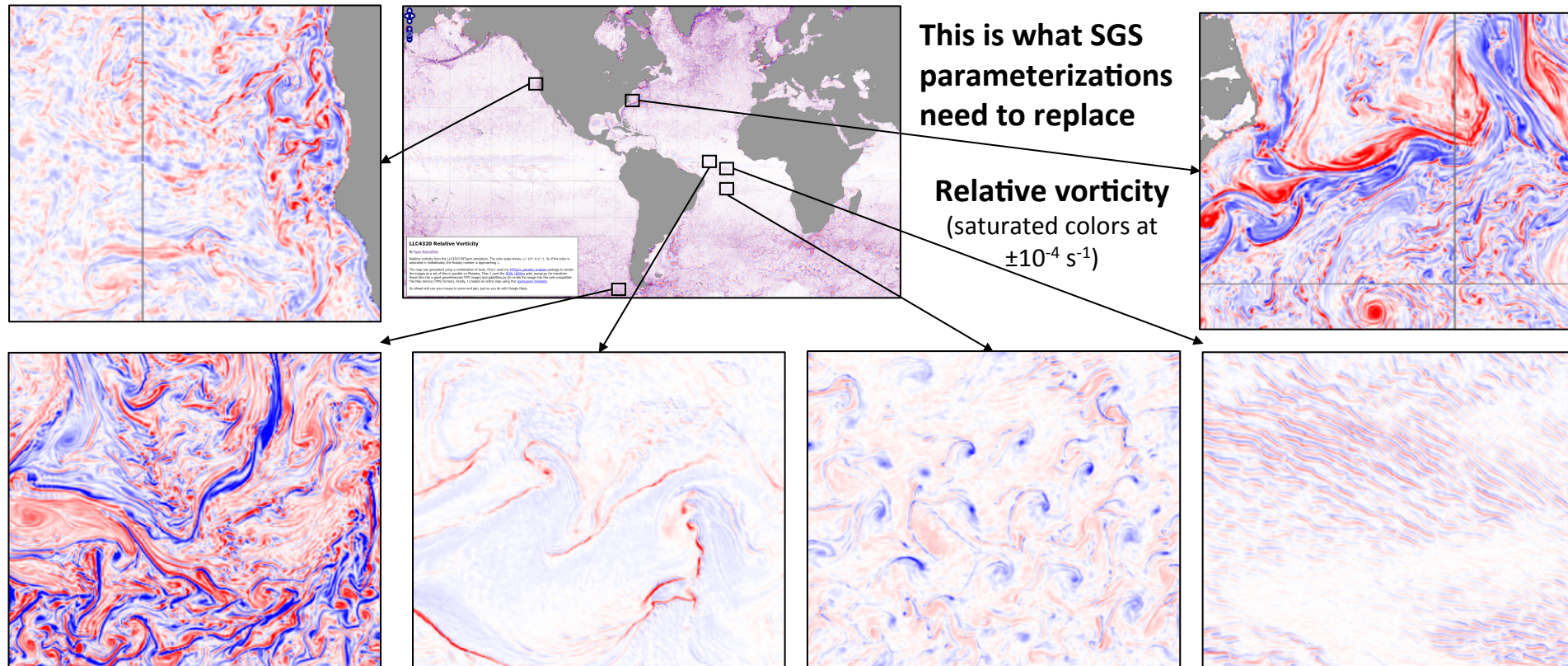


False color image from Aqua MODIS (NASA)



# Summary & New Avenues

- Limited computational resources for ocean-climate models
- Scales to resolve in the ocean smaller than in the atmosphere
- Zoo of unresolved scales not accounted for, or represented in their bulk form - crucial for determining the temperature at the surface of the ocean

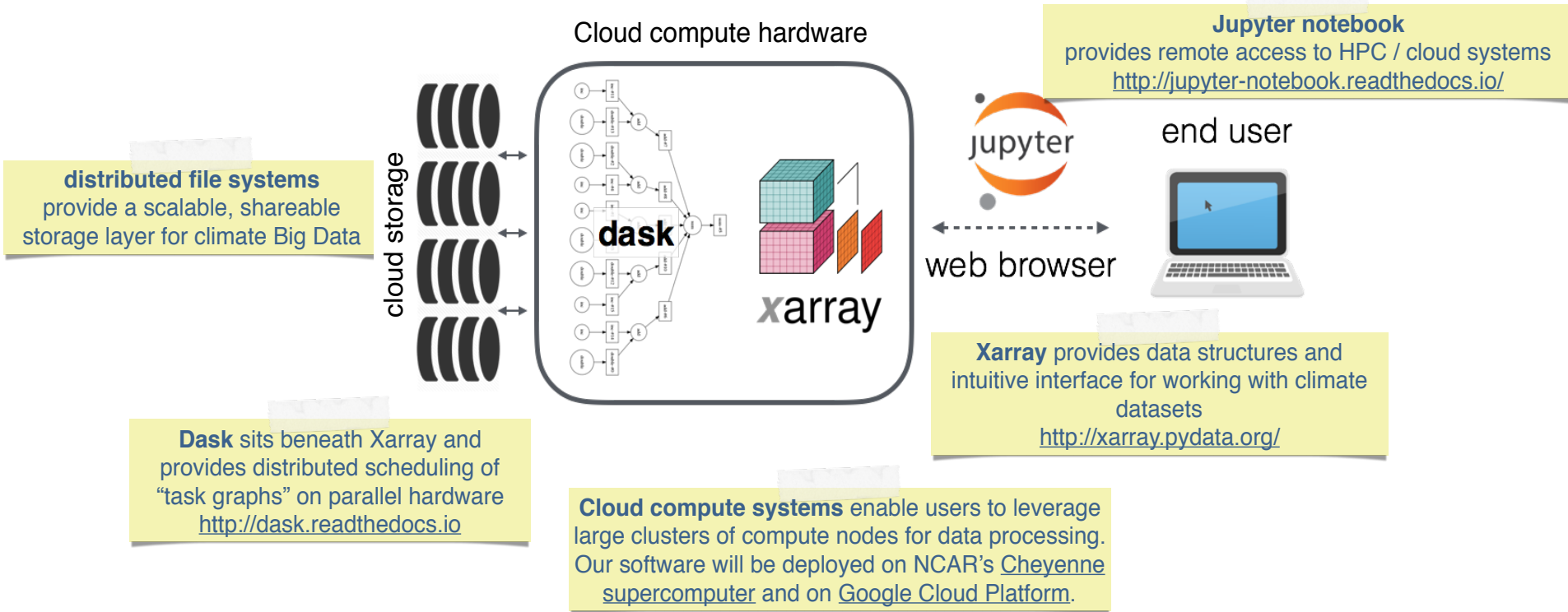


2km model from ECCO2 project (Menemenlis and Hill)

Thanks to Alistair Adcroft (Princeton/GFDL)

## Pangeo Architecture

Led by Ryan Abernathey  
(Columbia University)



EARTH CUBE



Google Cloud Platform

Lamont-Doherty Earth Observatory  
COLUMBIA UNIVERSITY | EARTH INSTITUTE

Ryan Abernathey, Chiara Lepore, Michael Tippet, Naomi Henderson, Richard Seager

Kevin Paul, Joe Hamman, Ryan May, Davide Del Vento



Matthew Rocklin



Powered by Continuum Analytics

Many other collaborators around the world (also outside of geoscience)



NASA Goddard Institute for Space Studies

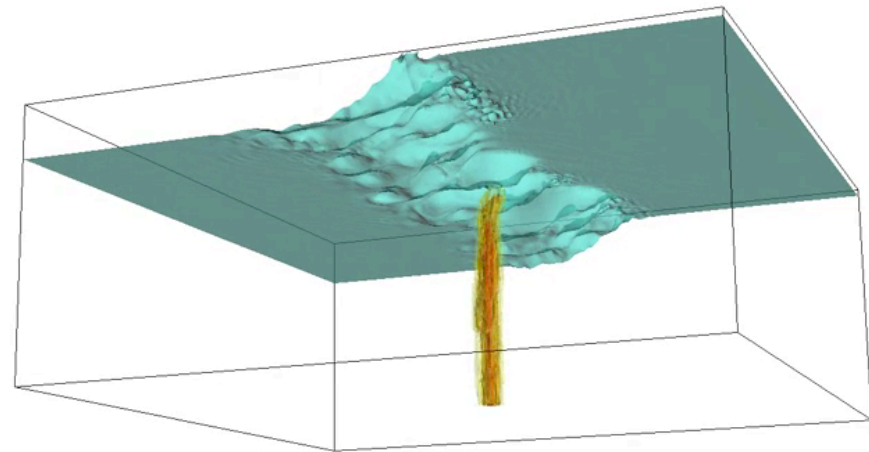
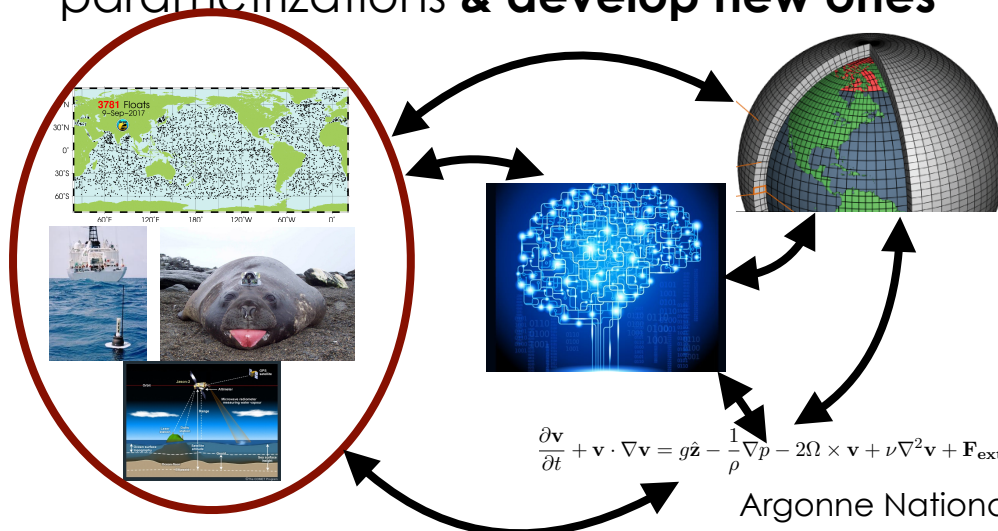
# Summary & New Avenues

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- The future: Increase Resolution ? Increase complexity? Increase ensemble size ?

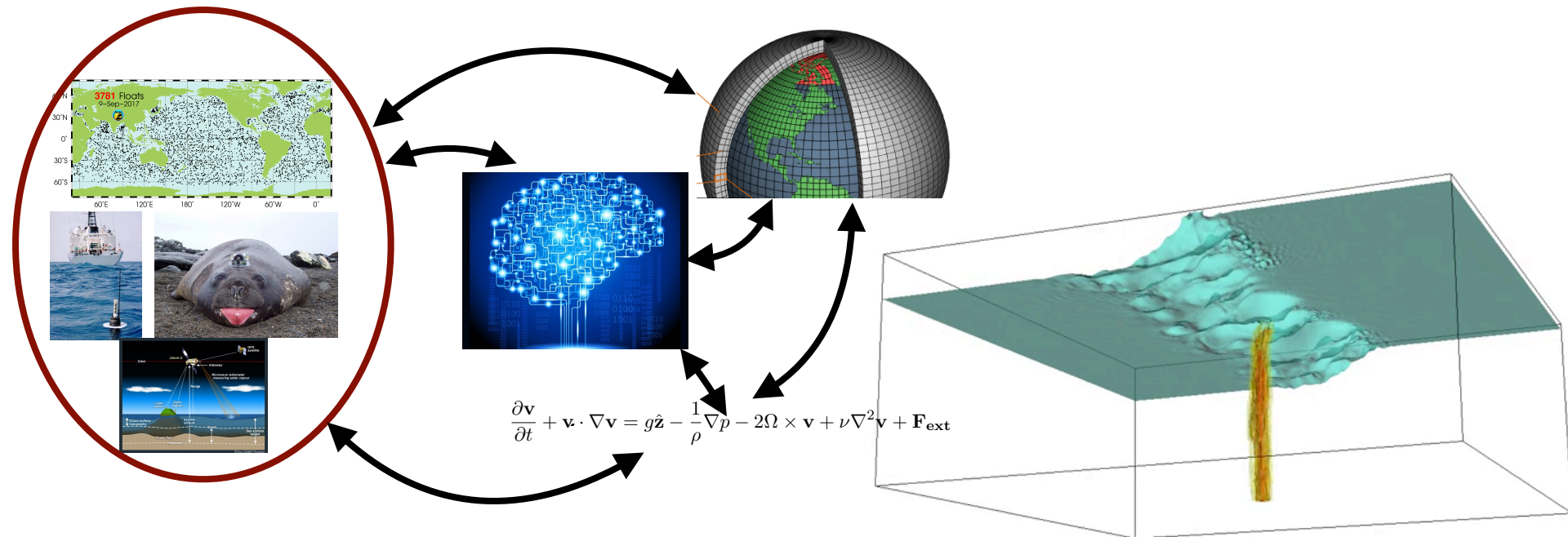
# Summary & New Avenues

- Limited computational resources for ocean-climate models
- Scales to resolve in the ocean smaller than in the atmosphere
- Zoo of unresolved scales not accounted for, or represented in their bulk form - crucial for determining the temperature at the surface of the ocean
- The future: Increased Resolution ? Increased complexity ? Increase ensemble size ?
- My philosophy = back to basics. Use (idealised) high-resolution simulations, together with theory/first principle, and observations to inform current parametrizations & **develop new ones**

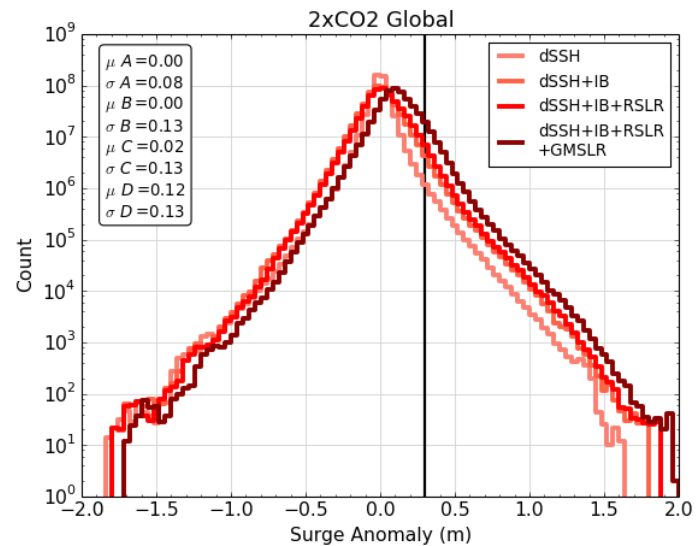
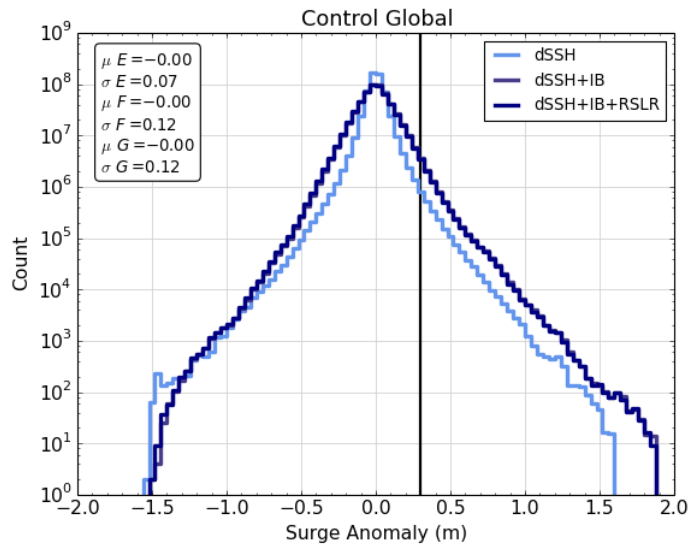
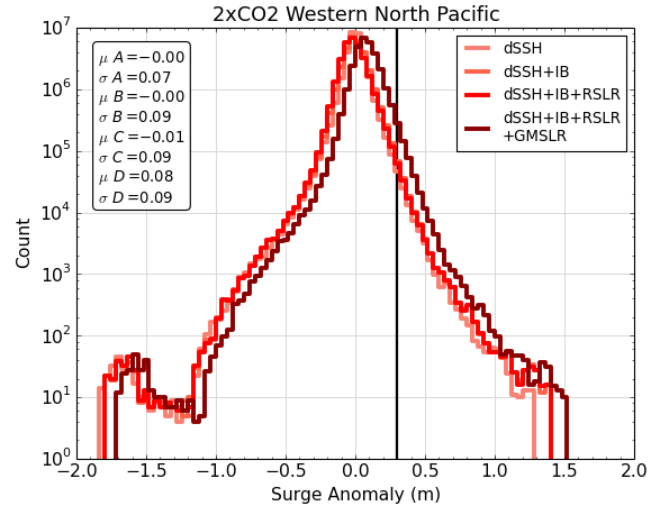
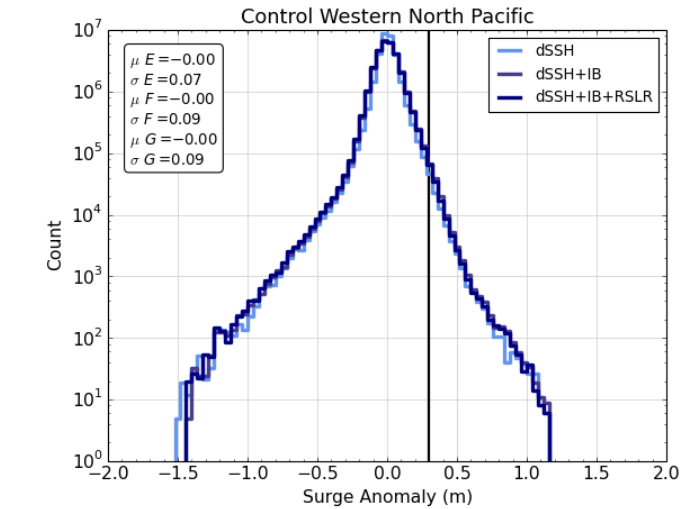


# Summary & New Avenues

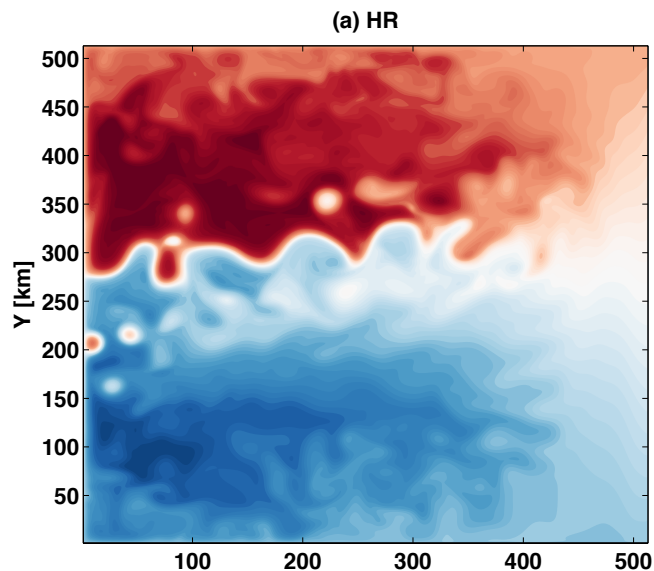
- Limited computational resources for ocean-climate models
- Scales to resolve in the ocean smaller than in the atmosphere
- Zoo of unresolved scales not accounted for, or represented in their bulk form - crucial for determining the temperature at the surface of the ocean
- Use (idealised) high-resolution simulations, together with theory/first principle, and obs to inform current parametrizations **& develop new ones**



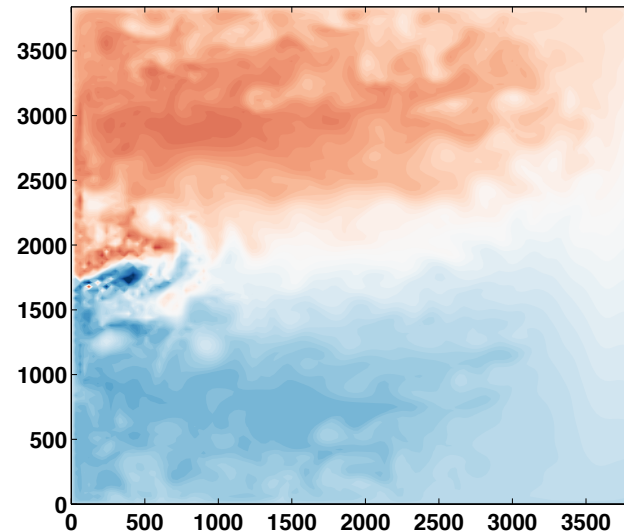
# Motivation: How to Solve the Climate Problem?



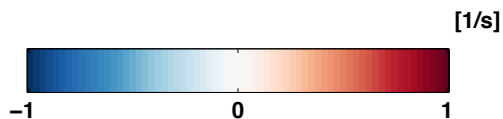
**7.5 km  
("truth")**



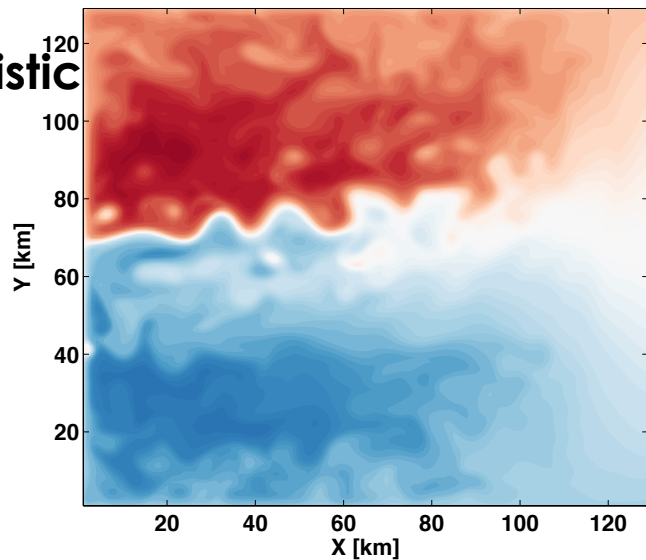
(b) LR



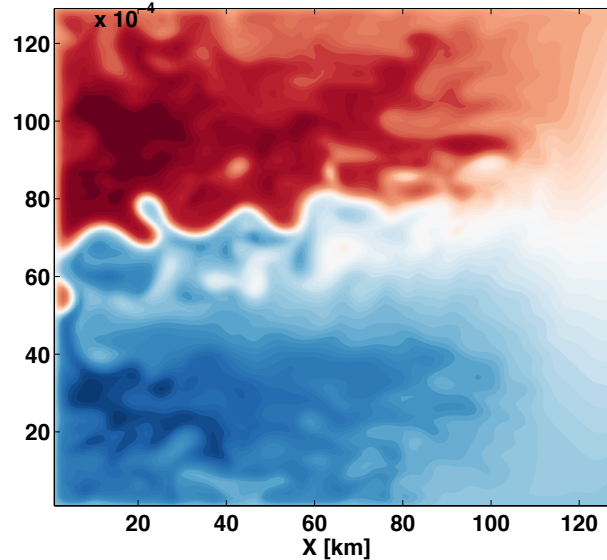
**30 km  
(no  
param)**



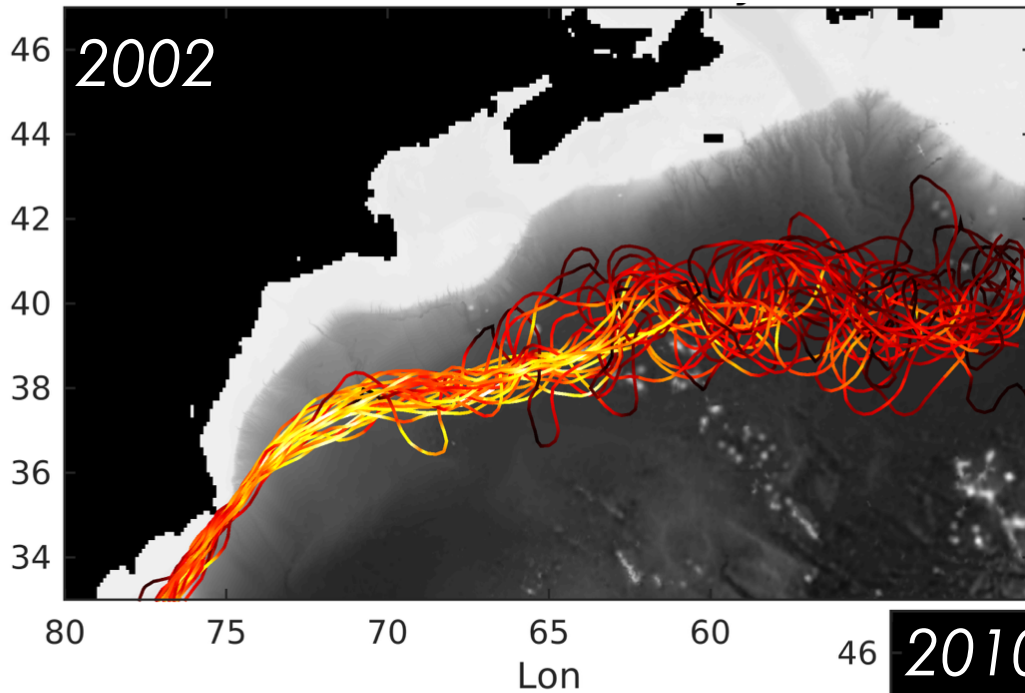
(c) LR + det



(d) LR + sto



# Ocean Turbulence in the Gulf Stream



*Bolton & Zanna, 2016*

- Year - to - Year variability in Gulf Stream speed due to scale interaction

- Do we expect changes in the turbulent behaviour of the Gulf Stream & other currents & associated mixing?

