Using Observations to Constrain the Yield Curve, Flow Rule and Mechanical Strength of Sea Ice

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Sea Ice Momentum Equation

Coriolis water drag rheology

$$\rho_i h \frac{Du}{Dt} = -\rho_i h f \hat{k} \times u + \tau_a - \tau_w - \rho_i h g \nabla H_d + \nabla \cdot \sigma$$

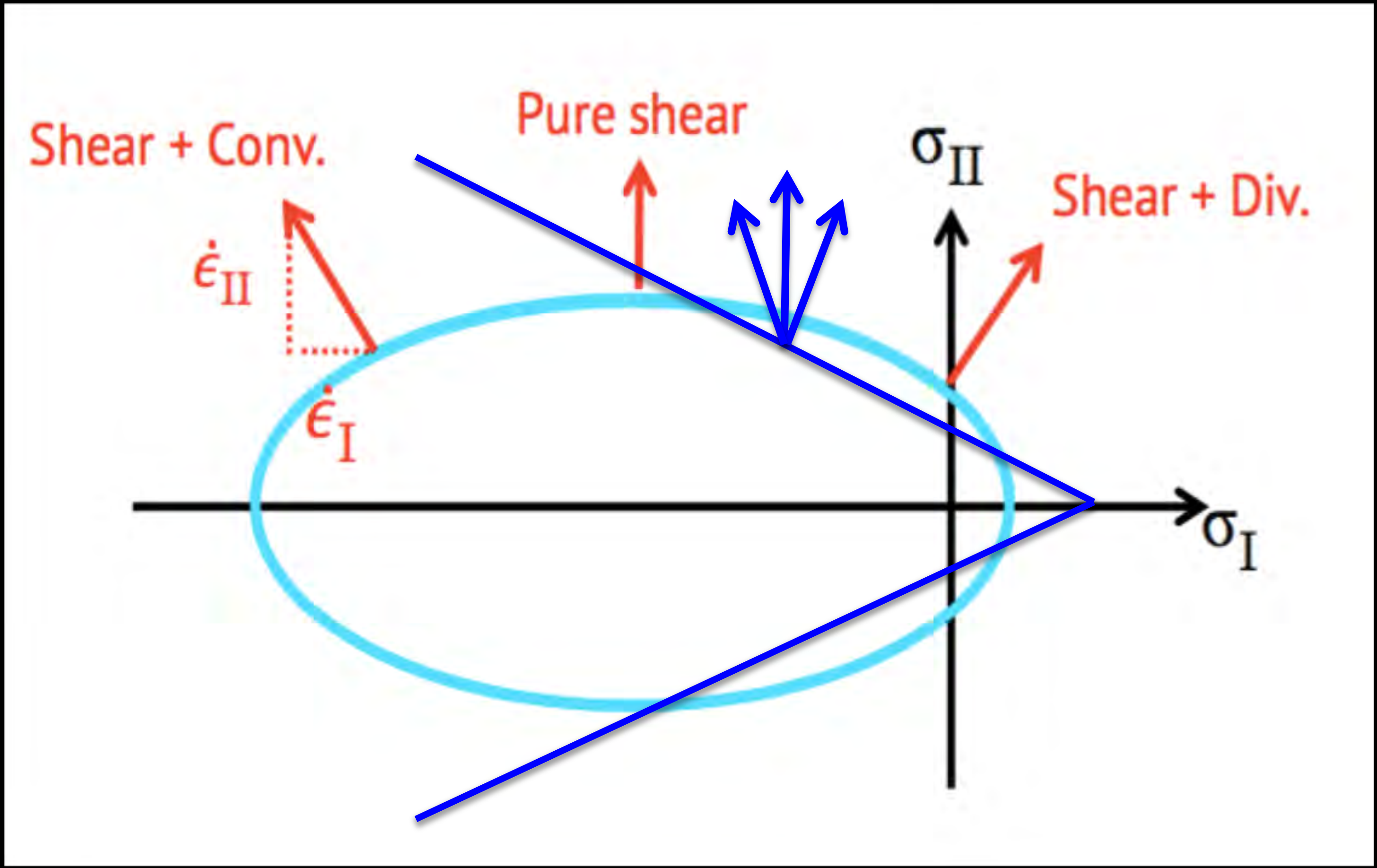
inertial air drag SSTilt

Constitutive Relations

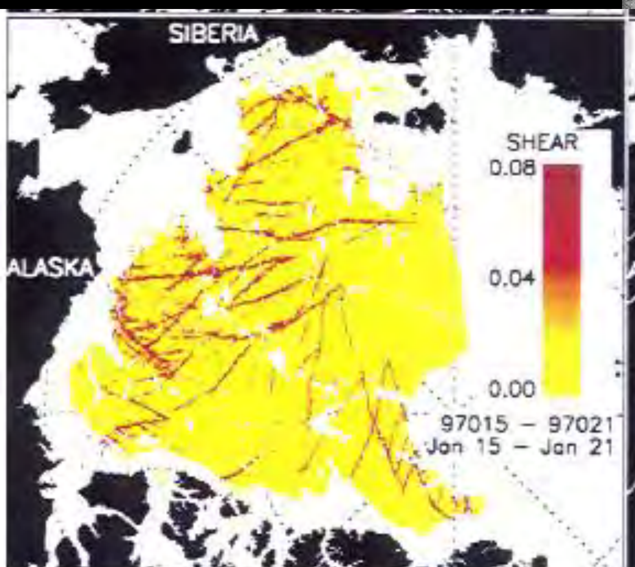
- Elastic
- Viscous (8 days)
- Plastic

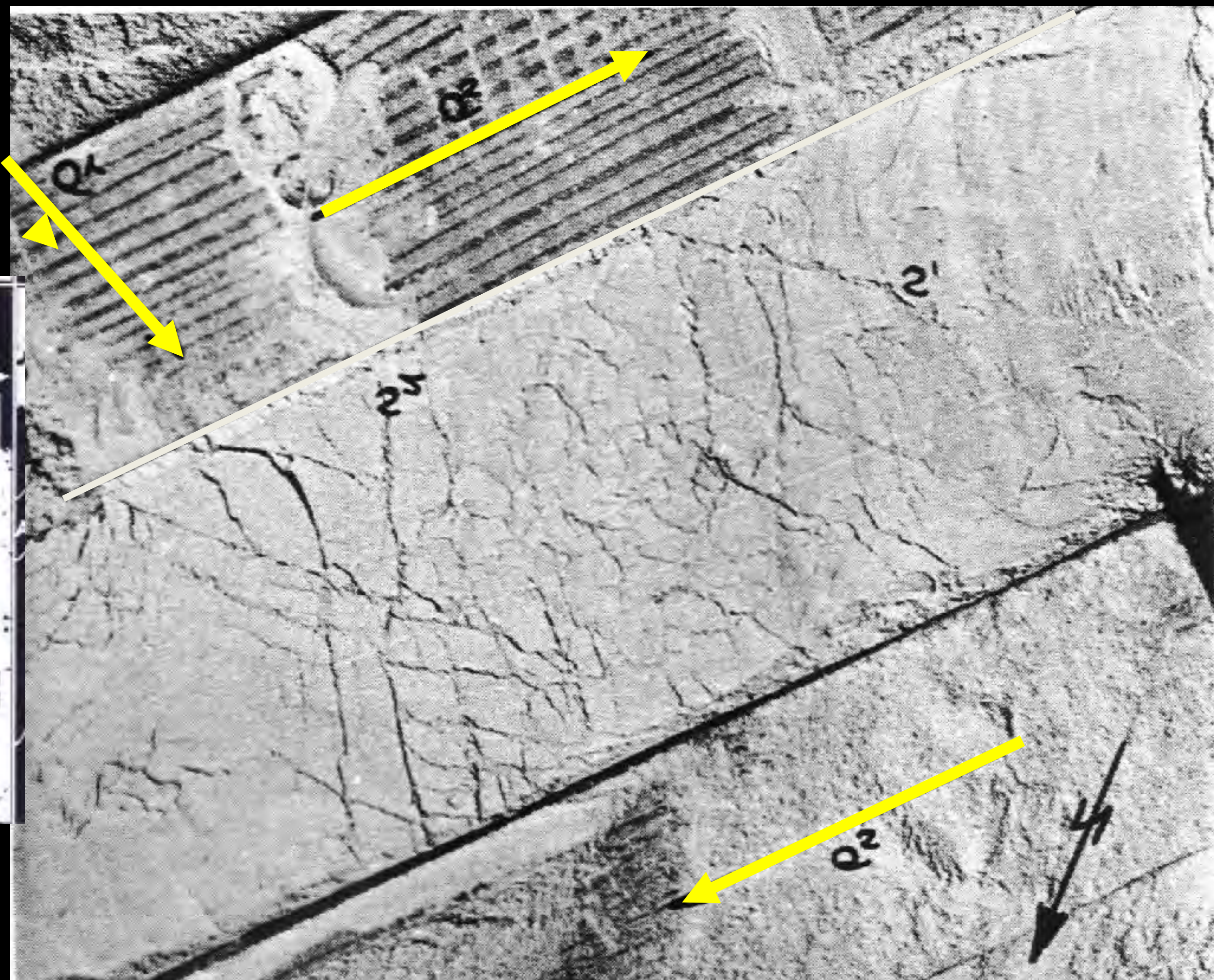
Plasticity

Yield Curve and Flow Rules

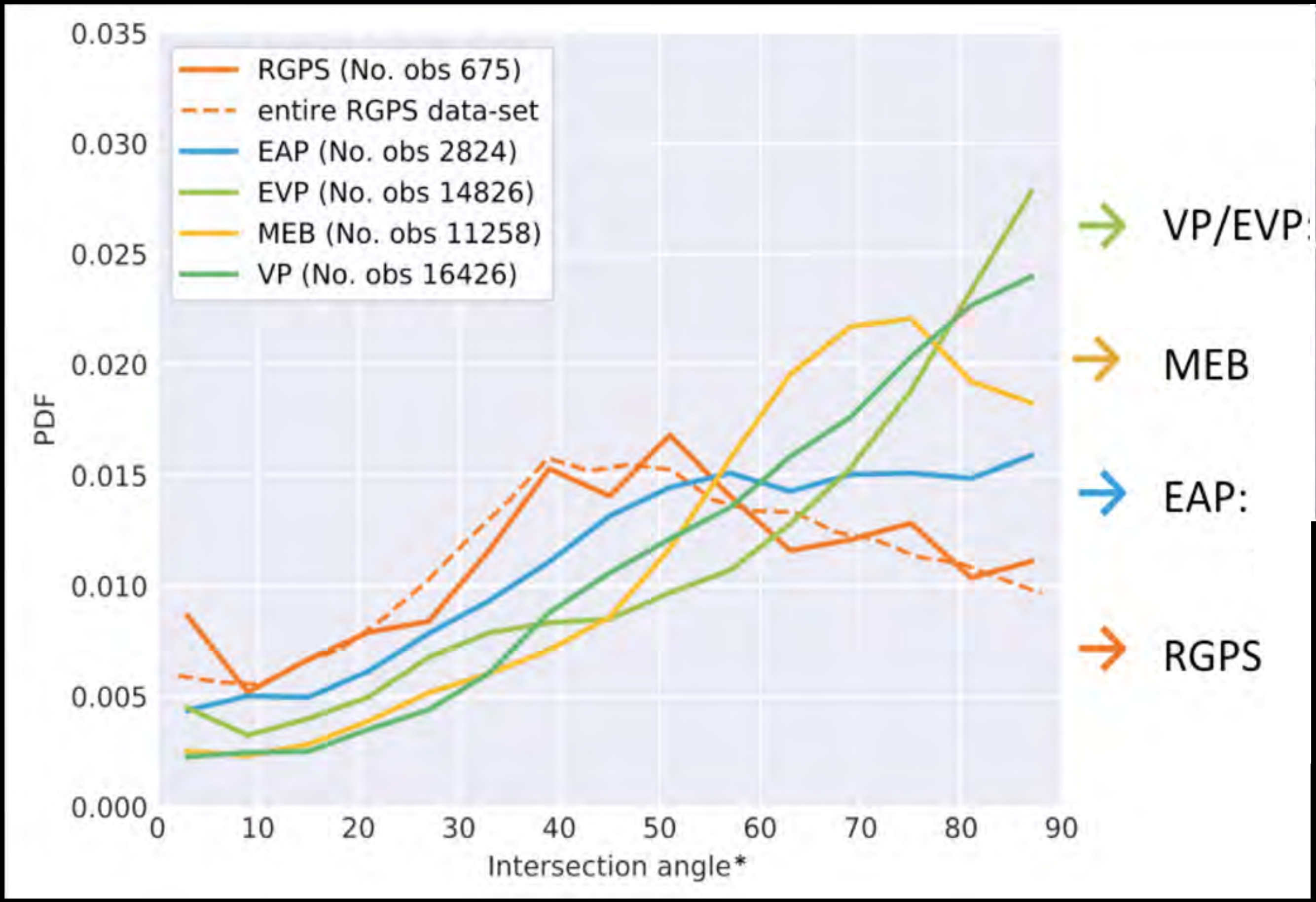


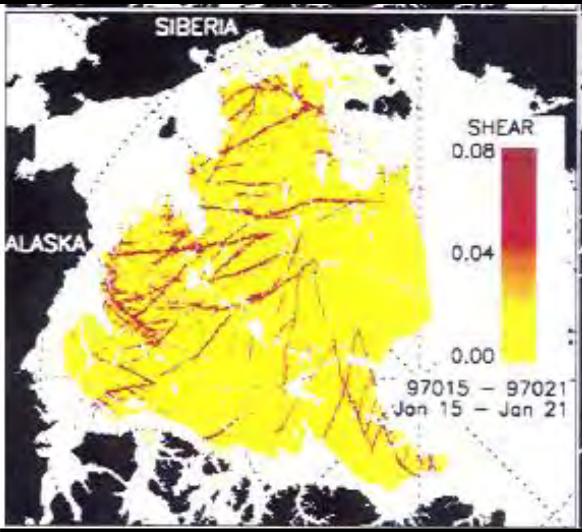
Clay under shear deformation





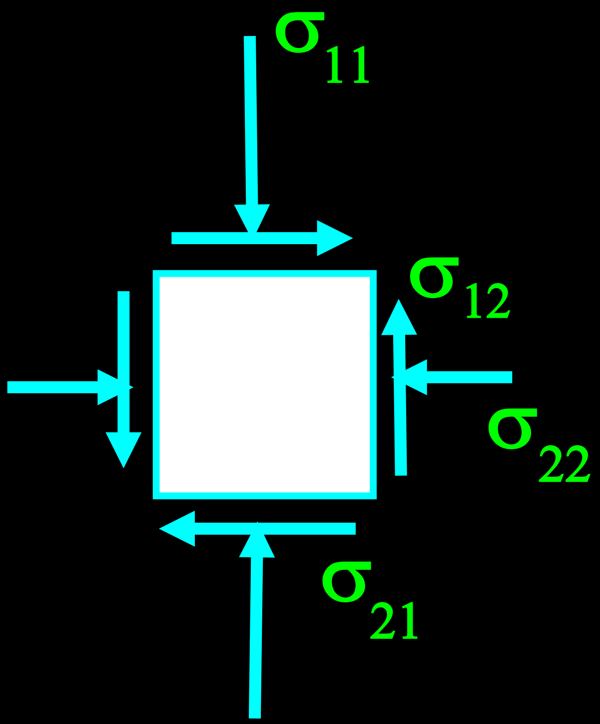
Intersection of Fracture Lines





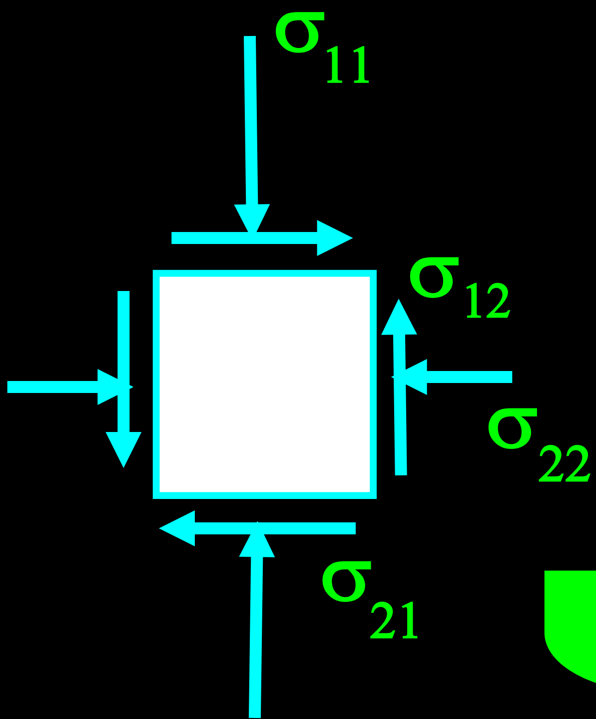
Hutter et al., 2018

Stress State

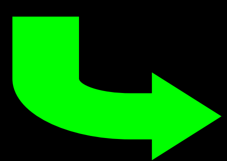


$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$

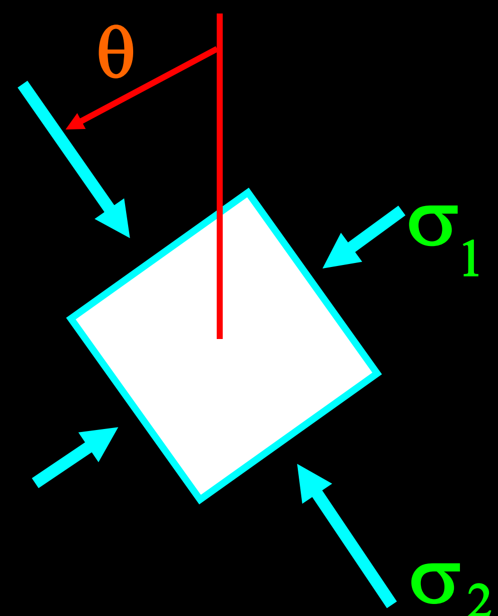
Stress State



$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$

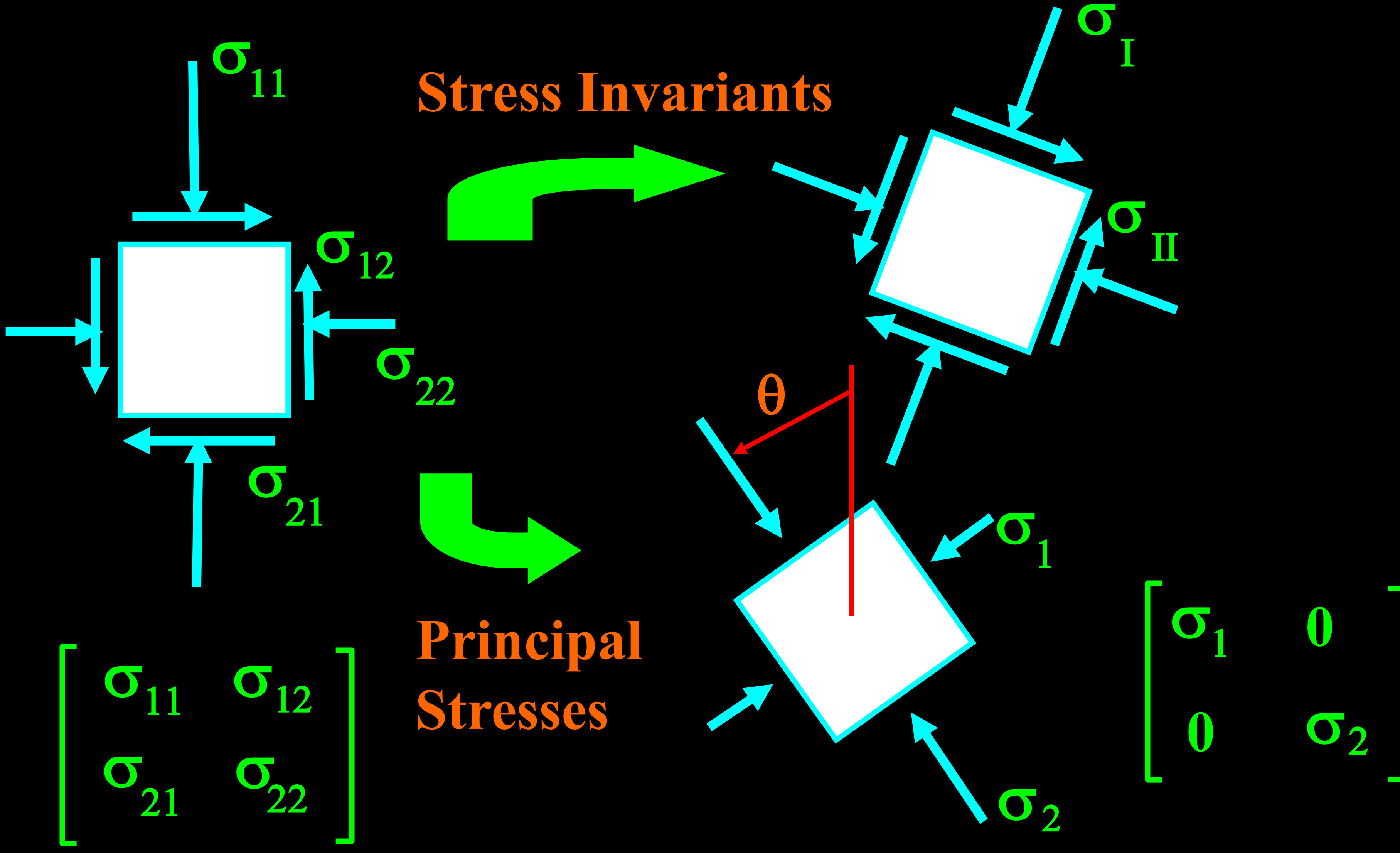


Principal Stresses

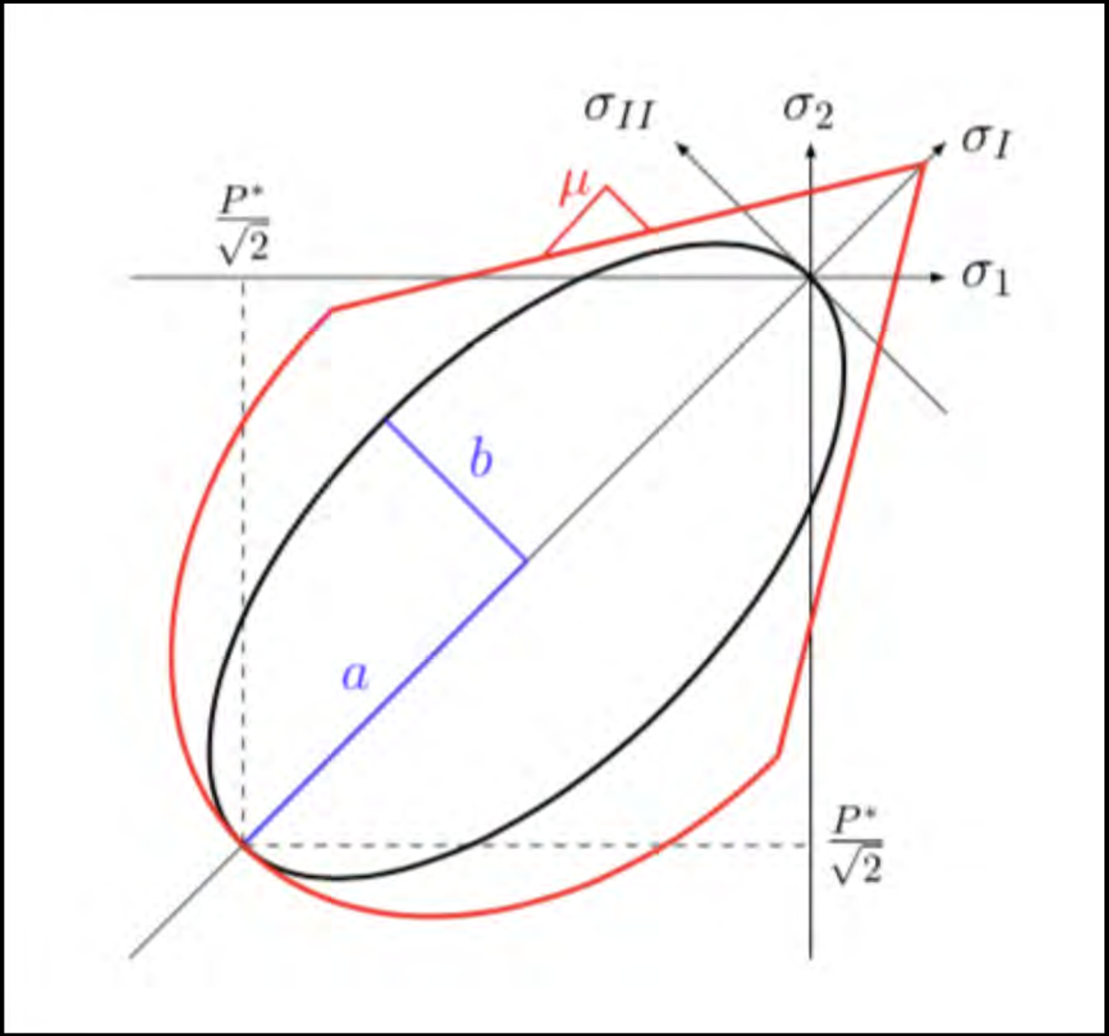


$$\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$$

Stress State



Two Yield Curves

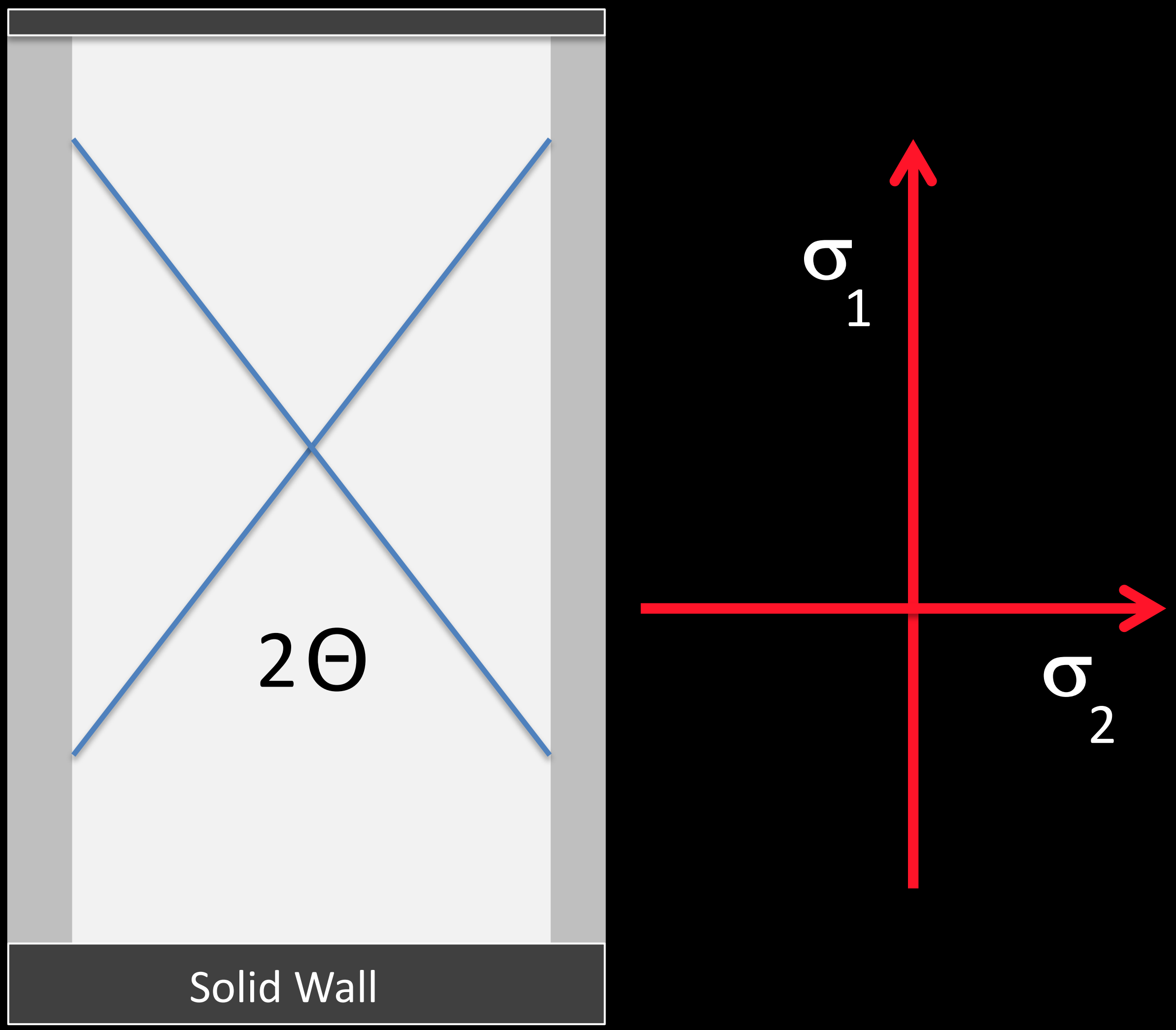


Hibler, 1979; Hibler and Schulson, 2000

Experimental Set-Up

Prescribe Strain rate

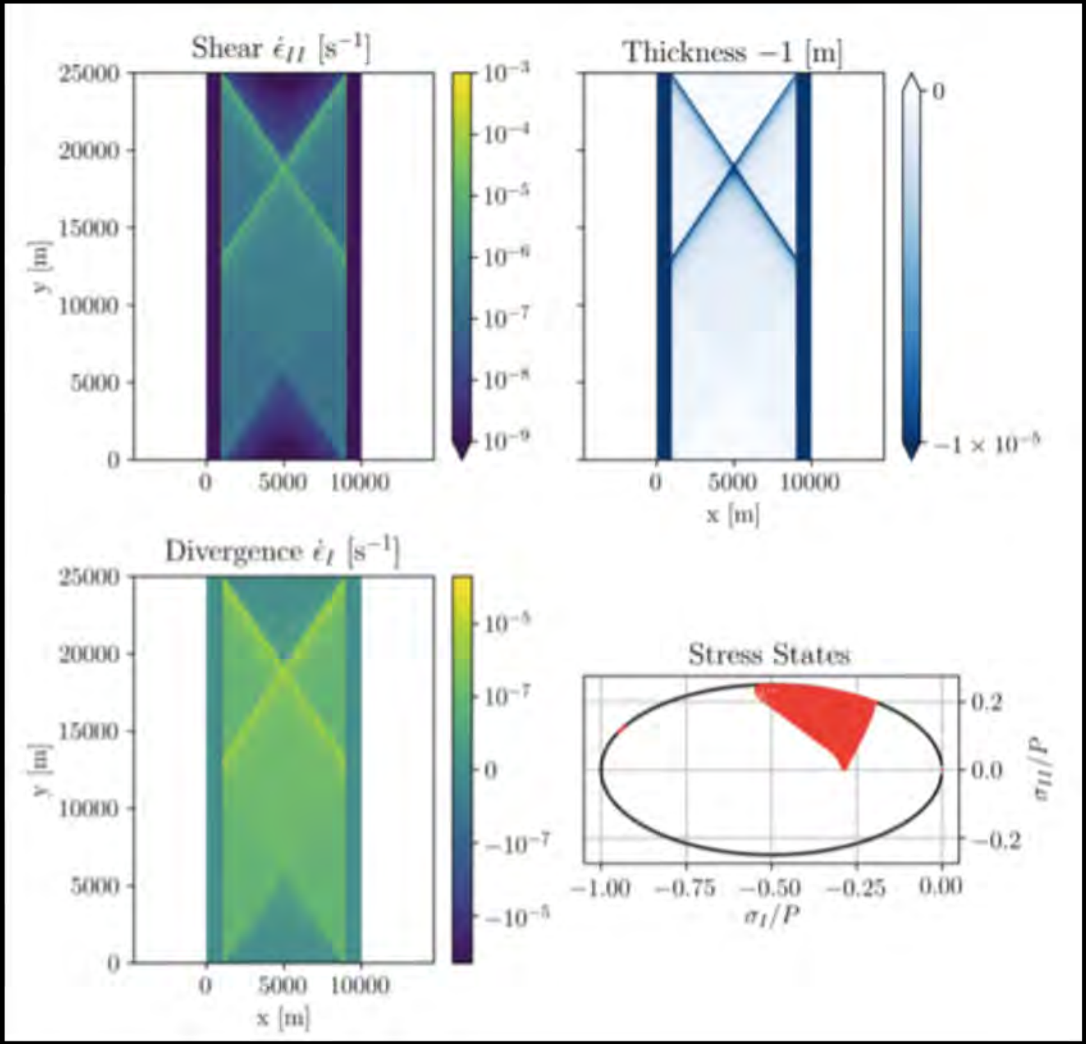
8 km x 25 km
dx = 25 m



Ringeisen et al., 2018

Axial Loading Test

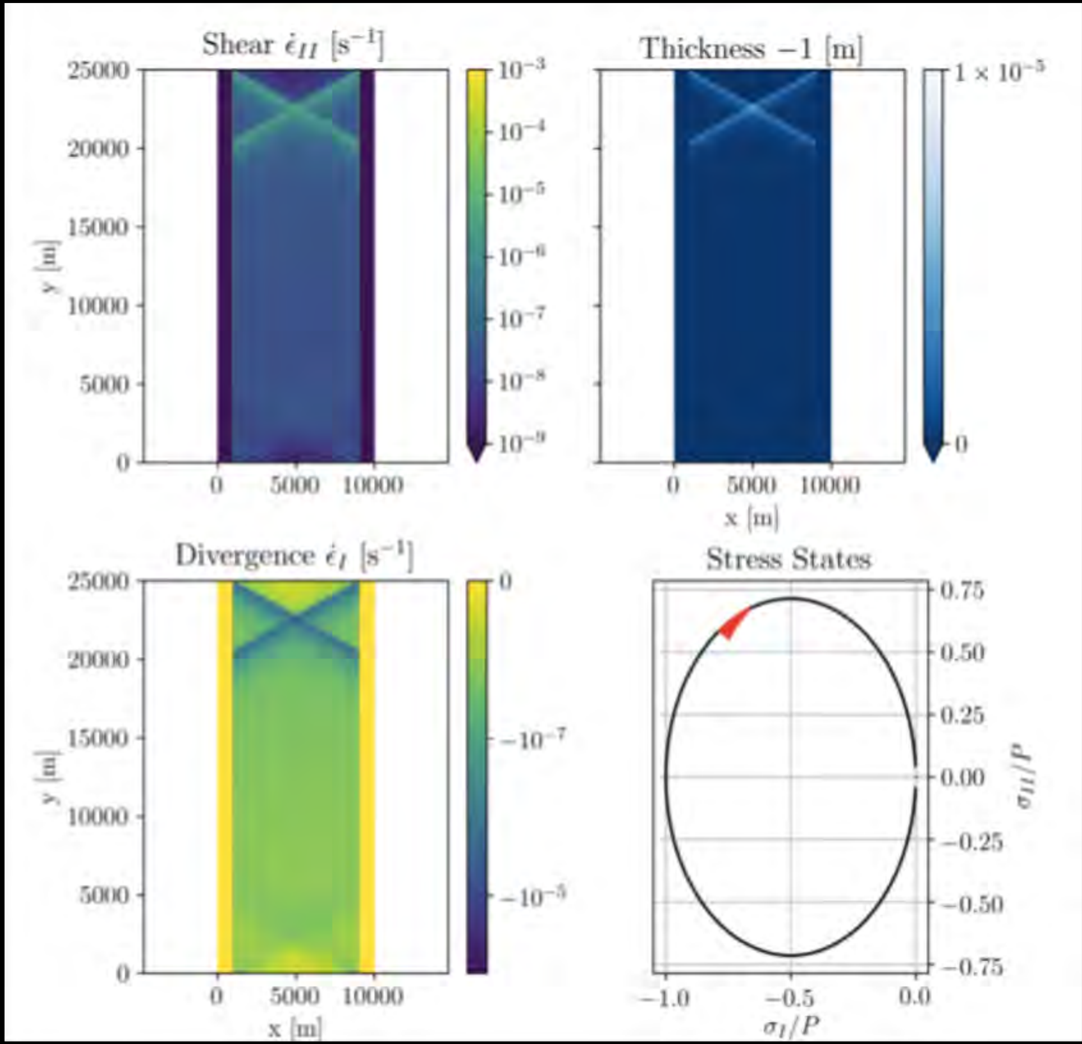
Control Run (e=2)



34 degrees

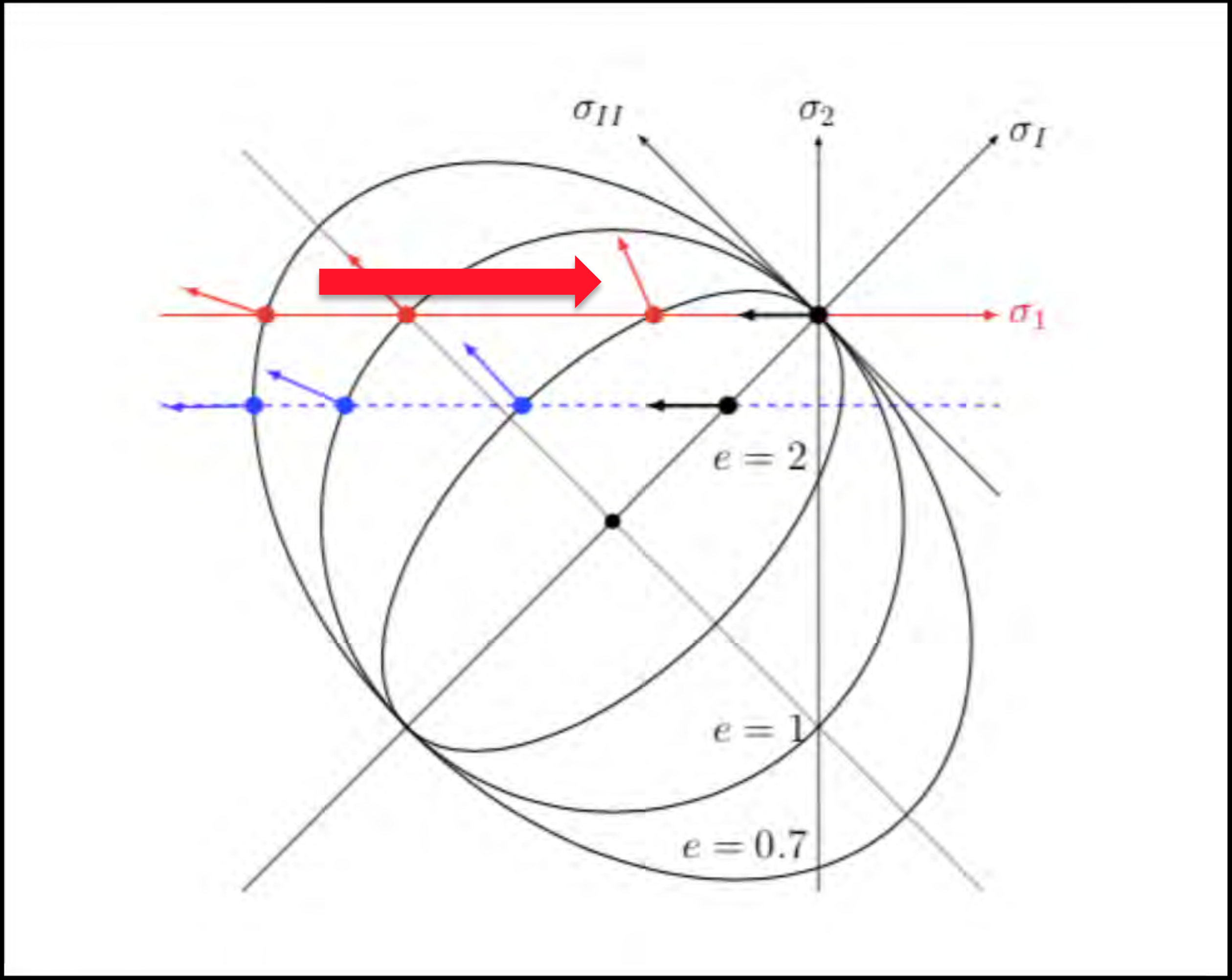
Axial Loading Test

Increased Shear Strength (e=0.7)

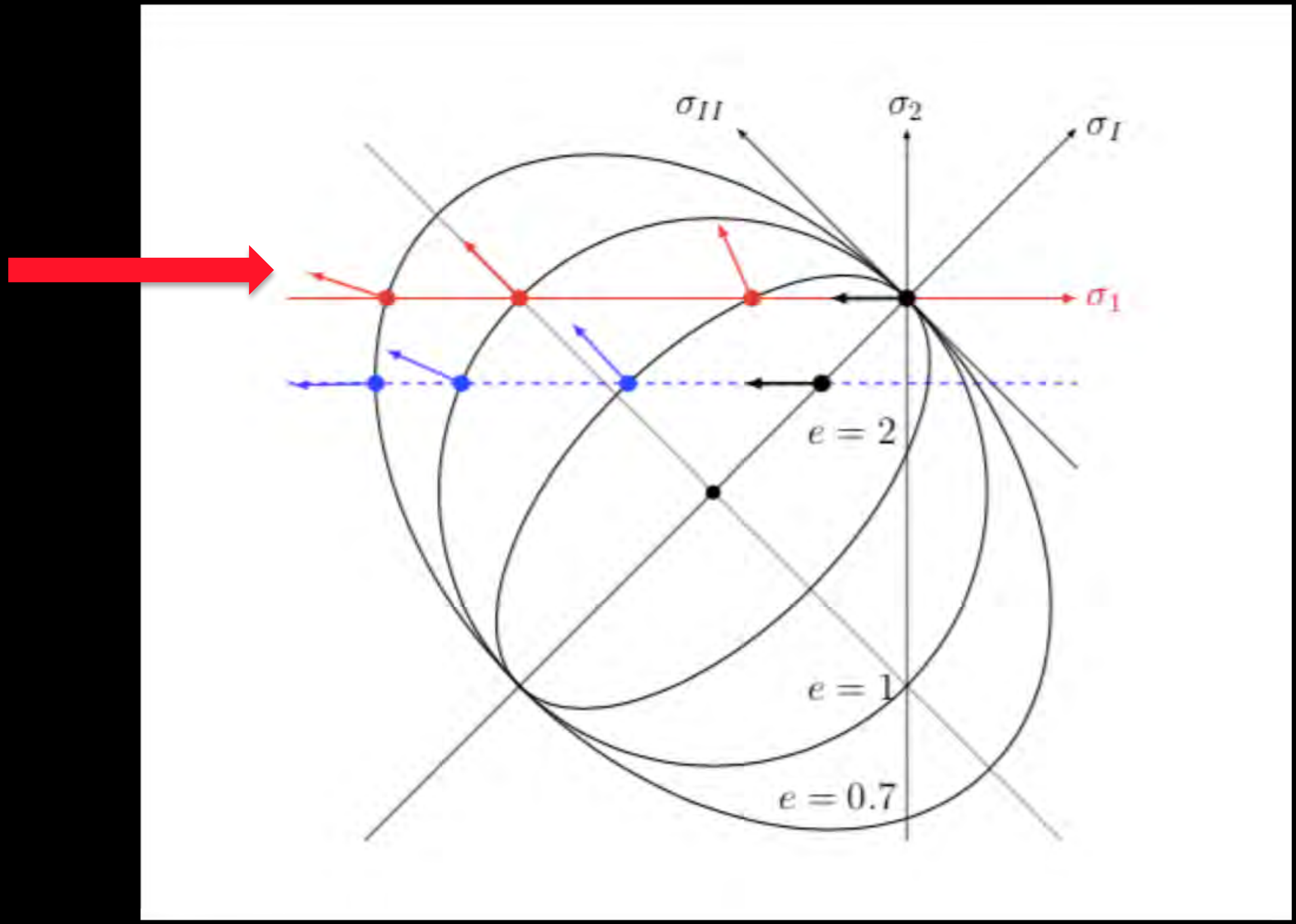


61 degrees

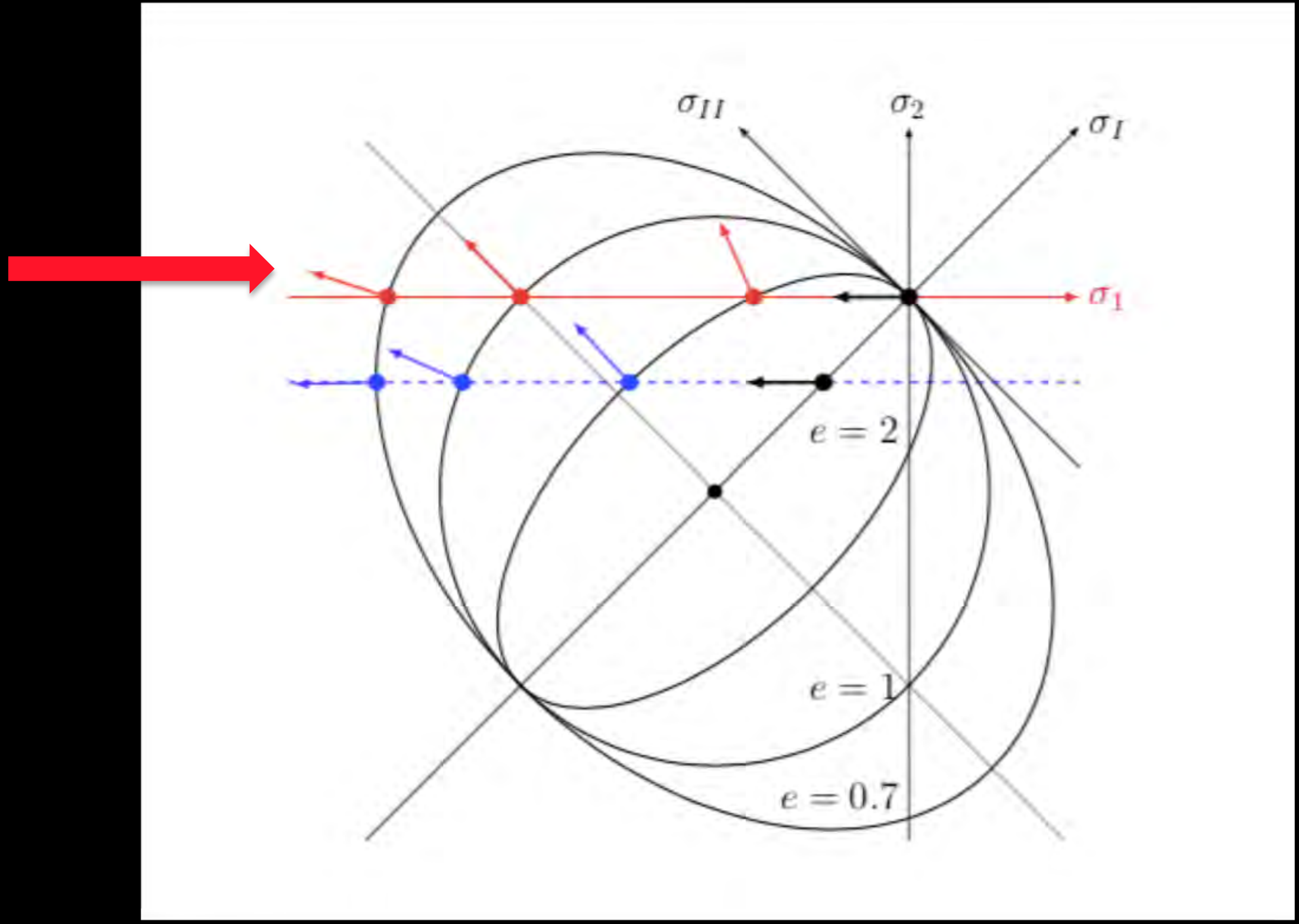
Axial Loading Test



Axial Loading Test

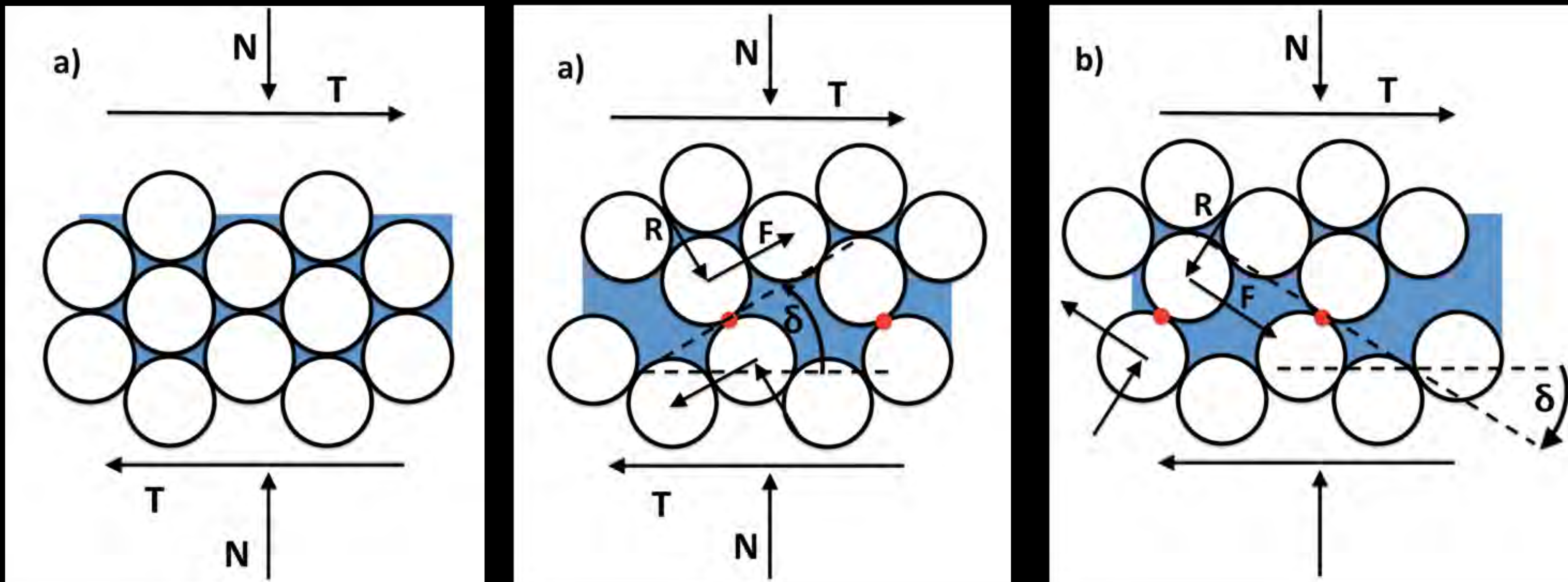


Axial Loading Test

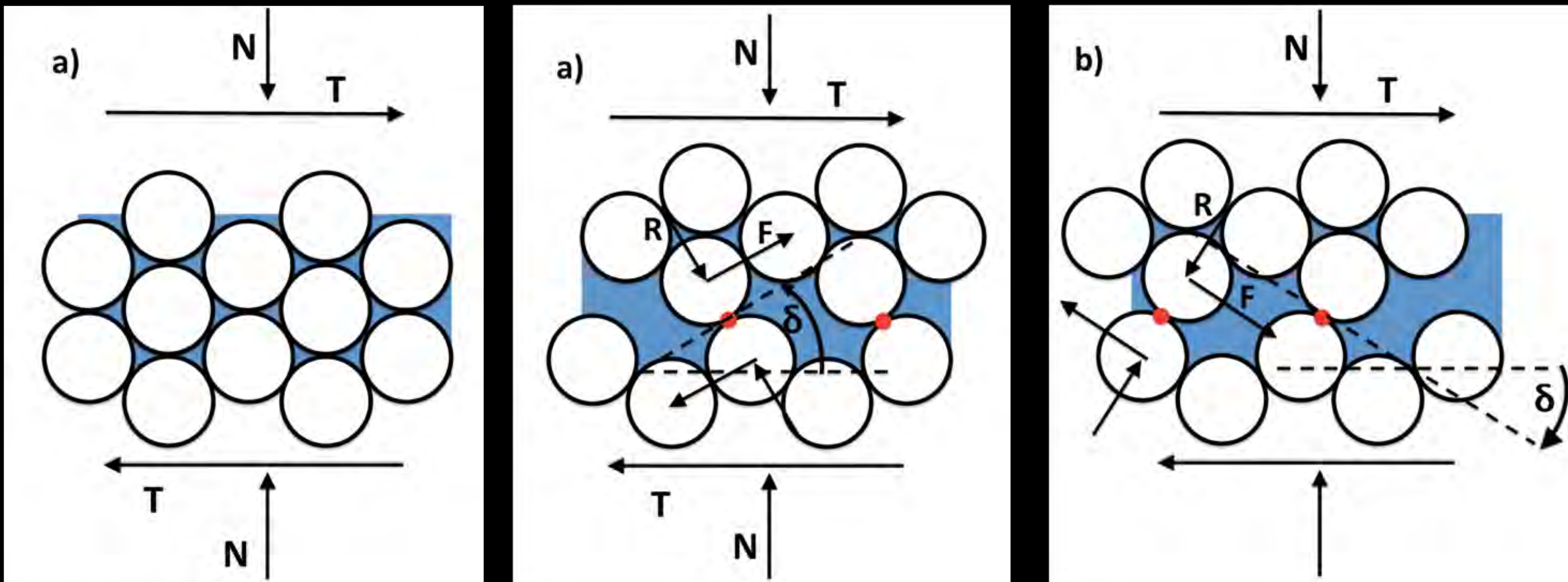


Divergence along ice fracture is set by the shear strength of the ice

Shear resistance -- Dilatation

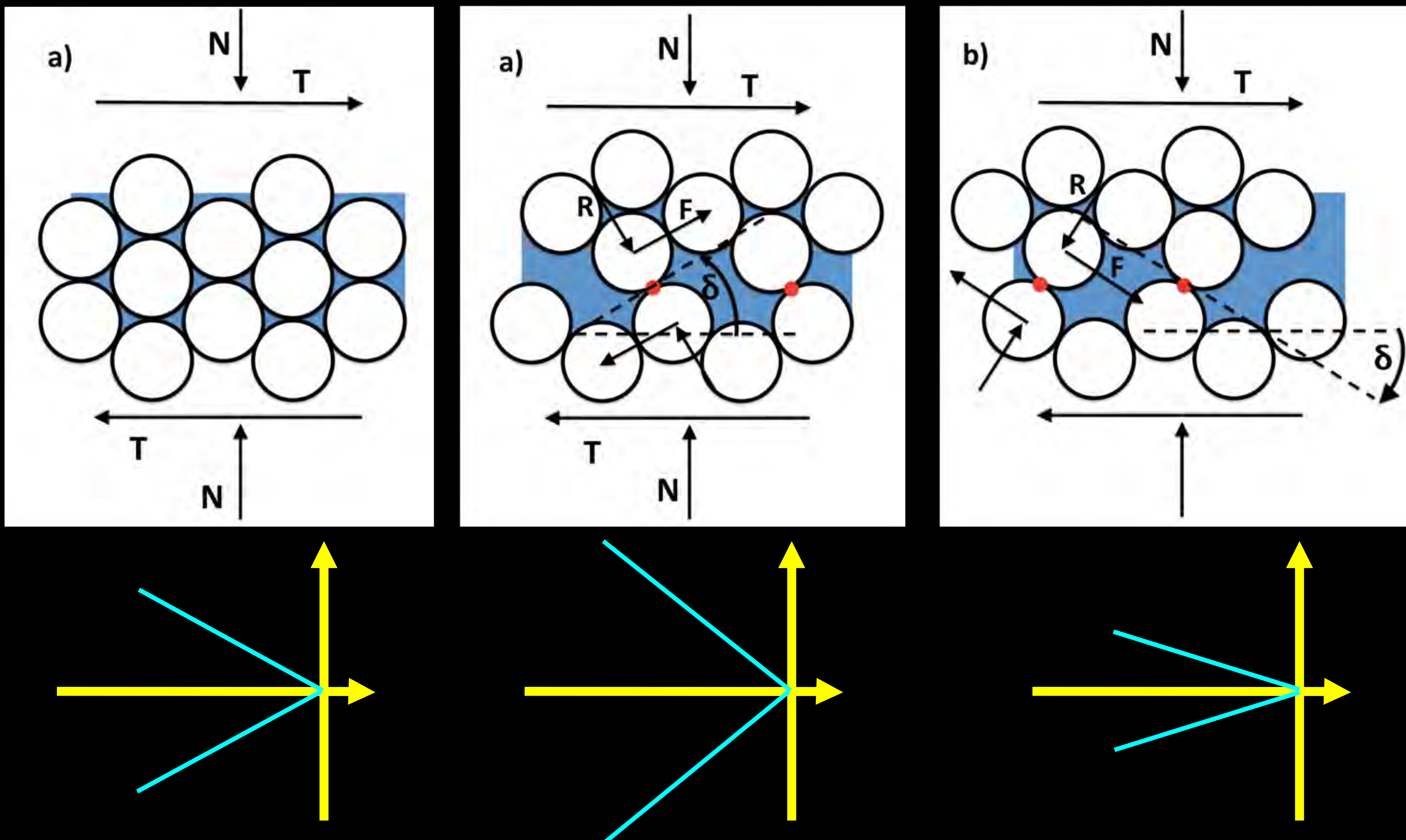


Shear resistance -- Dilatation



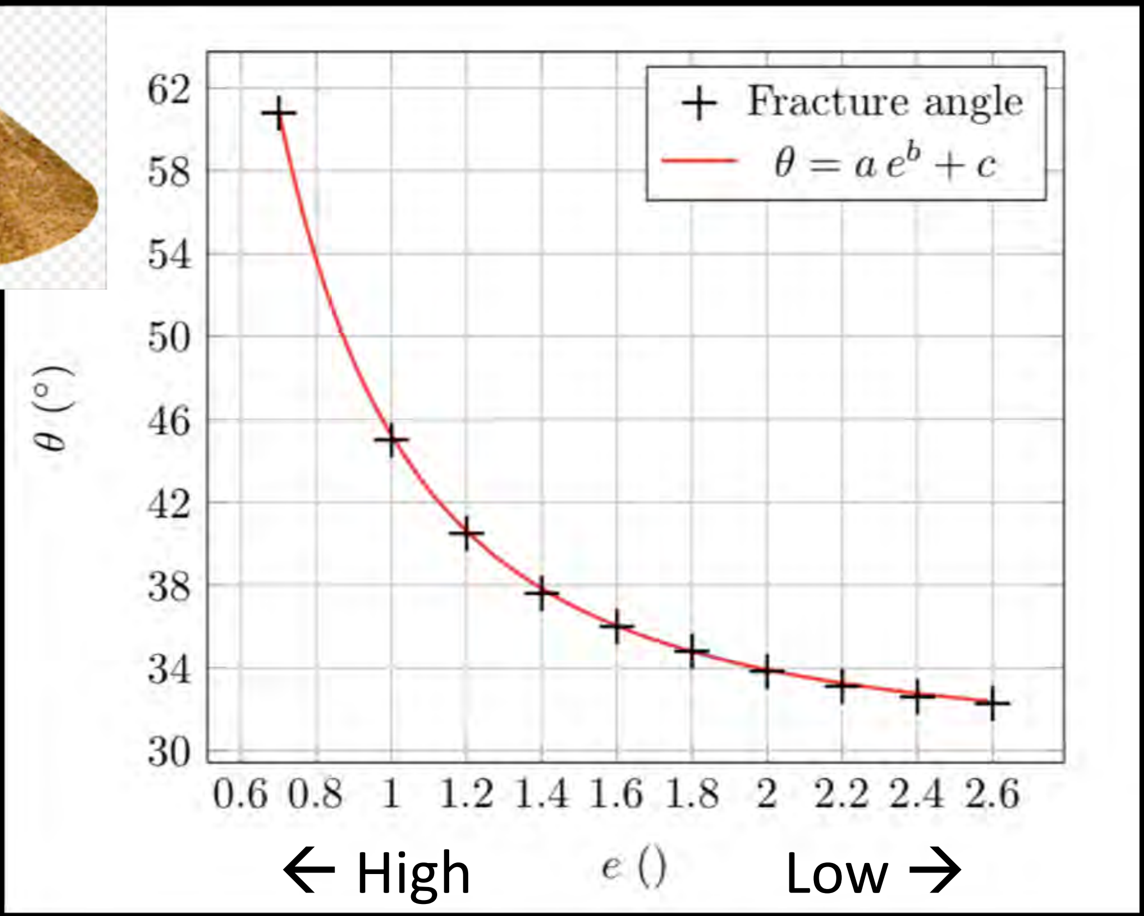
In a granular material:
high shear resistance leads to divergence
and
low shear resistance leads to convergence

Shear resistance -- Dilatation



Angle of Fracture vs Shear Strngth

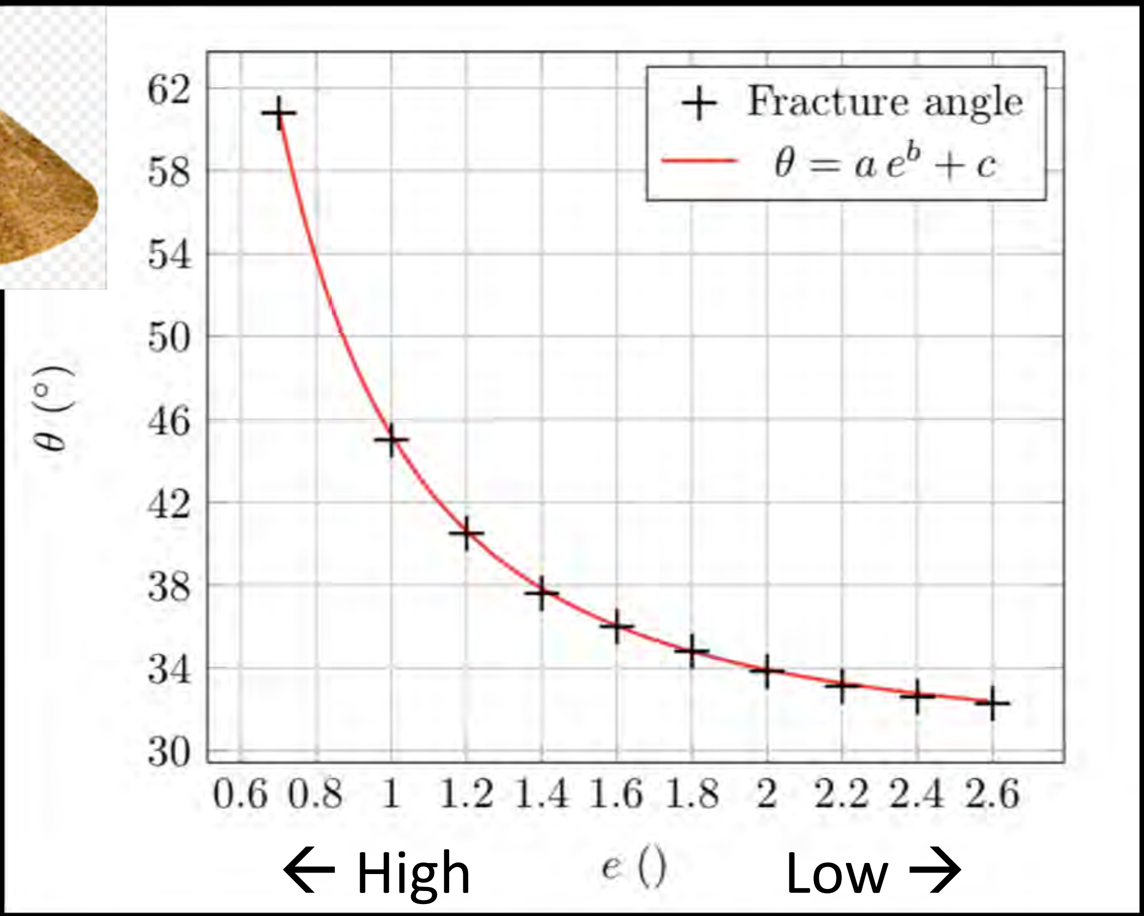
Angle larger than observations for all shear strength



Shear Strength

Angle of Fracture vs Shear Strngth

Angle larger than observations for all shear strength

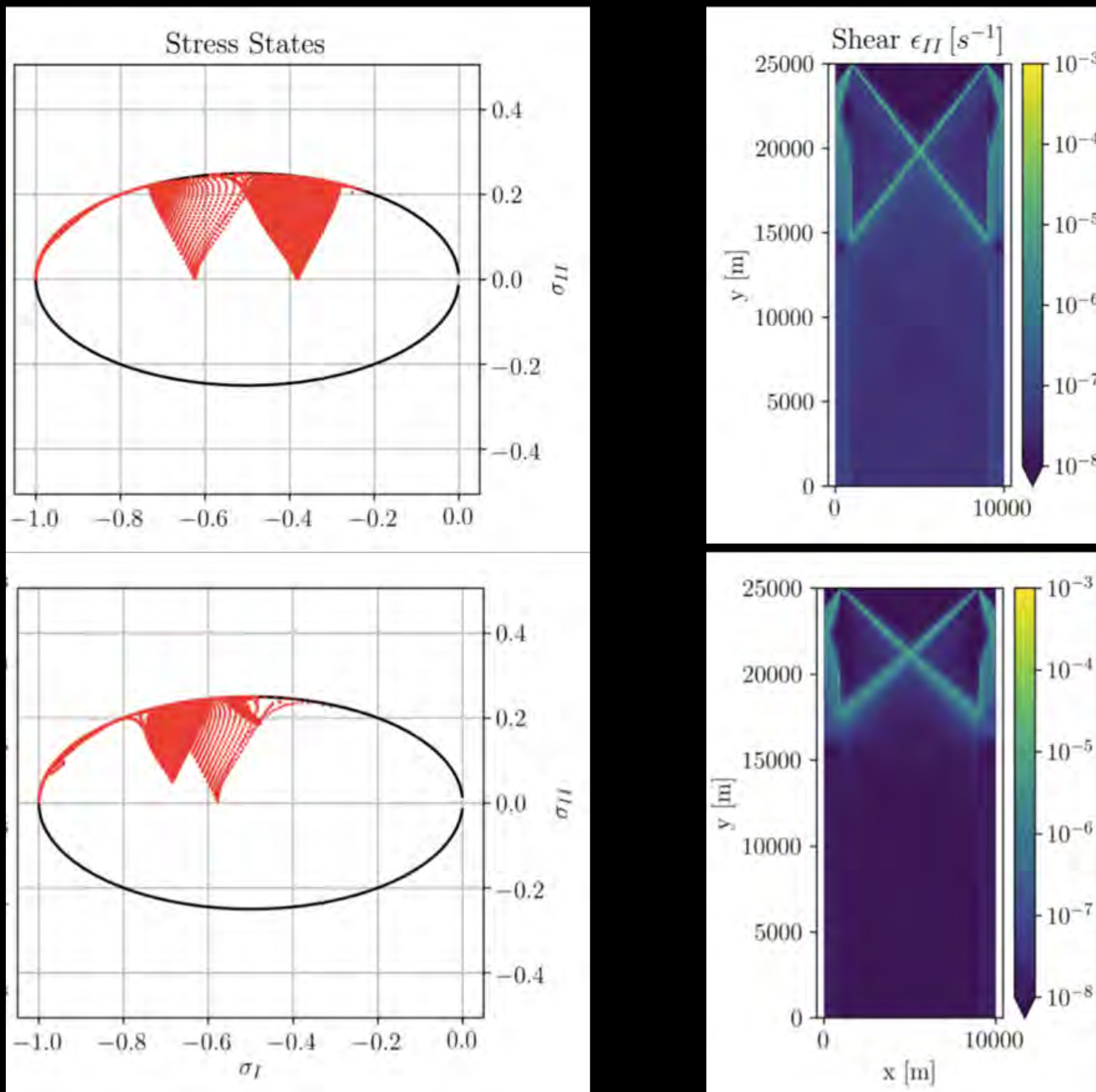


Shear Strength

Again contrary to the behavior of a granular material

Axial Loading Test

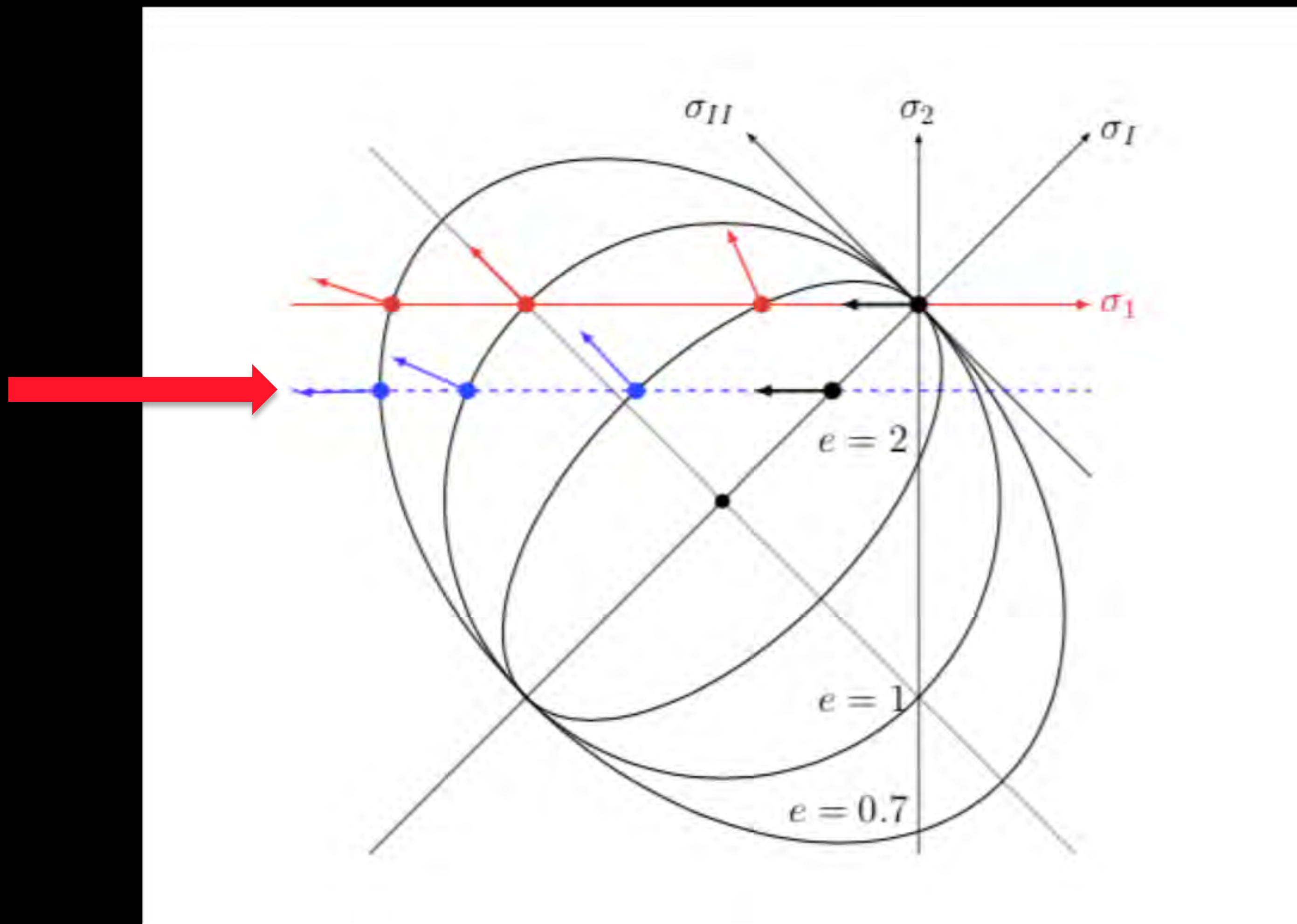
Confining Pressure



Low confining pressure

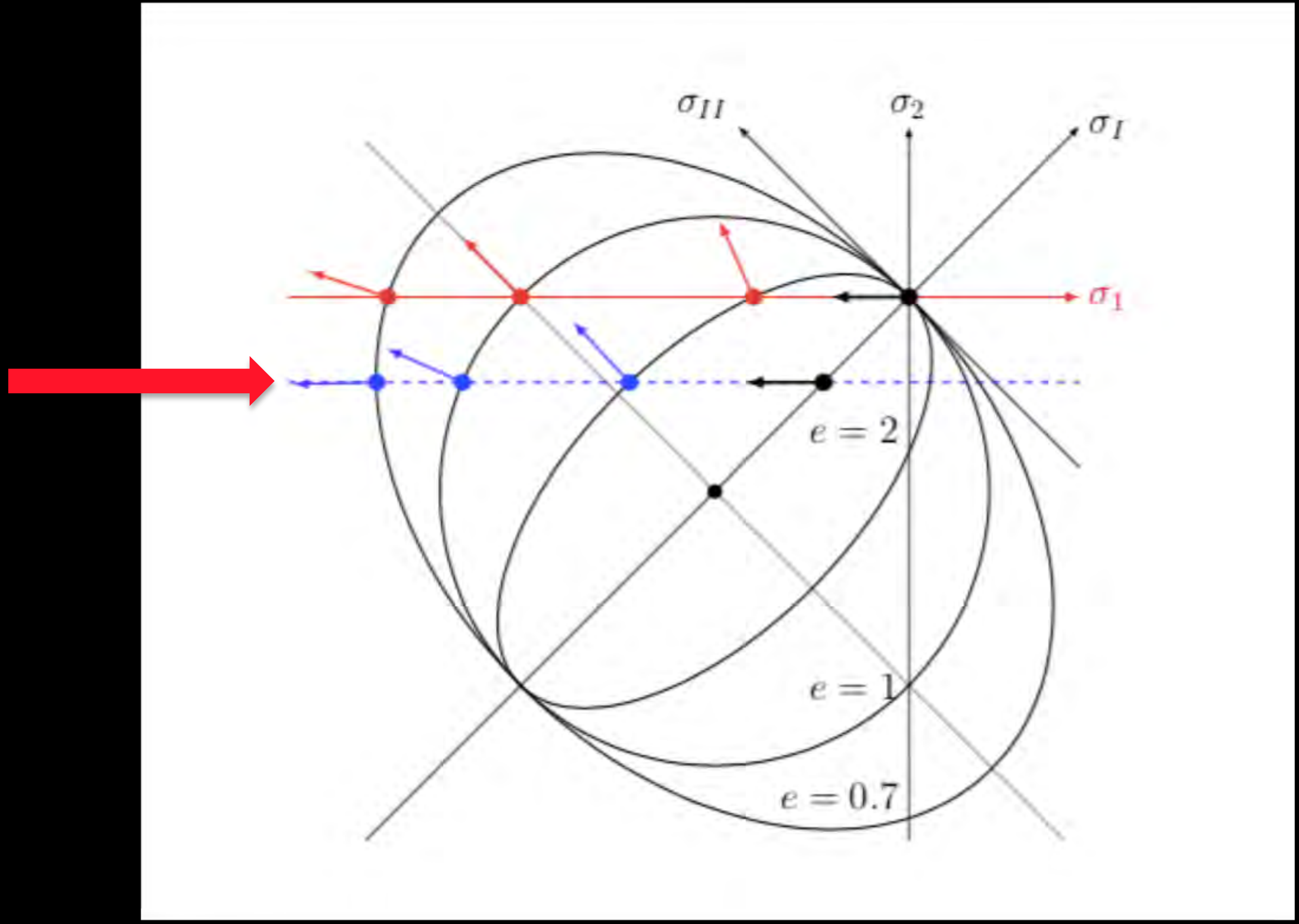
High confining pressure

Axial Loading Test



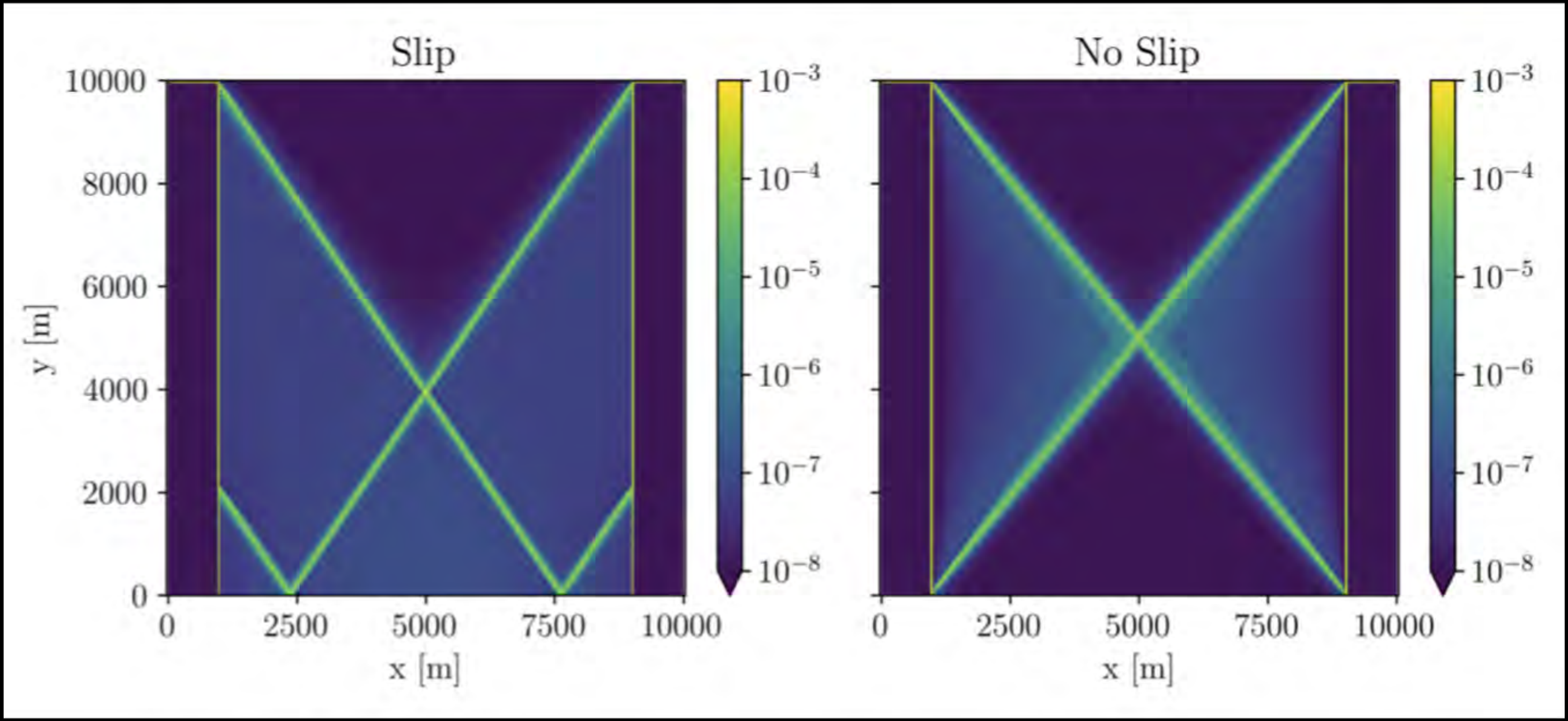
Divergence and angle of fracture depends on the confining pressure

Axial Loading Test



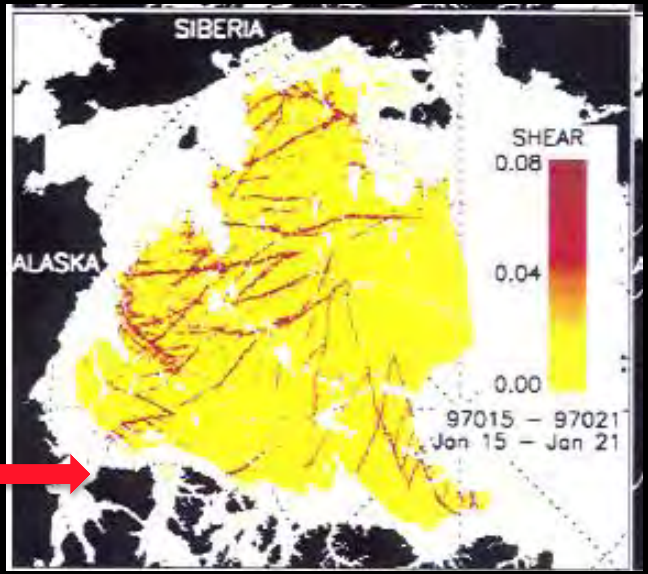
Again contrary to a granular material

Boundary Conditions



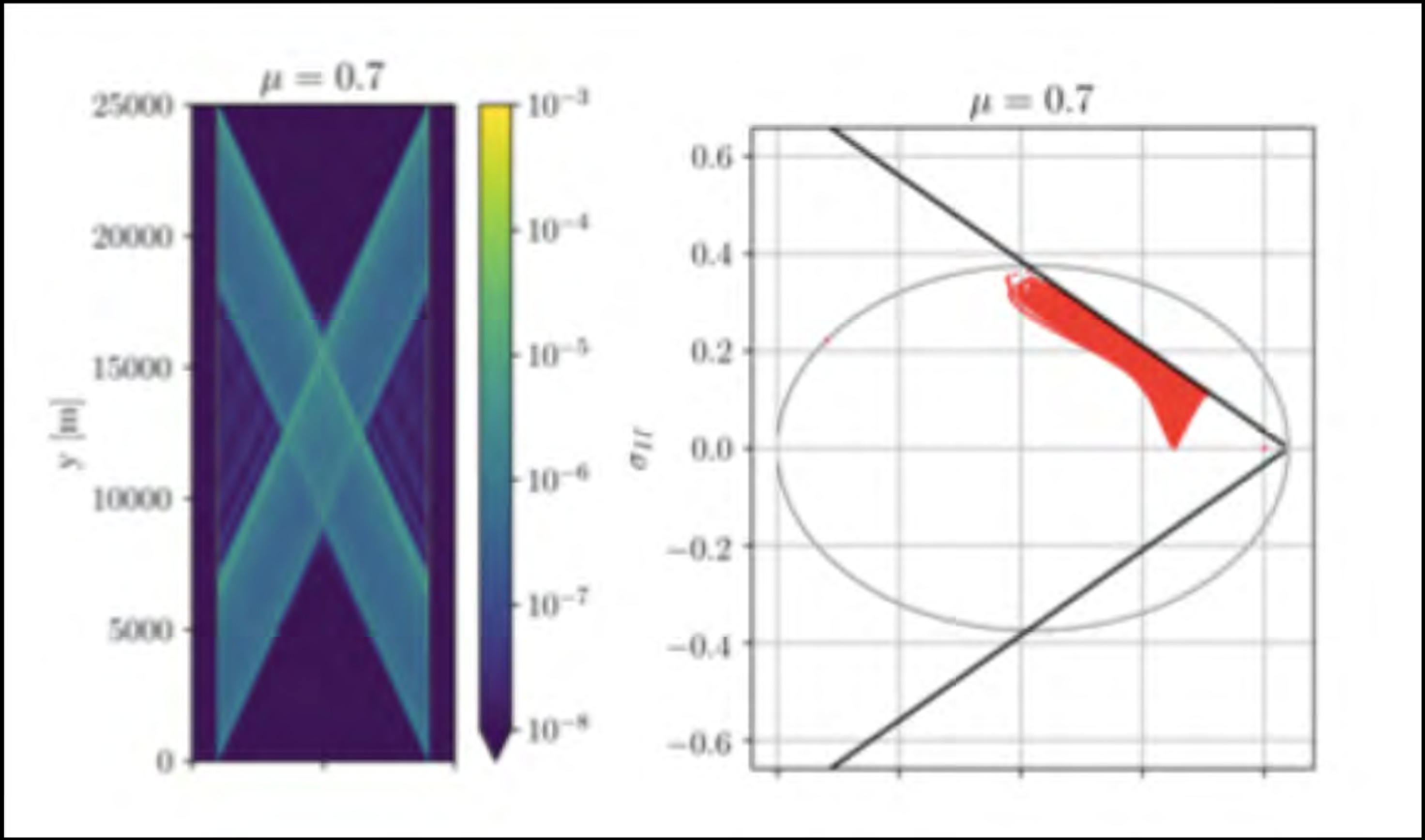
This re-opens the question of boundary conditions

- Currently we use no-slip u=v=0
- A more natural boundary condition is set on the stresses:
  - open boundary: stress free
  - Closed boundary: stress from yield curve



Axial Loading Test

Mohr-Coulomb



Angle of fracture is 23 deg in line with theory and RGPS observations

Conclusions

- The dependence of the fracture angle on shear strength is contrary to that of a granular material.
- The divergence along ice fracture is set by the shear strength contrary to granular material where shear strength and dilatation (divergence) evolve in time and are a function of the distribution of contact normals.
- The angle of fracture depends on the confining pressure contrary to granular material.
- The angle of fracture depends on the choice of boundary condition.

Way Forward

- Dissipation of energy with a Mohr-Coulomb rheology
- What is the implied flow rule in the when plastic deformation are simulated as elastic deformation with reduced elastic stiffness
- What is the fracture pattern simulated by the plastic constitute relation in the limit where eta and zeta reach their maximum values

Constitutive Relations

Viscous Plastic

- Elastic deformation are simulated as highly viscous (ideal plastic plastic)
- Plastic constitutive relations.
- Macroscopic angle of friction, dilatation and shear strength are related and time evolving.

Elasto Brittle

- Elastic constitutive relations.
- Plastic deformation are simulated as elastic deformation with a small Elastic Modulus (or spring constant)

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Discretization (FEM or FD), advection scheme (Lagrangian or Eulerian), dilatation, variable macroscopic angle of friction are not model specific

Constitutive Relations

3 x 10^{12} kg/s ~2 x 10^{12} kg/s

$$\sigma_{ij} = \boxed{2\eta}\dot{\epsilon}_{ij} + \left[\boxed{(\zeta - \eta)}\dot{\epsilon}_{kk} - \cancel{\frac{P}{2}}\right]\delta_{ij},$$

Viscous Plastic

$$\sigma_{ij} = \boxed{2G}\epsilon_{ij} + \boxed{\lambda\delta_{ij}}\epsilon_{kk}$$

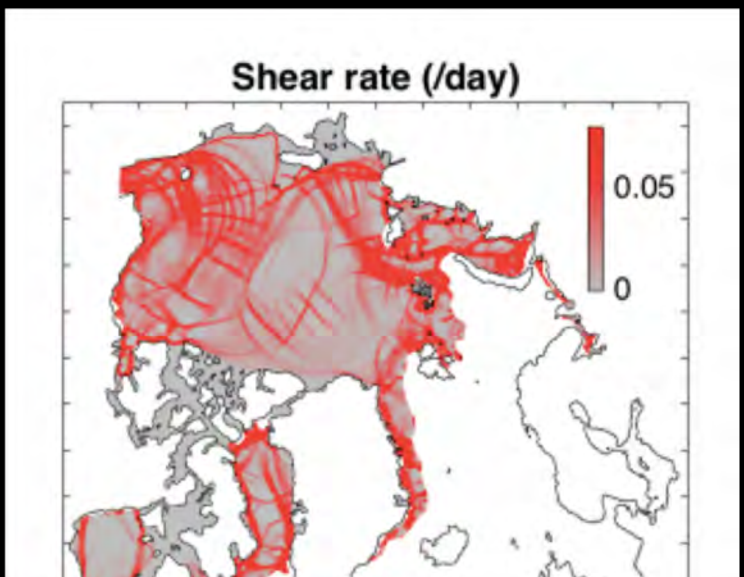
Elasto Brittle

8 x 10^7 kg/s^2

characteristic time of ~6 hours (~2 x 10^5 sec)

VP vs neXtSIM

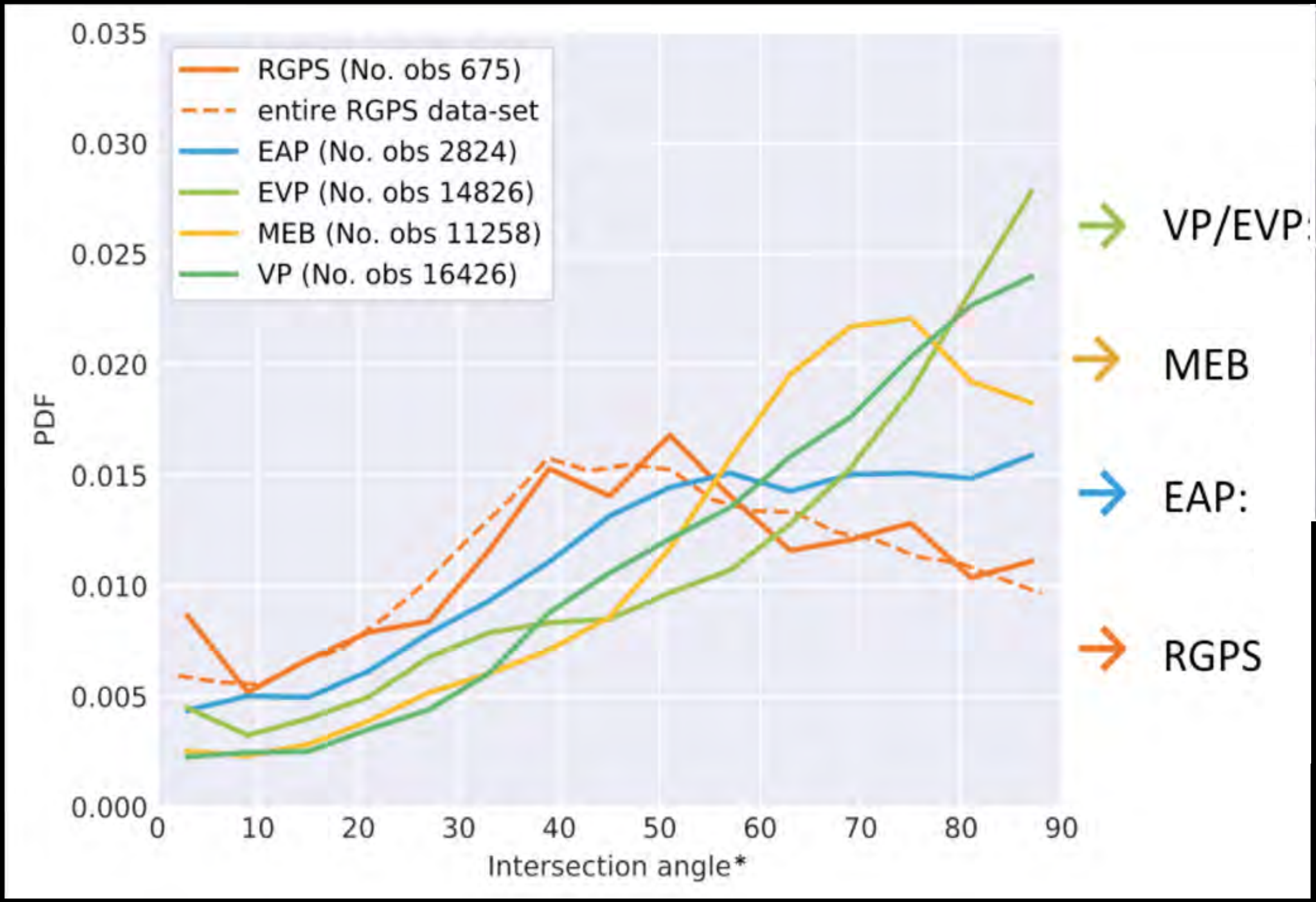
VP, 1km, 2 OL, Hutter

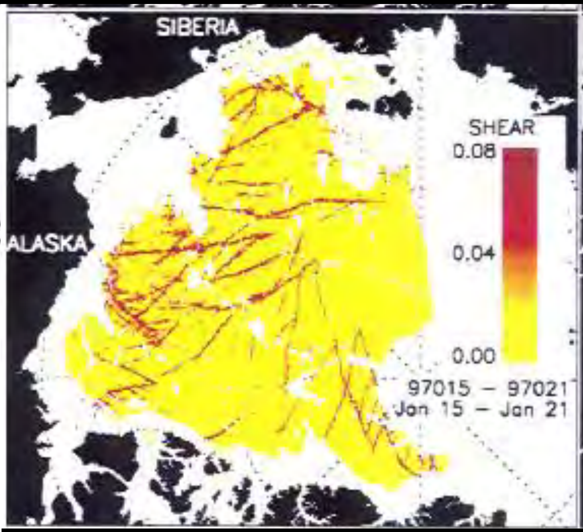


neXtSIM (EB), Rampal et al. 2016

LKFs appear to be a fundamental property of plasticity theory

Intersection angle of Fracture Lines





Range of intersection angle in line with a MC yield curve with Dilatation and variable macroscopic angle of friction

Hutter et al., 2018

Time scale associated with Viscous Deformation

Viscous time scale: 10^{-4} day^{-1} (Hibler, 1979)

→ ~30 years

Characteristic time scale for LKFs: hours or day

Outlook

- Important to continue to use multiple approach. We should encourage diversity rather than converging towards a single approach.
- It generates interesting questions and furthers our understanding all approaches used.

Outlook

- Use a sea ice rheology that includes all behavior of sea ice: elastic, viscous and plastic.
- Consider both thermal and mechanical stresses inside ice.
- Develop yield criteria based on the vertically dependent internal stress
- Move to Discrete Element Model and Lagrangian advection scheme