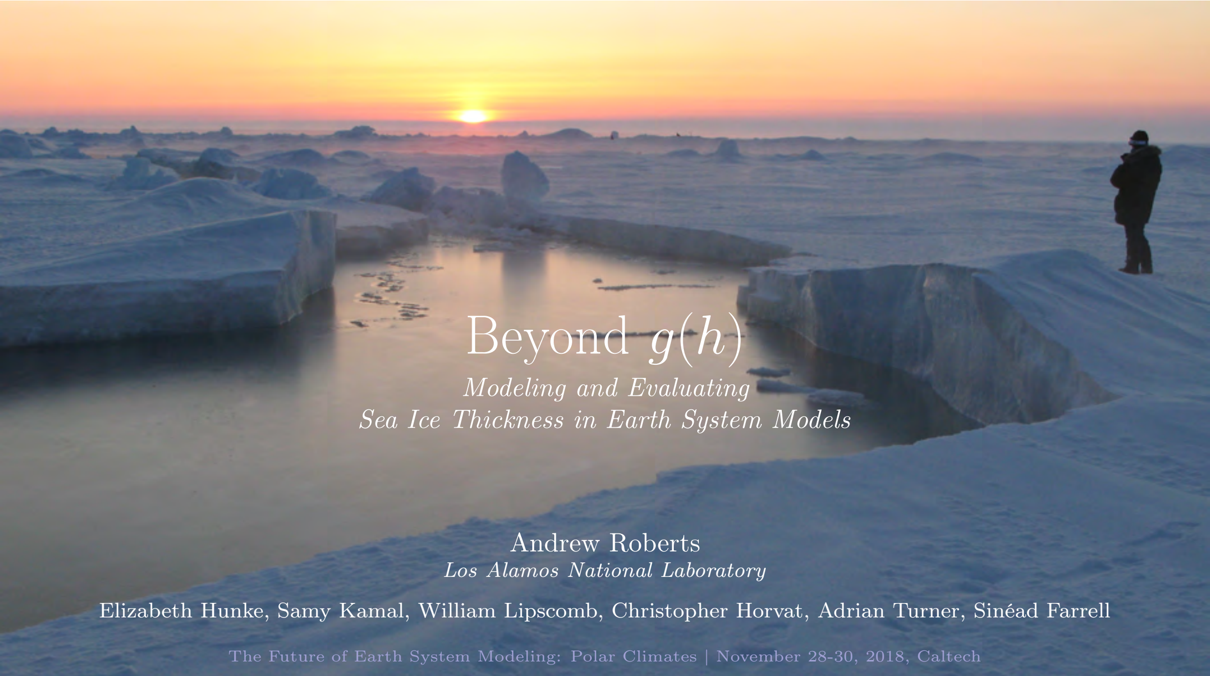

Beyond $g(h)$

Modeling and Evaluating
Sea Ice Thickness in Earth System Models

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Los Alamos National Laboratory

Elizabeth Hunke, Samy Kamal, William Lipscomb, Christopher Horvat, Adrian Turner, Sinéad Farrell

The Future of Earth System Modeling: Polar Climates | November 28-30, 2018, Caltech

Outline

1. How should we design sea ice models to obtain better predictions of polar climate?
2. How can we better integrate observations with models?
3. What additional observations would help improve models?

Outline

1. How should we design coupled sea ice models to obtain better predictions of sea ice thickness, concentration and drift?
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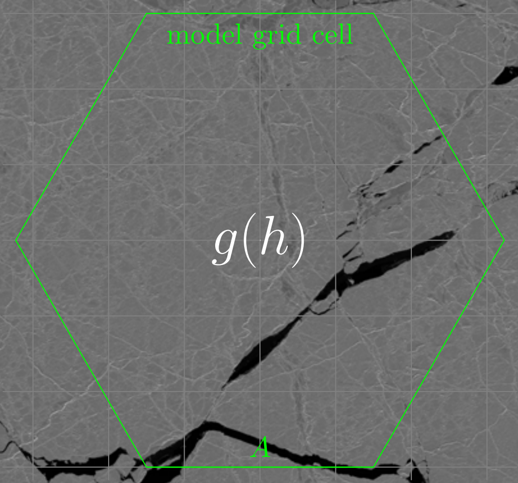
Outline

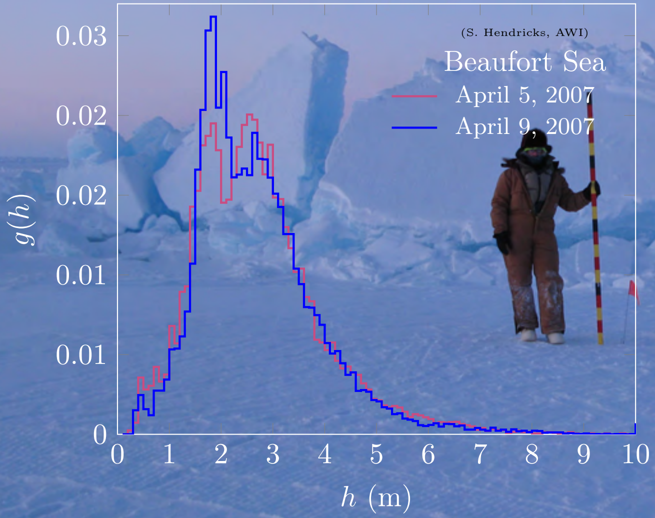
1. How can we better integrate thickness observations with models?
2. What additional thickness observations would help improve models?
3. How should we design coupled sea ice models to obtain better predictions of sea ice thickness, concentration and drift?

Brief Introduction to the Thickness Distribution $g(h)$



USGS Global Fiducials Library

Sea Ice Thickness Distribution



$$m = \rho \int_0^\infty g(h)\, h\, dh$$

$g(h)$ is used to describe mass conservation in sea ice models:

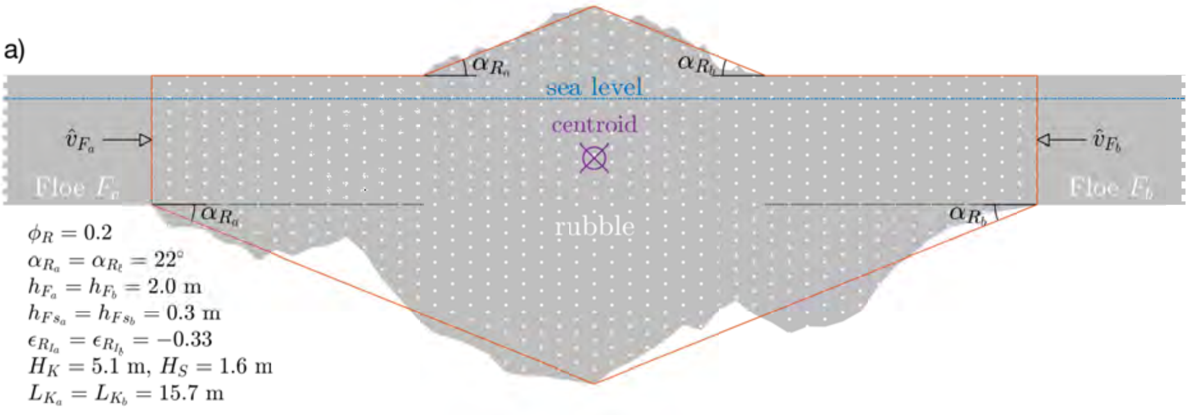
$$\frac{dg}{dt} = \Psi + \Theta - g(\nabla \cdot \dot{\boldsymbol{x}})$$

Ψ Dynamic Redistribution,
Θ Thermodynamic Redistribution

Introduction to $g(h)$

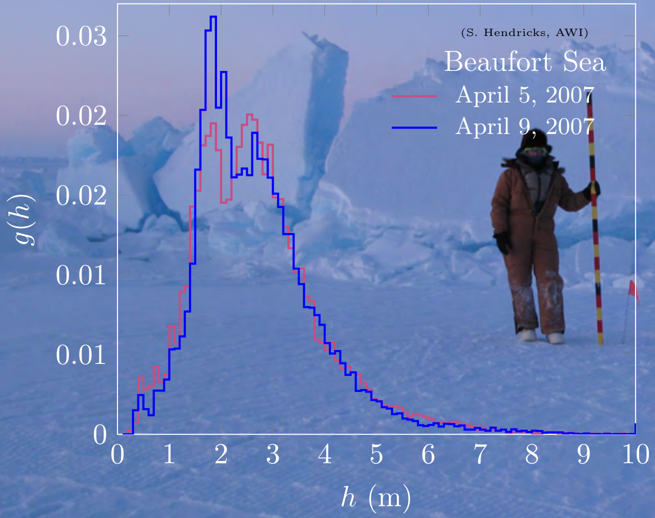
Beaufort Sea 2007

Ridge Cross Section



Roberts, A. F., E. C. Hunke, S. M. Kamal, W. H. Lipscomb, C. Horvat, W. Maslowski (2018): A Variational Method for Sea Ice Ridging in Earth System Models, in review for J. Adv. Model. Earth Sy.

Sea Ice Thickness Distribution



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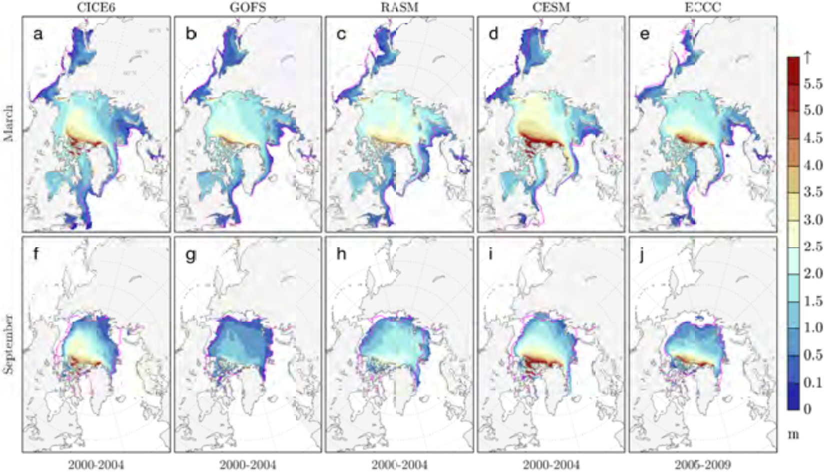
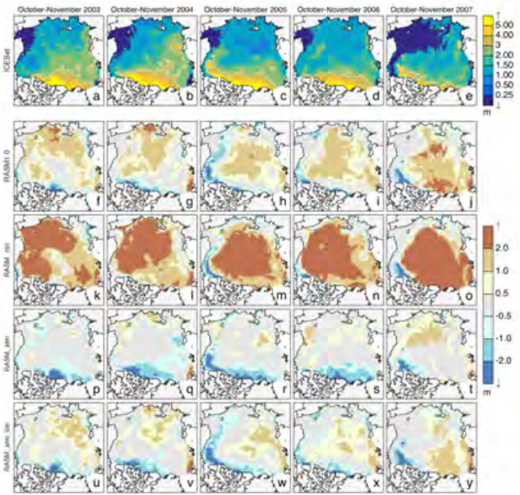
Introduction to $g(h)$



Roberts, A. F., Hunke, E. C., Allard, R., Bailey, D. A., Craig, A. P., Lemieux, J., Turner, M. D. (2018). Quality control for community-based sea-ice model development. *Philos. Trans. Royal Soc. A*, 376, 17. doi:10.1098/rsta.2017.0344

1. How can we better integrate thickness observations with models?

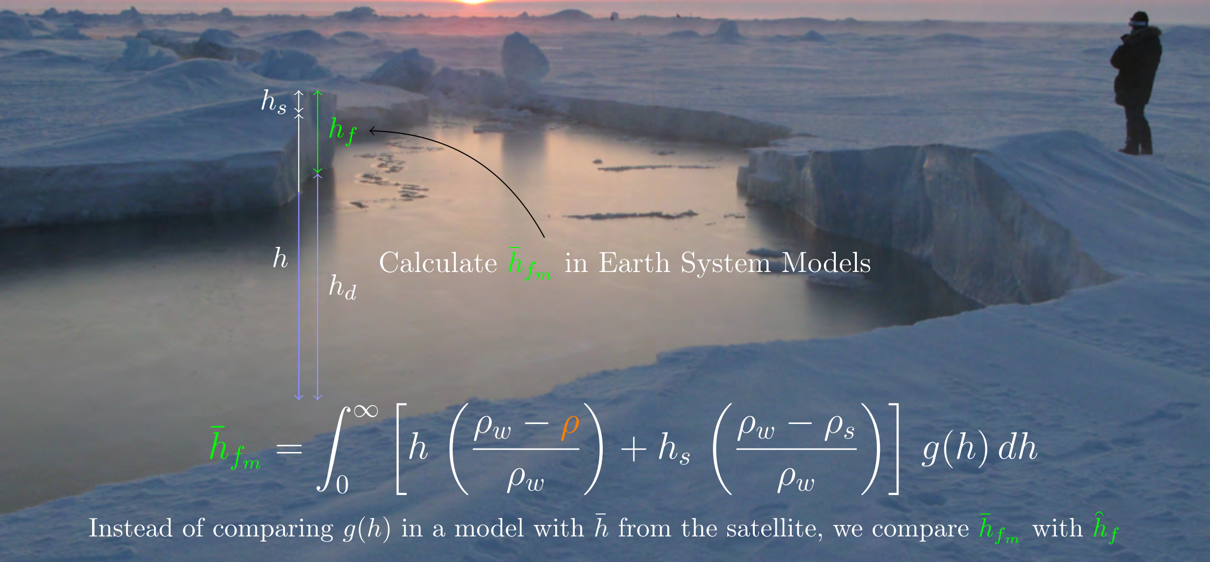


- Thickness (h) is an important variable to evaluate well, because it carries the dynamic and thermodynamic history of the pack.
- A robust method for evaluating modeled thickness has been elusive, partly due to uncertainty in observed snow thickness (h_s) and sea ice density (ρ)
- The method used opposite is not the answer to our problems.

Cassano, J. J., DuVivier, A., Roberts, A., Hughes, M., Seefeldt, M., Brunke, M., et al. (2017). Development of the Regional Arctic System Model (RASM): Near-surface atmospheric climate sensitivity. *J. Climate*, 30(15). doi:10.1175/JCLI-D-15-0775.1

1. How can we better integrate thickness observations with models?

Answer: Evaluate and train against freeboard, not thickness



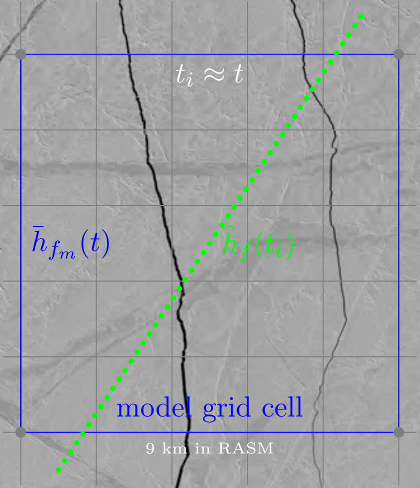
Calculate $\bar{h}_{f_m}$ in Earth System Models

$$\bar{h}_{f_m} = \int_0^\infty \left[h \left(\frac{\rho_w - \rho}{\rho_w} \right) + h_s \left(\frac{\rho_w - \rho_s}{\rho_w} \right) \right] g(h)\, dh$$

Instead of comparing $g(h)$ in a model with $\bar{h}$ from the satellite, we compare $\bar{h}_{f_m}$ with $\hat{h}_f$

1. How can we better integrate thickness observations with models?

We can evaluate: $\mathrm{Bias}[\bar{h}_{f_m}(t)] \approx \bar{h}_{f_m}(t) - \frac{1}{n}\sum_{i=1}^{n} \hat{h}_f(t_i)$



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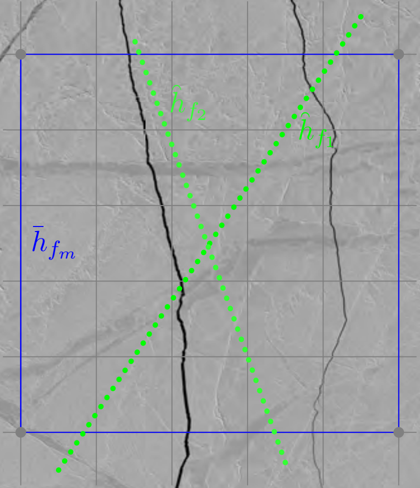

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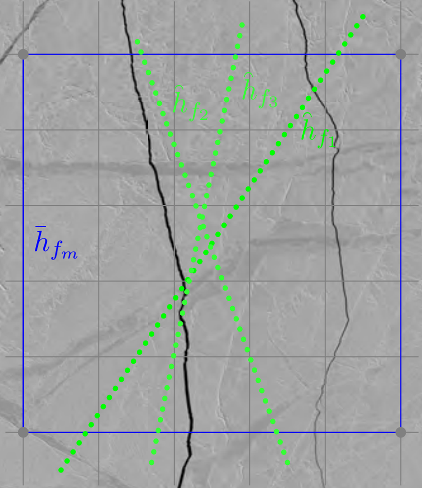
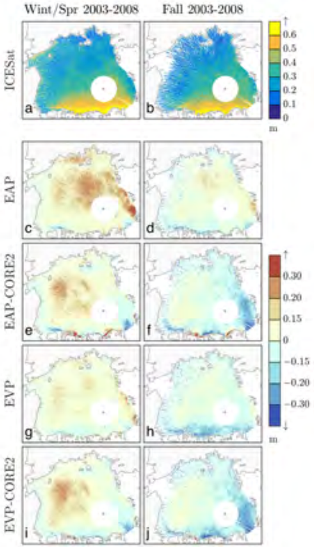
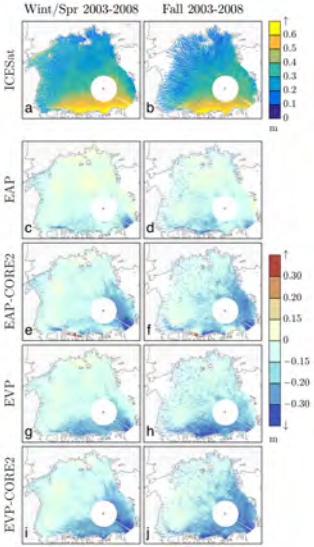


Evaluation of the Regional Arctic System Model (RASM) Freeboard

$\mathrm{Bias}[\bar{h}_{f_m}]$



$\mathrm{Bias}[\bar{h}_{f_m}, h_s \to 0]$



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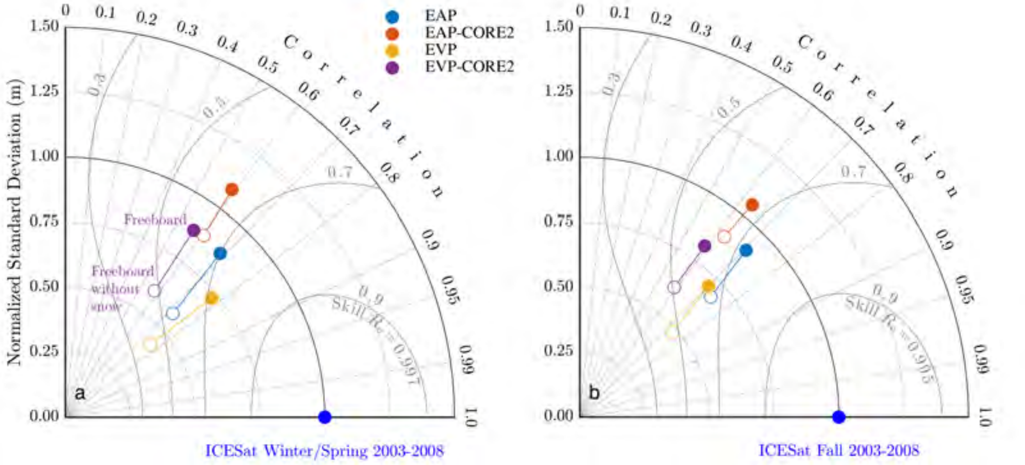
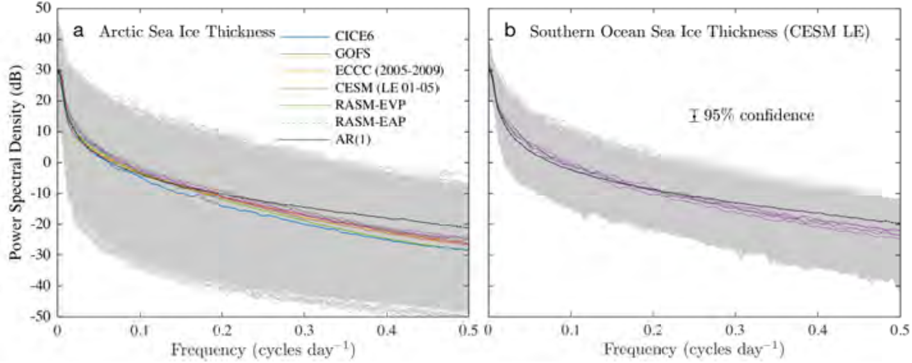


Details of the skill score in:
Roberts, A. F., Hunke, E. C., Allard, R., Bailey, D. A., Craig, A. P., Lemieux, J., Turner, M. D. (2018). Quality control for community-based sea-ice model development. *Philos. Trans. Royal Soc. A*, 376, 17. doi:10.1098/rsta.2017.0344

2. What additional freeboard observations would help improve models?

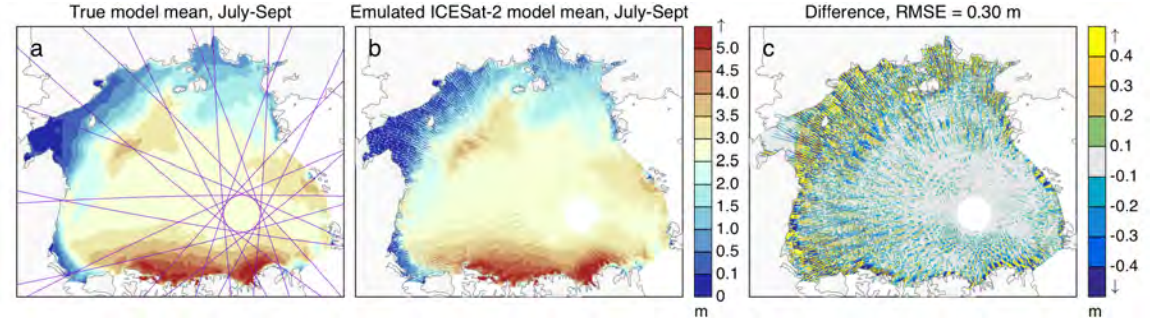


$$\text{AR(1) series: } h_i = 0.994\, h_{i-1} + \varepsilon_i$$

Due to the high degree of autocorrelation in sea ice thickness time series, we need regular seasonal measurements of sea ice freeboard taken at the top of the snow layer, just as provided by ICESat and to be provided by ICESat-2 with a 91-day repeat orbit.

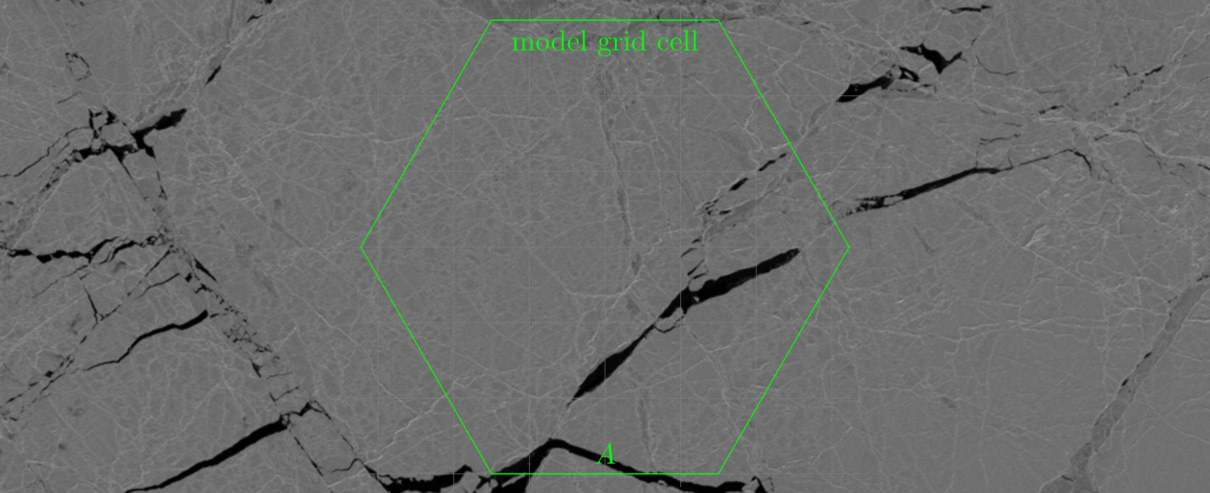
2. What additional freeboard observations would help improve models?

Answer: Seasonal *absolute* freeboard for as long as possible



ICESat, ICESat-2, ..., ICESat-N

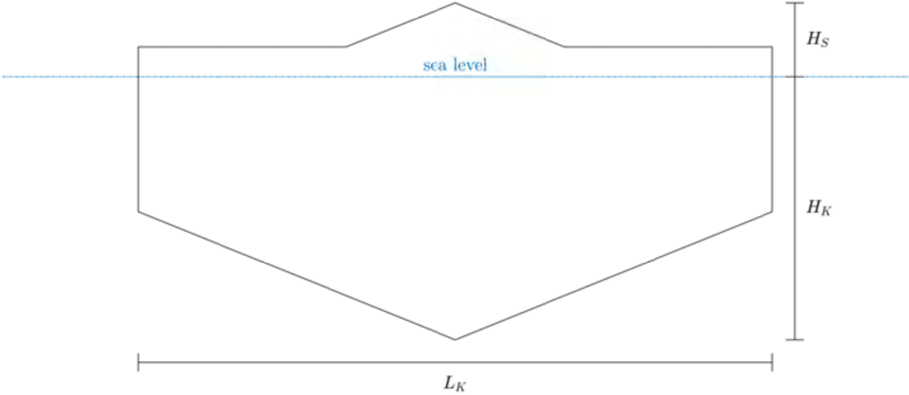
3. How should we design models to better predict sea ice thickness?



Roberts, A.F. (2018). Audiovisual Vignettes of Sea Ice Ridging in the Beaufort Sea in 2007. Zenodo.
Download the 8-minute movie: doi:10.5281/zenodo.1252414

3. How should we design models to better predict sea ice thickness?

Information about the local organization of ridges is missing from $g(h)$



3. How should we design models to better predict sea ice thickness?

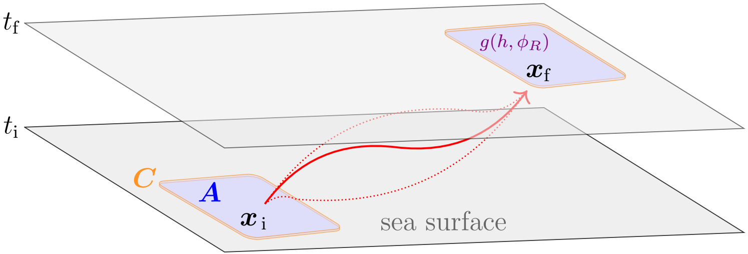
Information about the local organization of ridges is missing from $g(h)$

Model Element	Variable or Component	
Compressive Strength	Internal Stress	σ
Scale Aware Density	Mass per unit area	m
Form Drag	External Stress	τ_a, τ_w
Landfast Ice	Internal Stress	σ
Snow Morphology	Albedo and Enthalpy	α, ε
Melt Pond Drainage	Albedo and Enthalpy	α, ε
Biogeochemistry	Carbon Cycle	

The ‘Barrow List’ of parts of a modern sea ice model that need more than $g(h)$

‘We haven’t got any money, so we’ve got to think’
- Ernest Rutherford

The Variational Method: Building up from a ridge, not down from a grid cell



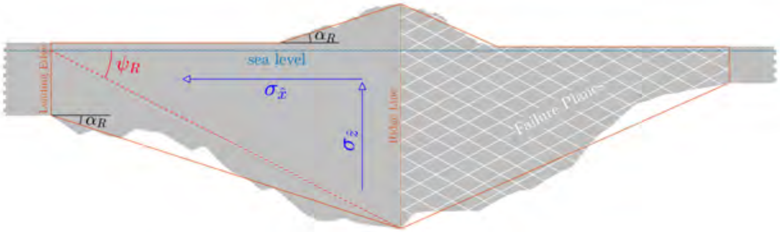
$$m \frac{d\dot{\boldsymbol{x}}}{dt} = \nabla \cdot \bar{\boldsymbol{\sigma}} + \boldsymbol{F}_b$$

$$\int_{t_i}^{t_f} \int_A \left(\nabla \cdot \bar{\boldsymbol{\sigma}} - m \frac{d\dot{\boldsymbol{x}}}{dt} \right) \cdot \delta \boldsymbol{x} \, dA \, dt = 0$$

Roberts, A. F., E. C. Hunke, S. M. Kamal, W. H. Lipscomb, C. Horvat, W. Maslowski (2018): A Variational Method for Sea Ice Ridging in Earth System Models, in review for J. Adv. Model. Earth Sy.

Step 1: Coarse-grain ridging using simple polygons

Define a relationship between horizontal stress $\bar{\boldsymbol{\sigma}}$ and potential energy density $\mathcal{V}$



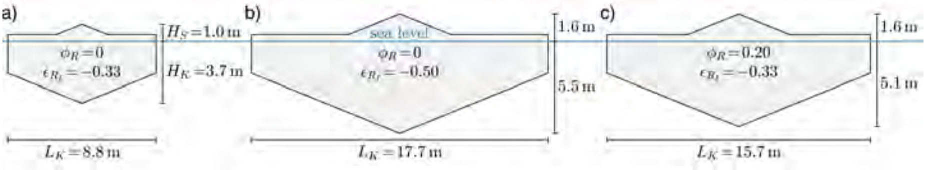
$$0 = \frac{1}{3}\frac{d}{dt}\frac{\partial \mathcal{T}}{\partial \dot{\hat{x}}} - \frac{\partial \mathcal{V}}{\partial \hat{x}}$$

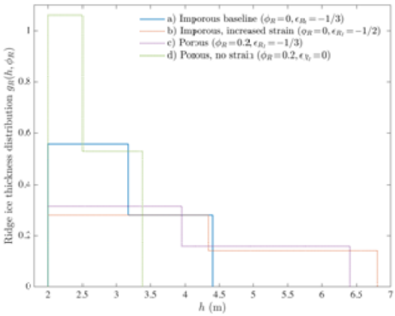
Imposing Coulombic failure, we isolate the conservative part of the system

($\mathcal{T}$ and $\mathcal{V}$ are kinetic and potential energy density, respectively)

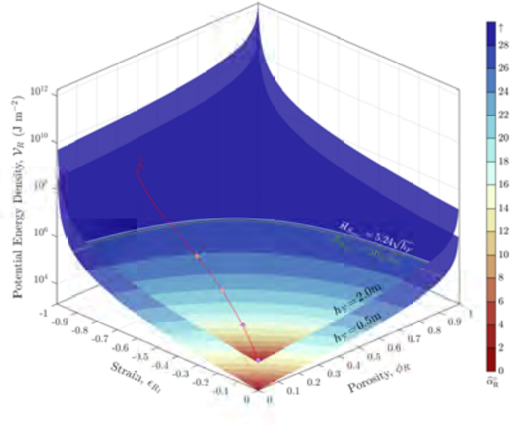
Demonstration of the influence of macroporosity

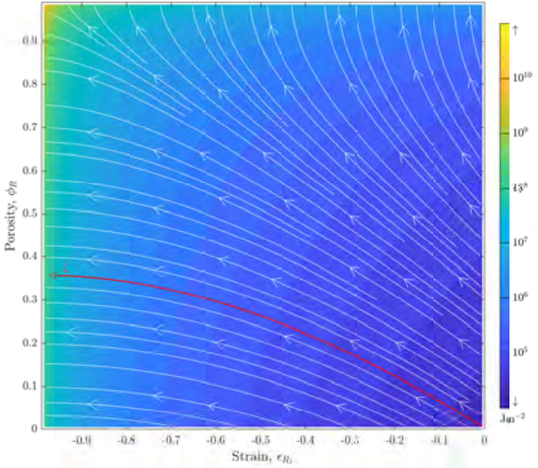
Comparative effect of strain ϵ_{R_I} and macroporosity ϕ_R





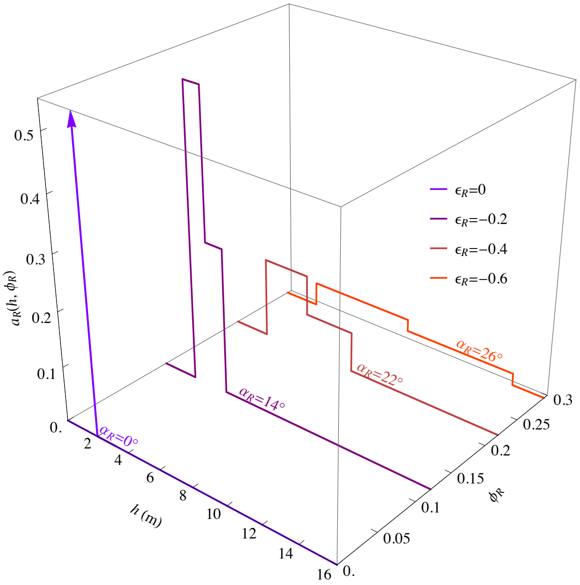
Step 2: Apply the variational constraint to the ridge, giving $\nabla_R \cdot \mathbf{đ} = 0$





We now have a solution to the initial value problem, with a state space trajectory (red).

The bivariate thickness distribution $g(h, \phi)$

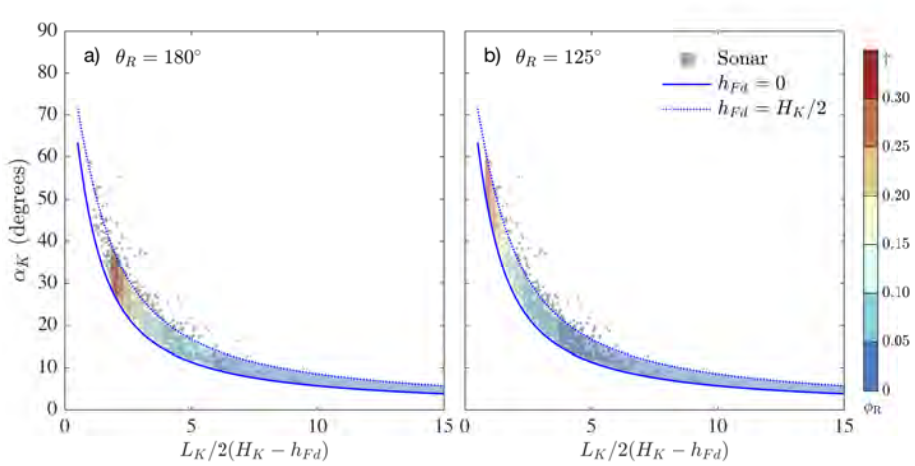


Redistribution of an individual ridge:

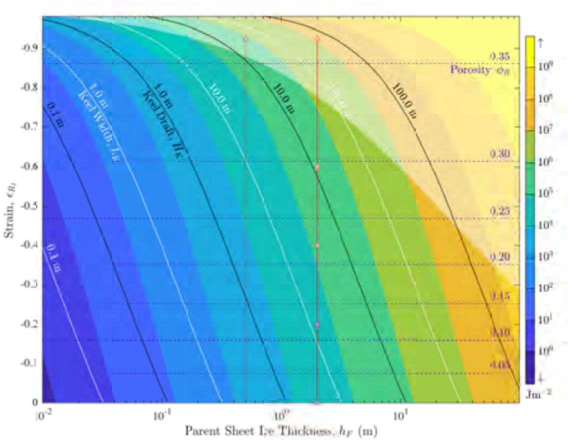
$$m = \rho \int_0^\infty \int_0^1 g(h, \phi)\ (1-\phi)\ h\ d\phi\ dh$$

Ice density ρ includes microporosity. Ridges defined by strain ϵ_R, porosity ϕ_R, and angle of repose α_R.

Shear and Porosity



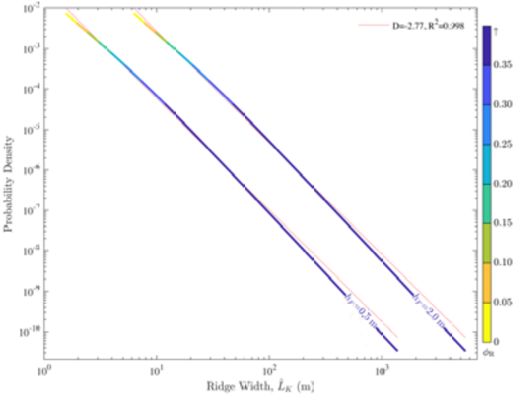
Step 3: Derive the resulting ridge frequency statistics



The $\hat{\zeta}$-plane, along the initial value state space trajectory on the previous slide.

Consequently, stationarity applies to collections of ridges

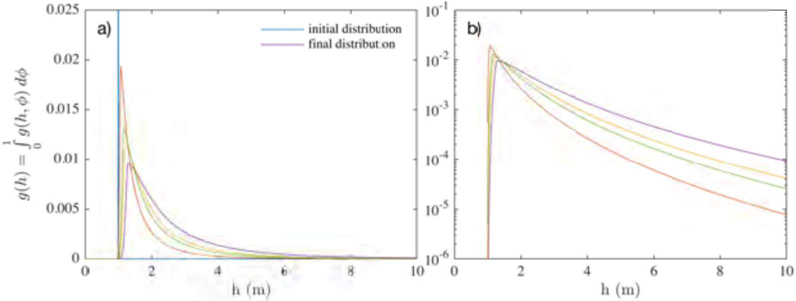
$$0 = \delta \iint_A \mathcal{V}_R \, dA$$



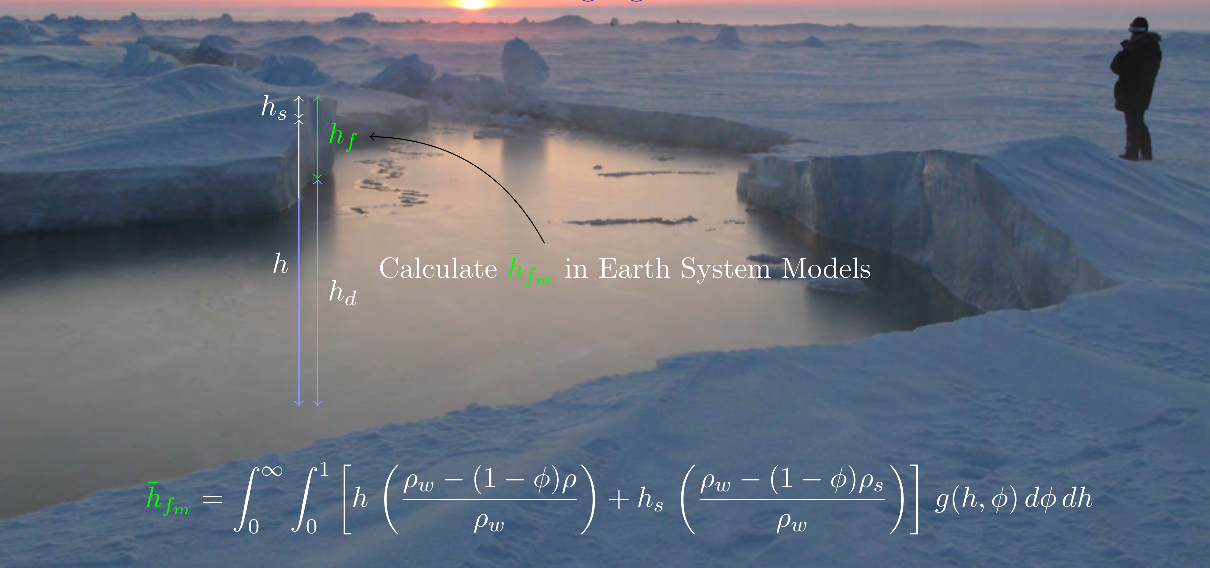
Which defines redistribution of $g(h)$ analytically

$$g(h) = \int g(h, \phi)\, d\phi$$

$$\frac{dg(h, \phi)}{dt} = \Psi(h, \phi) - g(h, \phi)(\nabla \cdot \dot{\boldsymbol{x}})$$



Combined variational ridging and ICESat-2 emulator



$$\bar{h}_{f_m} = \int_0^\infty \int_0^1 \left[h \left(\frac{\rho_w - (1-\phi)\rho}{\rho_w} \right) + h_s \left(\frac{\rho_w - (1-\phi)\rho_s}{\rho_w} \right) \right] g(h,\phi)\, d\phi\, dh$$

Conclusions

1 How can we better integrate thickness observations with models?

- Use satellite freeboard measurements with an emulator.

2 What additional freeboard observations would help improve models?

- Ongoing laser altimetry using a seasonal repeat orbit: ICESat-N

3 How should we design sea ice models to obtain better predictions of sea ice thickness, concentration and drift?

- Expand the model state space to accommodate the local organization of ridges and floes, e.g. $g(h, \phi)$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