Modeling the Future of the Ice Sheets

Lessons Learned from the Development of ISSM

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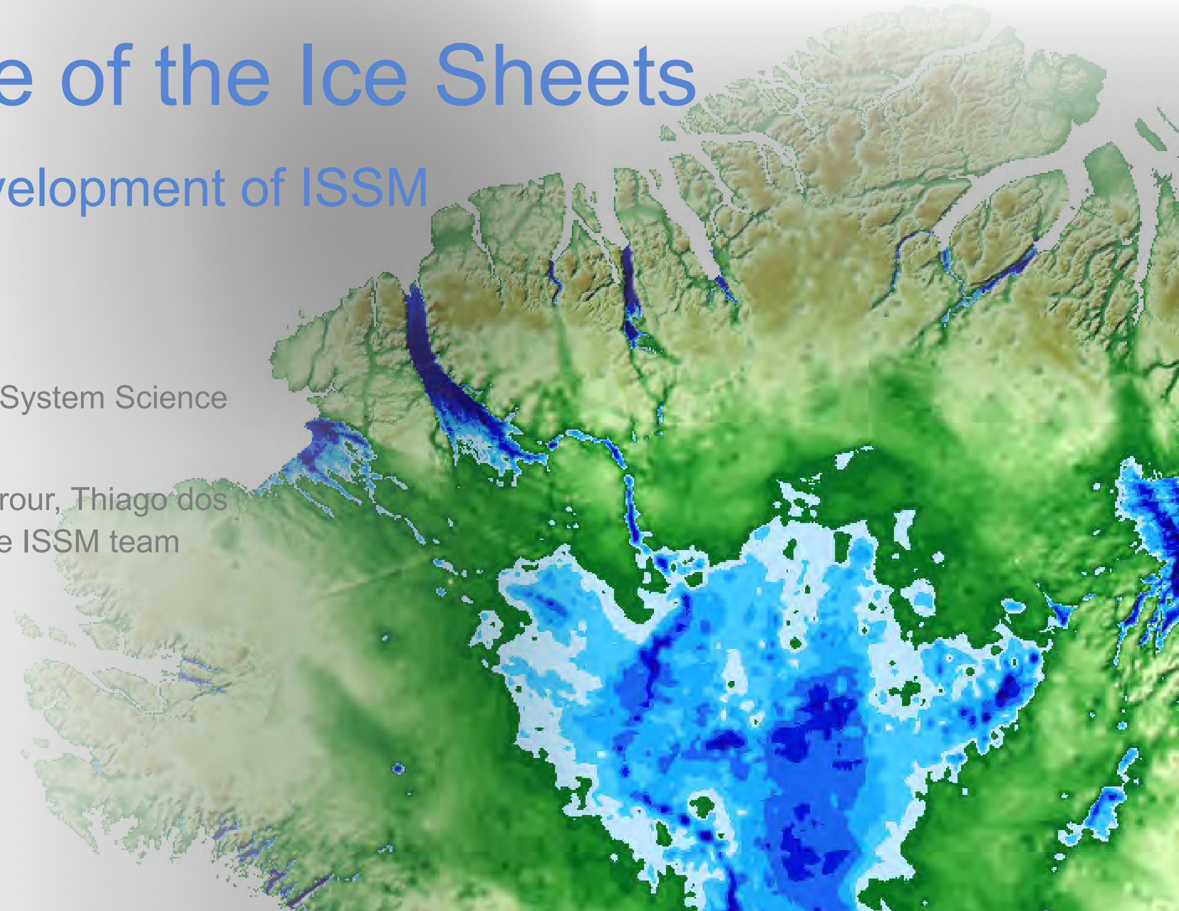
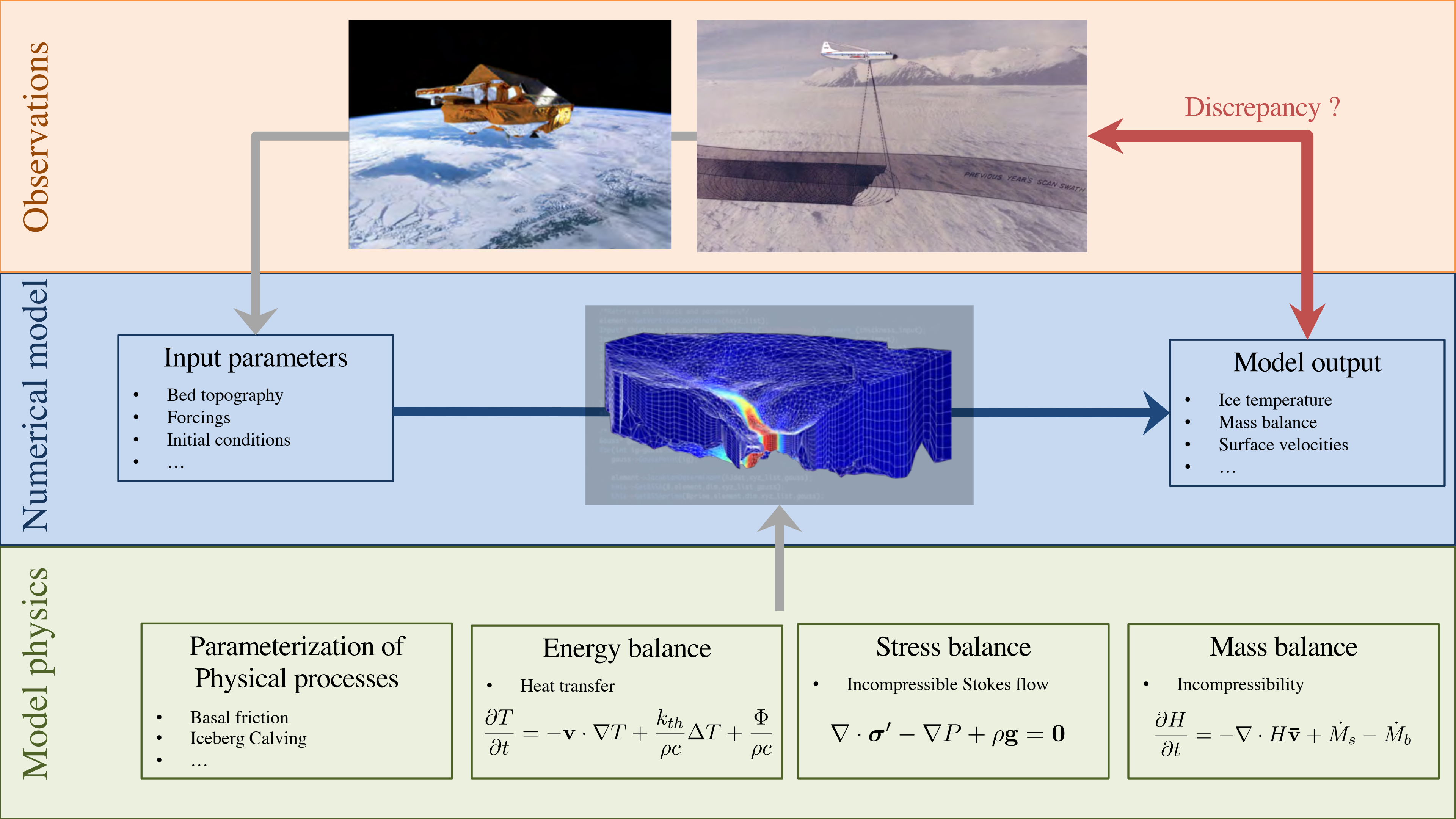


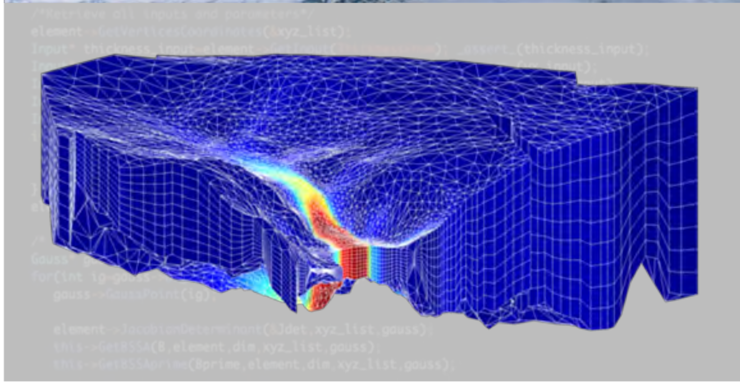

Observations
PREVIOUS YEAR'S SCAN SWATH
Discrepancy ?
Numerical model
Input parameters
Bed topography
Forcings
Initial conditions
…
Model output
Ice temperature
Mass balance
Surface velocities
…
Model physics
Parameterization of Physical processes
Basal friction
Iceberg Calving
…
Energy balance
Heat transfer
$\frac{\partial T}{\partial t} = -\mathbf{v} \cdot \nabla T + \frac{k_{th}}{\rho c} \Delta T + \frac{\Phi}{\rho c}$
Stress balance
Incompressible Stokes flow
$\nabla \cdot \boldsymbol{\sigma}' - \nabla P + \rho \mathbf{g} = \mathbf{0}$
Mass balance
Incompressibility
$\frac{\partial H}{\partial t} = -\nabla \cdot H\bar{\mathbf{v}} + \dot{M}_s - \dot{M}_b$

Outline

1. Physics of ice sheet flow
   - Momentum balance and possible approximations
   - Key physical processes:
     - Grounding line dynamics
     - Calving dynamics

2. Numerical strategies
   - Mesh refinement and resolution of PDEs
   - Verification and Validation
   - Dealing with moving boundaries

3. Project management
   - Code development and architecture
   - Regression tests
   - Project management web interface

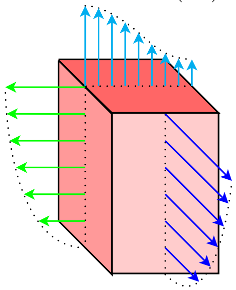


Hierarchy of ice flow models

$$
\begin{cases}
\dfrac{\partial}{\partial x}\left(2\mu\dfrac{\partial v_x}{\partial x}\right)+\dfrac{\partial}{\partial y}\left(\mu\dfrac{\partial v_x}{\partial y}+\mu\dfrac{\partial v_y}{\partial x}\right)+\dfrac{\partial}{\partial z}\left(\mu\dfrac{\partial v_x}{\partial z}+\mu\dfrac{\partial v_z}{\partial x}\right)-\dfrac{\partial p}{\partial x}=0\\[2ex]
\dfrac{\partial}{\partial x}\left(\mu\dfrac{\partial v_x}{\partial y}+\mu\dfrac{\partial v_y}{\partial x}\right)+\dfrac{\partial}{\partial y}\left(2\mu\dfrac{\partial v_y}{\partial y}\right)+\dfrac{\partial}{\partial z}\left(\mu\dfrac{\partial v_y}{\partial z}+\mu\dfrac{\partial v_z}{\partial y}\right)-\dfrac{\partial p}{\partial y}=0\\[2ex]
\dfrac{\partial}{\partial x}\left(\mu\dfrac{\partial v_x}{\partial z}+\mu\dfrac{\partial v_z}{\partial x}\right)+\dfrac{\partial}{\partial y}\left(\mu\dfrac{\partial v_y}{\partial z}+\mu\dfrac{\partial v_z}{\partial y}\right)+\dfrac{\partial}{\partial z}\left(2\mu\dfrac{\partial v_z}{\partial z}\right)-\dfrac{\partial p}{\partial z}-\rho g=0
\end{cases}
$$

$$
\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0
$$

Full-Stokes (FS)



(v_x, v_y, v_z, P)

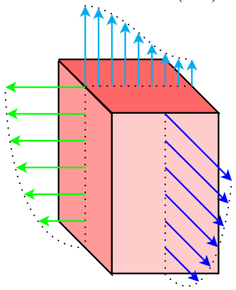
Hierarchy of ice flow models

$$\frac{\partial}{\partial x}\left(2\mu\frac{\partial v_x}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial v_x}{\partial y}+\mu\frac{\partial v_y}{\partial x}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial v_x}{\partial z}+\mu\frac{\partial v_z}{\partial x}\right)-\frac{\partial p}{\partial x}=0$$

$$\mathbf{v}(z)=\mathbf{v}_b-2\left(\rho g\right)^n\left\|\nabla s\right\|^{n-1}\nabla s\int_b^z\left(s-z\right)^n A\,dz$$

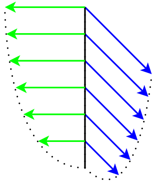
$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0$$

Full-Stokes (FS)



(v_x, v_y, v_z, P)

Hutter (SIA)



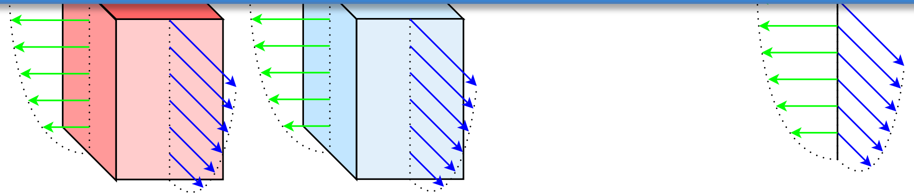
(v_x, v_y)

Hierarchy of ice flow models

$$\nabla \cdot (2\mu\dot{\boldsymbol{\varepsilon}}_{HO1}) = \rho g \frac{\partial s}{\partial x}$$

$$\nabla \cdot (2\mu\dot{\boldsymbol{\varepsilon}}_{HO2}) = \rho g \frac{\partial s}{\partial y}$$

$$\dot{\boldsymbol{\varepsilon}}_{HO1} = \begin{bmatrix} 2\dfrac{\partial v_x}{\partial x} + \dfrac{\partial v_y}{\partial y} \\[2ex] \dfrac{1}{2}\left(\dfrac{\partial v_x}{\partial y} + \dfrac{\partial v_y}{\partial x}\right) \\[2ex] \dfrac{1}{2}\dfrac{\partial v_x}{\partial z} \end{bmatrix} \qquad \dot{\boldsymbol{\varepsilon}}_{HO2} = \begin{bmatrix} \dfrac{1}{2}\left(\dfrac{\partial v_x}{\partial y} + \dfrac{\partial v_y}{\partial x}\right) \\[2ex] \dfrac{\partial v_x}{\partial x} + 2\dfrac{\partial v_y}{\partial y} \\[2ex] \dfrac{1}{2}\dfrac{\partial v_y}{\partial z} \end{bmatrix}$$



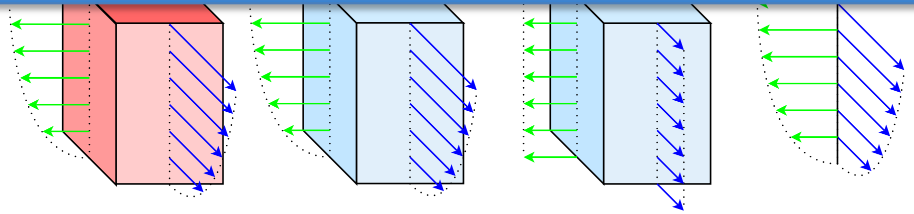
(v_x, v_y, v_z, P) (v_x, v_y) (v_x, v_y)

Hierarchy of ice flow models

$$\nabla \cdot (2\bar{\mu} H \dot{\boldsymbol{\varepsilon}}_{SSA1}) - \alpha^2 v_x = \rho g H \frac{\partial s}{\partial x}$$

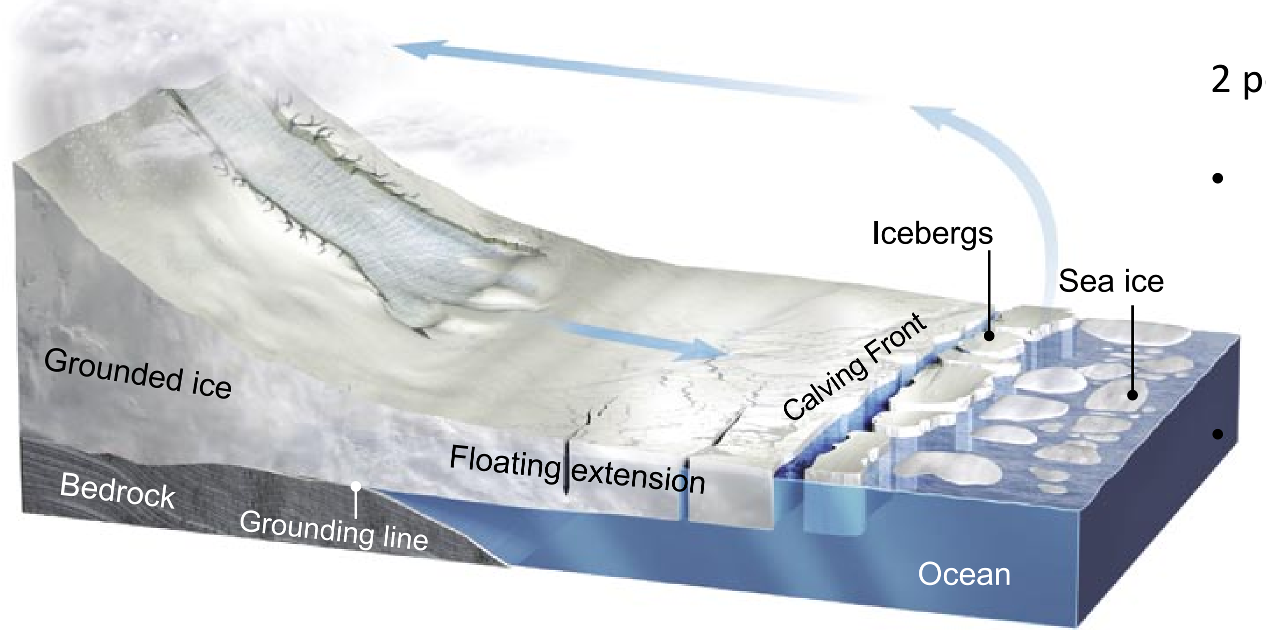
$$\nabla \cdot (2\bar{\mu} H \dot{\boldsymbol{\varepsilon}}_{SSA2}) - \alpha^2 v_y = \rho g H \frac{\partial s}{\partial y}$$

$$\dot{\boldsymbol{\varepsilon}}_{SSA1} = \begin{bmatrix} 2\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \\ \frac{1}{2}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) \end{bmatrix} \quad \dot{\boldsymbol{\varepsilon}}_{SSA2} = \begin{bmatrix} \frac{1}{2}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) \\ \frac{\partial v_x}{\partial x} + 2\frac{\partial v_y}{\partial y} \end{bmatrix}$$



(v_x, v_y, v_z, P) (v_x, v_y) (v_x, v_y) (v_x, v_y)

Grounding Line Dynamics (MISI)

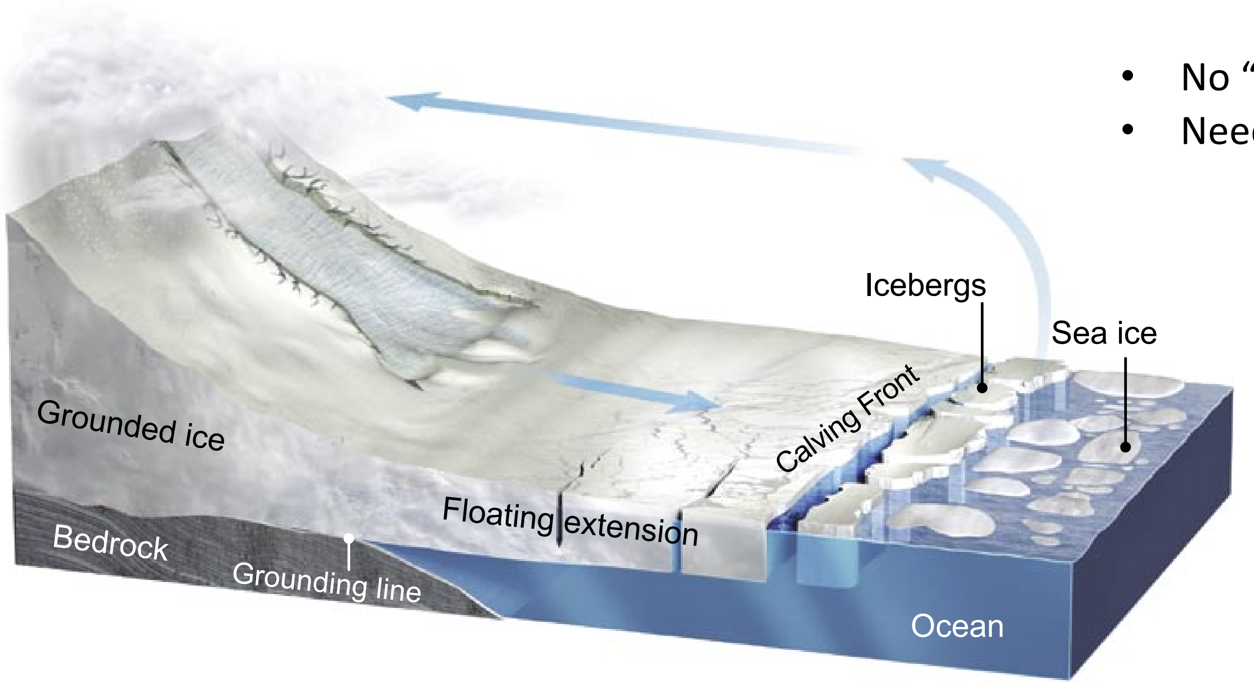


Lots of research over the past 10 years
(See MISMIP, MISMIP3d, MISMIP+,…)

2 possible strategies:

- Based on hydrostatic equilibrium (SSA, HO, L1L2, …)
    - Subelement parameterization
    - Mesh requirements: ~500 m
- Based on contact mechanics (FS)
    - No subelementparam. parameterization
    - Mesh requirements: <100 m

Calving Dynamics (MICI?)



Currently very active area of research

- No "universal" calving law currently exists
- Needs to be included in ice sheet models

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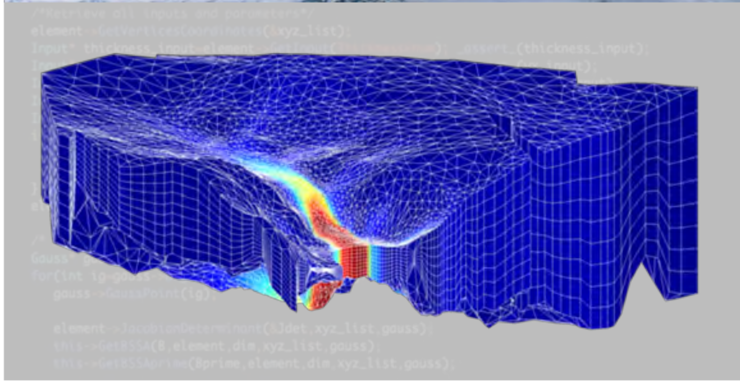
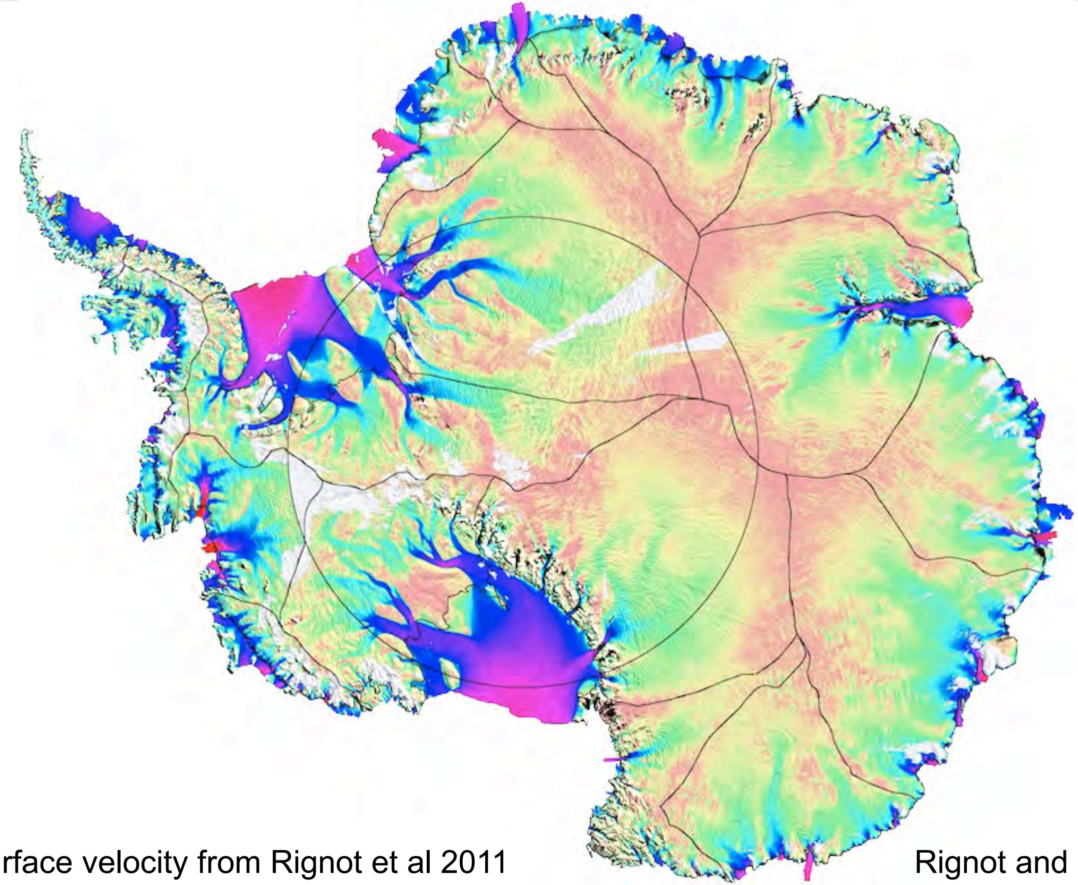
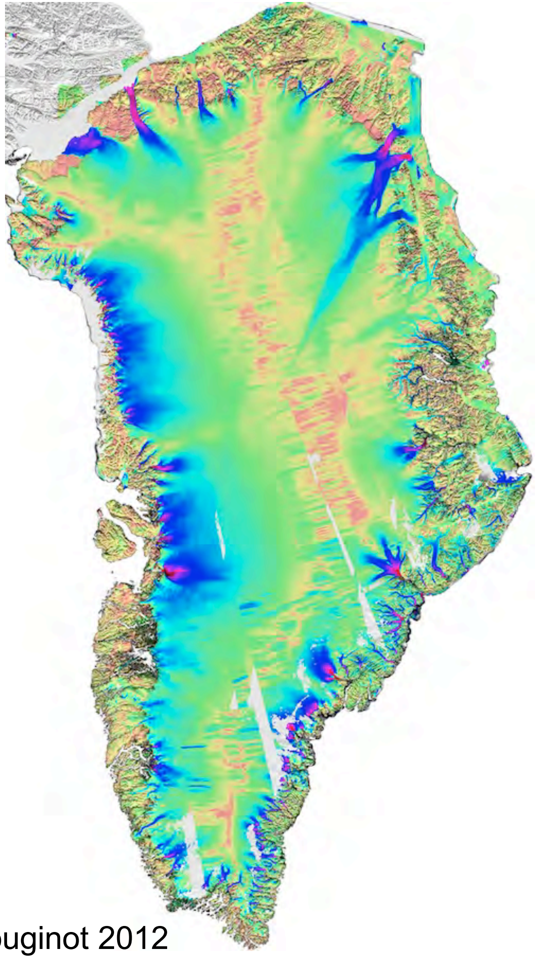


Ice sheet flow

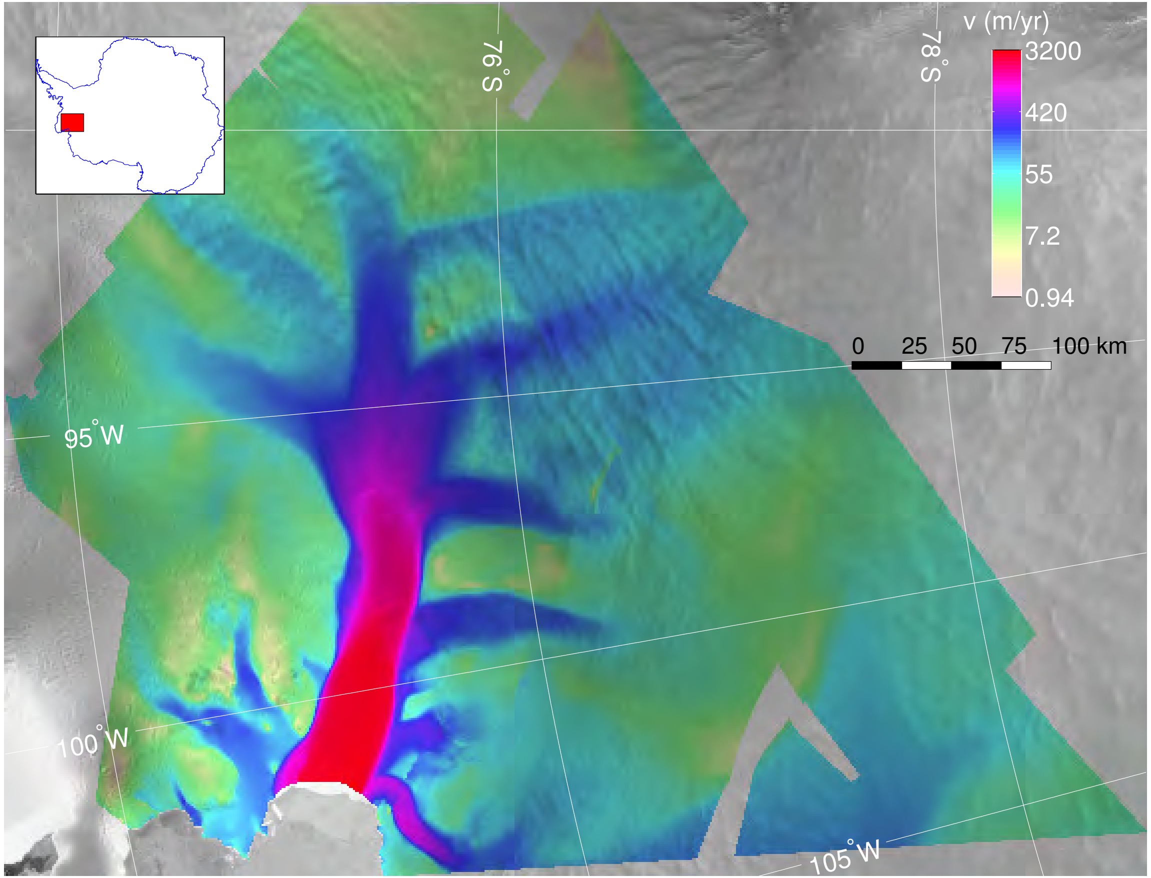


Surface velocity from Rignot et al 2011

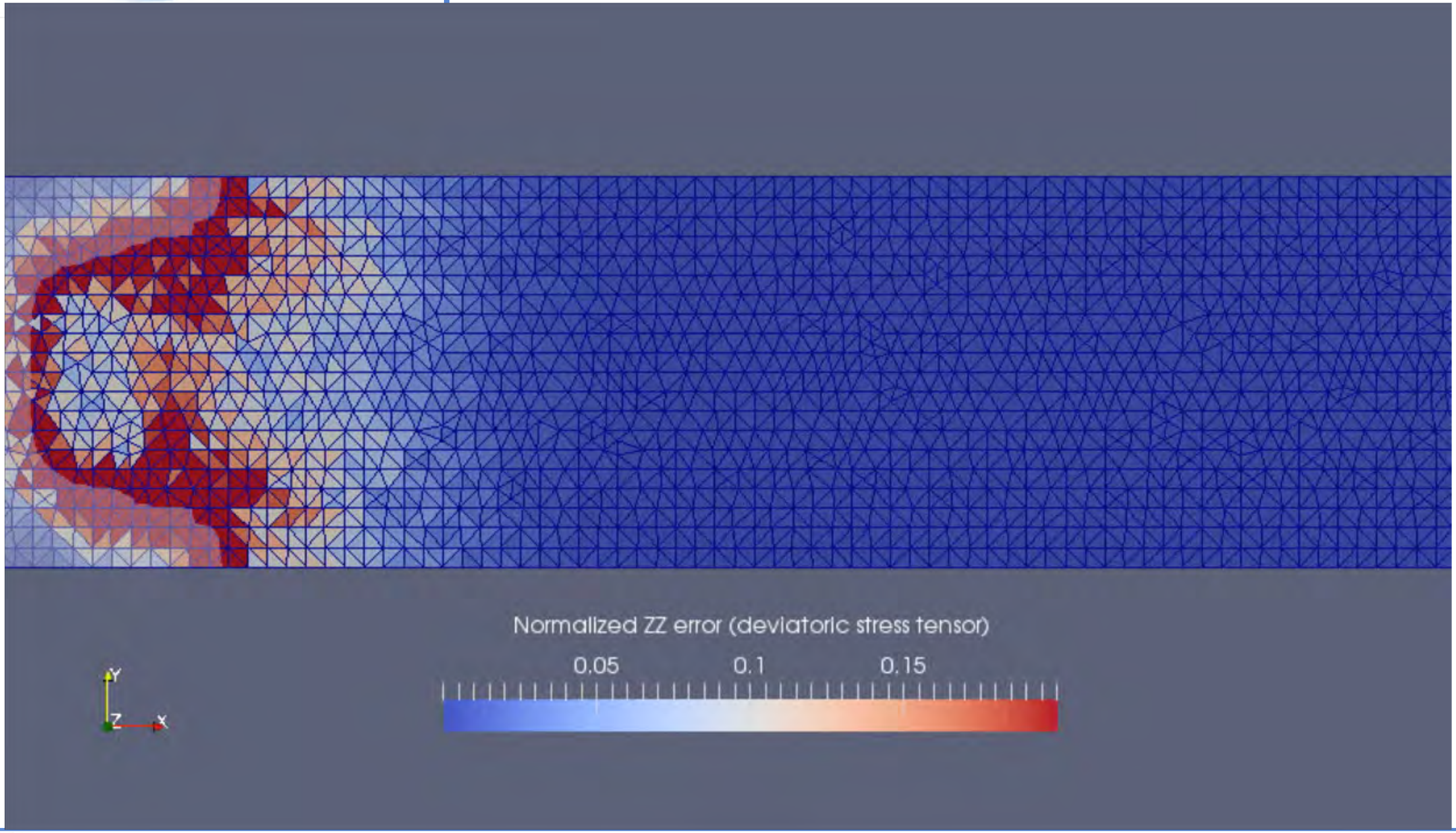


Rignot and Mouginot 2012

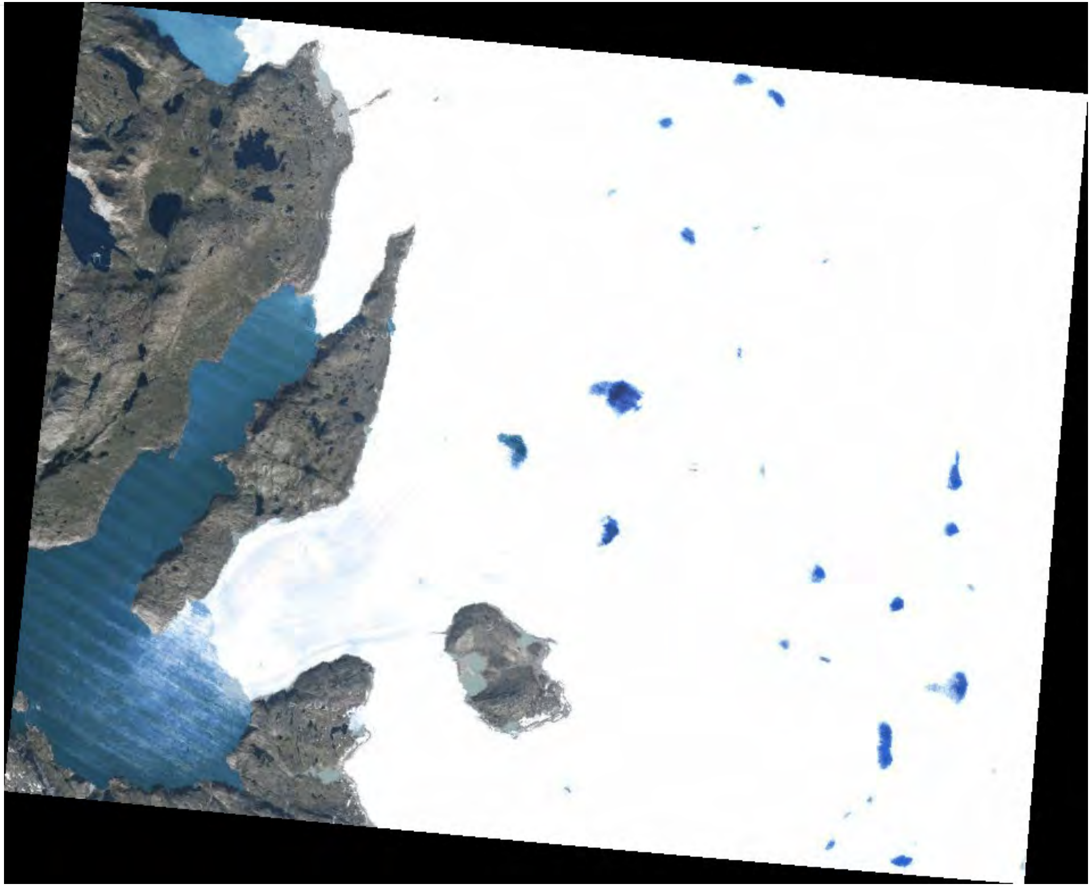
Anisotropic Mesh refinement



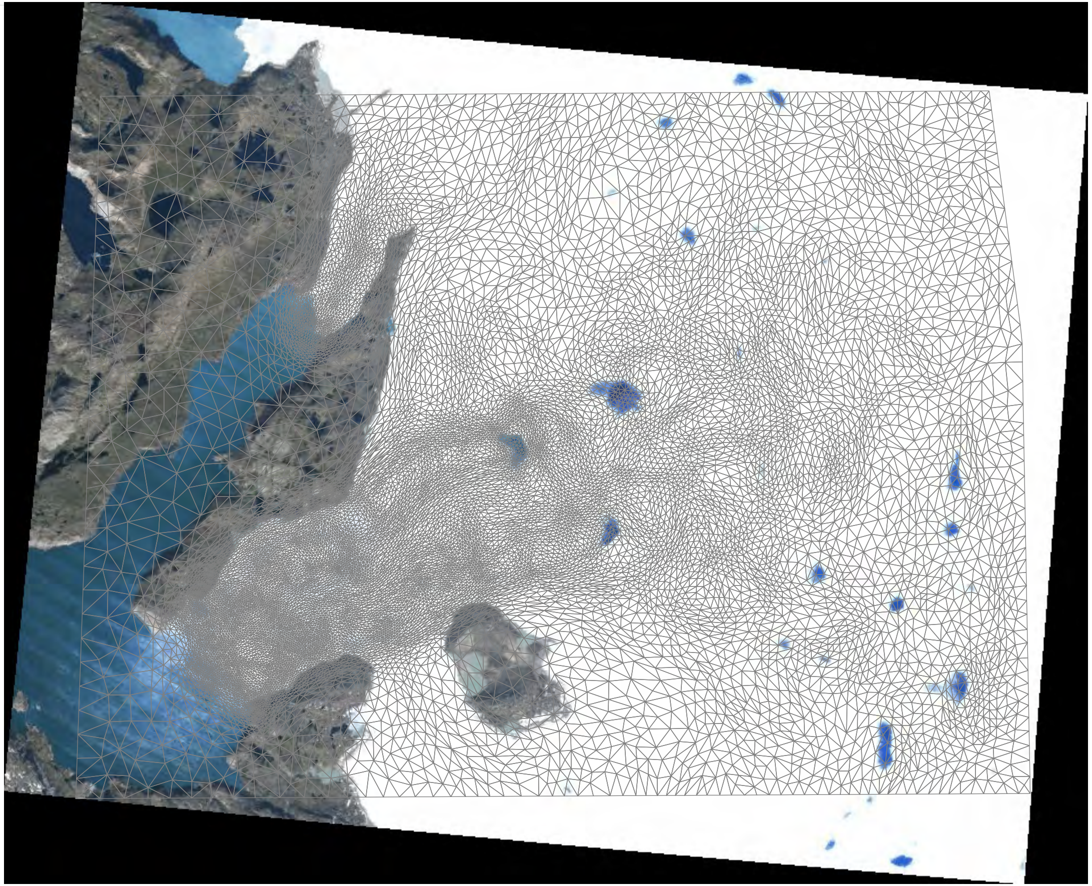
Adaptive Mesh Refinement



Store Glacier



Store Glacier



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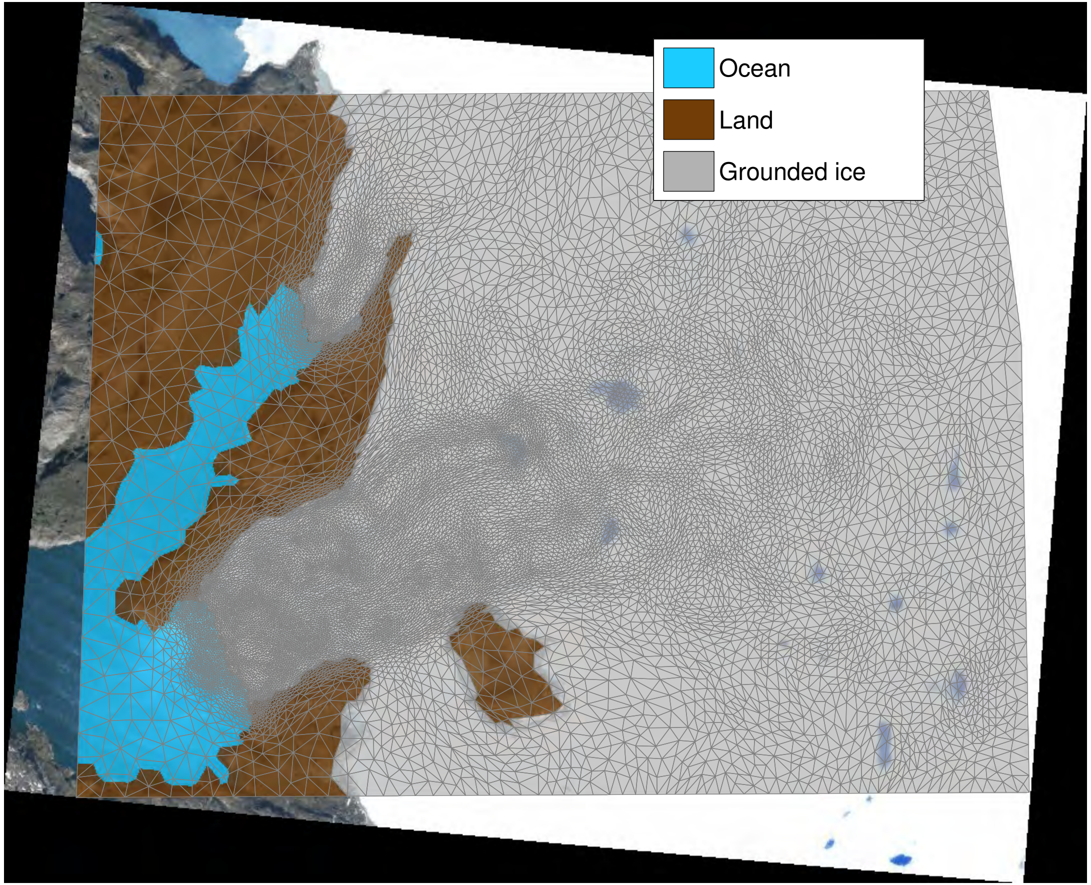

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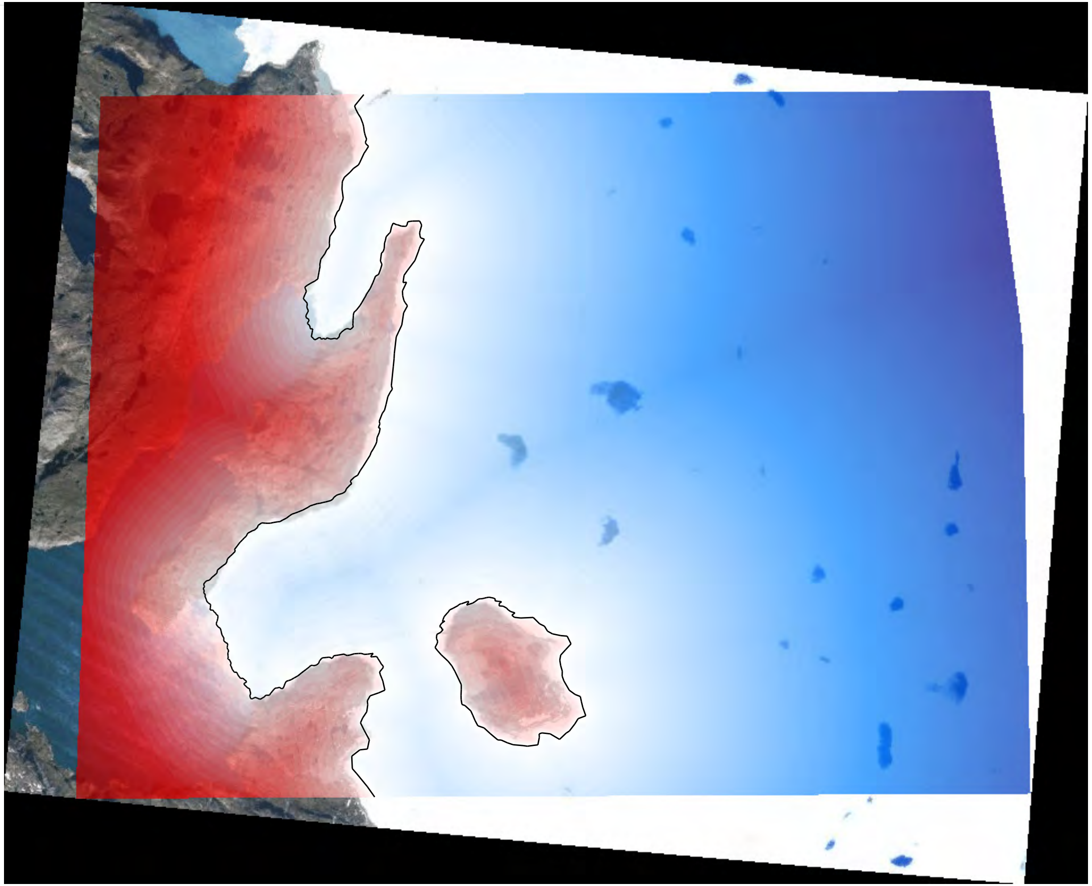
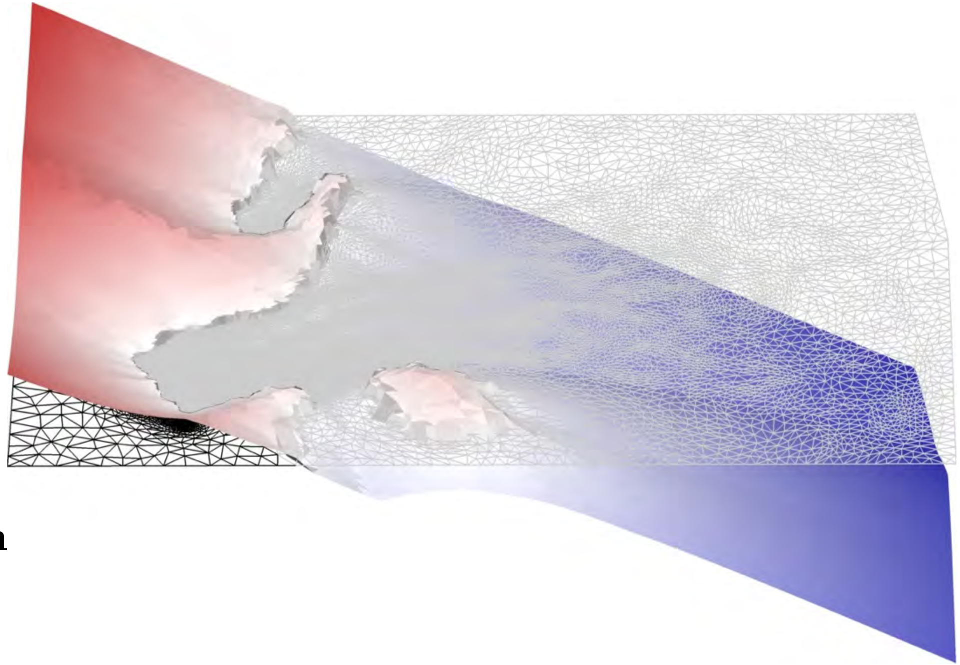


Level set Method

- Tracks model boundary implicitly

$$\begin{cases} \phi(x) < 0 & \text{If } x \text{ is on the ice} \\ \phi(x) > 0 & \text{If } x \text{ is not on the ice} \\ \phi(x) = 0 & \text{If } x \text{ is on the ice boundary} \end{cases}$$



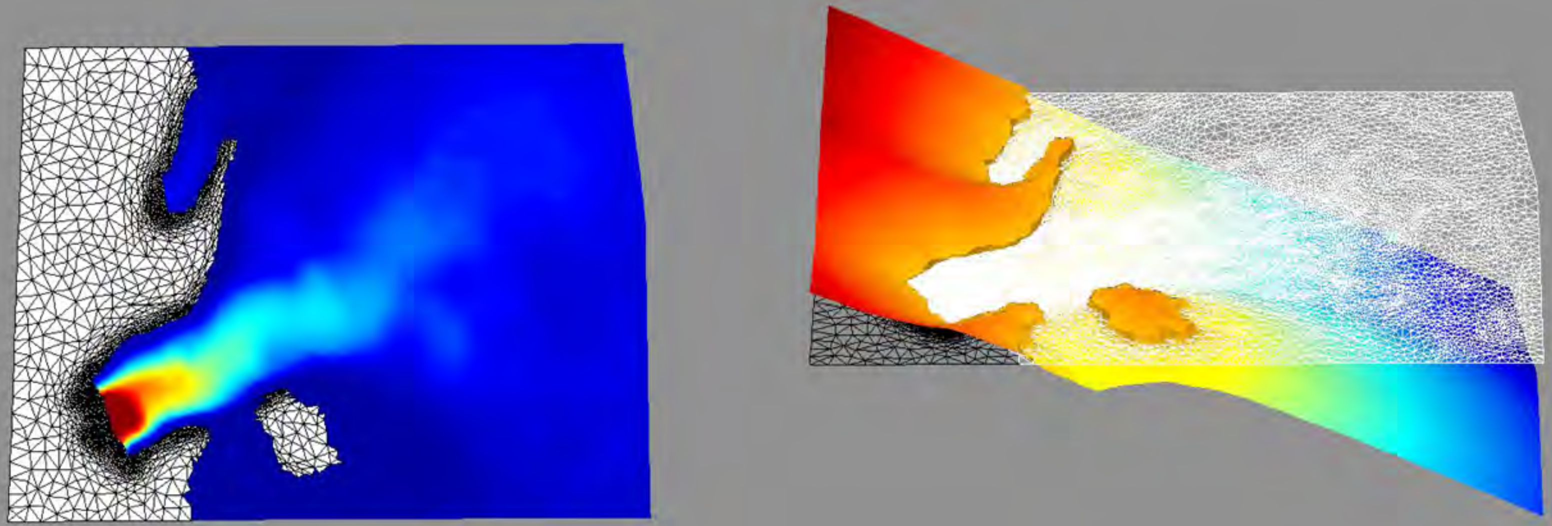
- Ice front moves with a speed v_{front}

$$\mathbf{v}_{\text{front}} = \mathbf{v} - \left(c + \dot{M}\right)\mathbf{n}$$

- Level set equation

$$\frac{\partial \phi}{\partial t} = -\mathbf{v}_{\text{front}} \cdot \nabla \phi$$

Level set Method c = 0, M = 10 km/yr



Example of ice front dynamics parameterization

$$\mathbf{v}_{\text{front}} = \mathbf{v} - \left(c + \dot{M}\right)\mathbf{n}$$

- Calving law (e.g., Morlighem et al. 2016)

$$c = \|\mathbf{v}\| \frac{\tilde{\sigma}}{\sigma_{\max}}$$

- Tensile von-Mises stress

$$\tilde{\sigma} = \sqrt{3}\, B\, \tilde{\dot{\varepsilon}}_e^{1/n}$$

- Threshold: σ_{max} calibrated

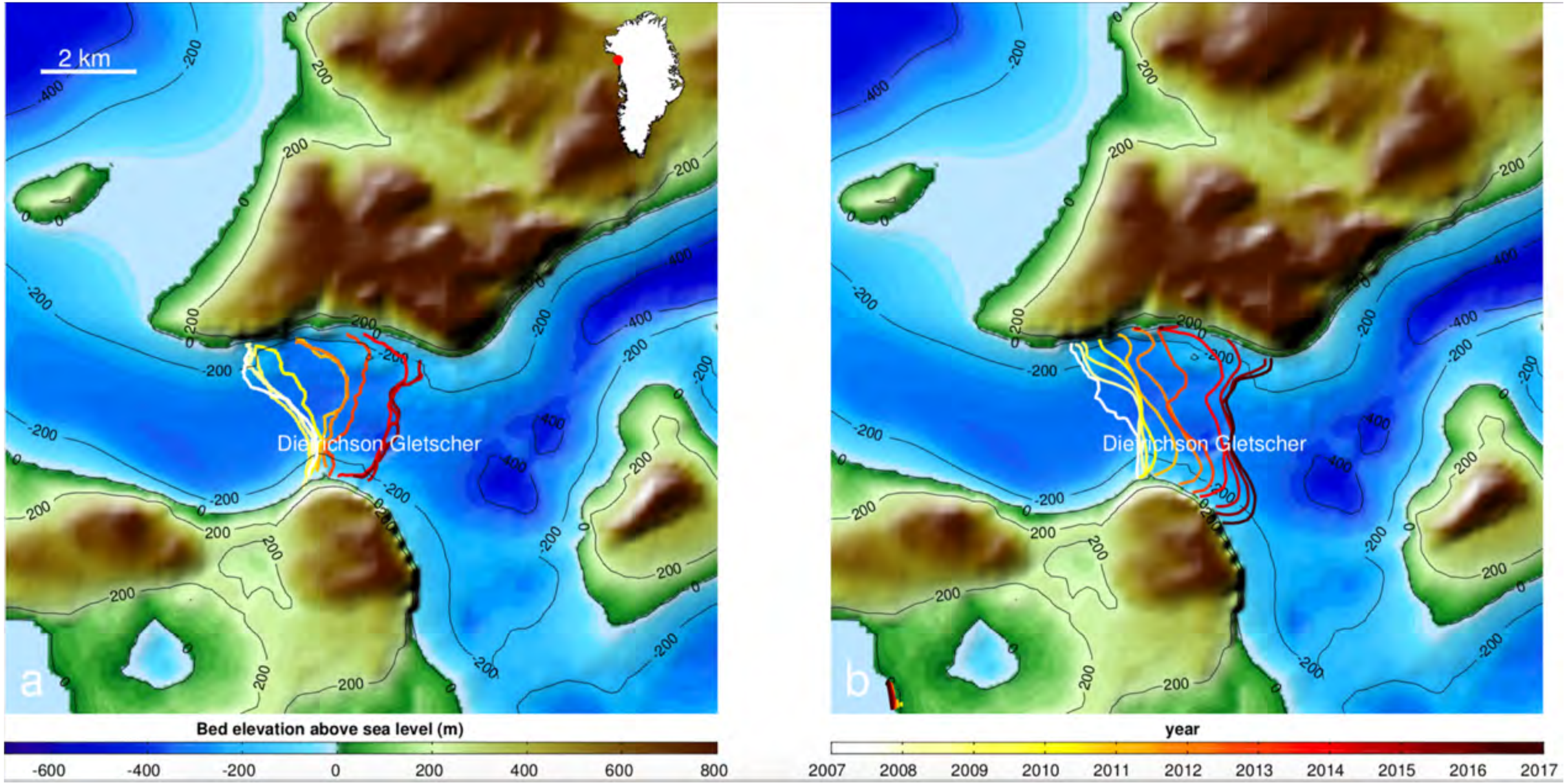
- Melting rate (Rignot et al. 2016)

$$\dot{M} = \left(A\, h\, q_{sg}^{\alpha} + B\right) \mathrm{TF}^{\beta}$$

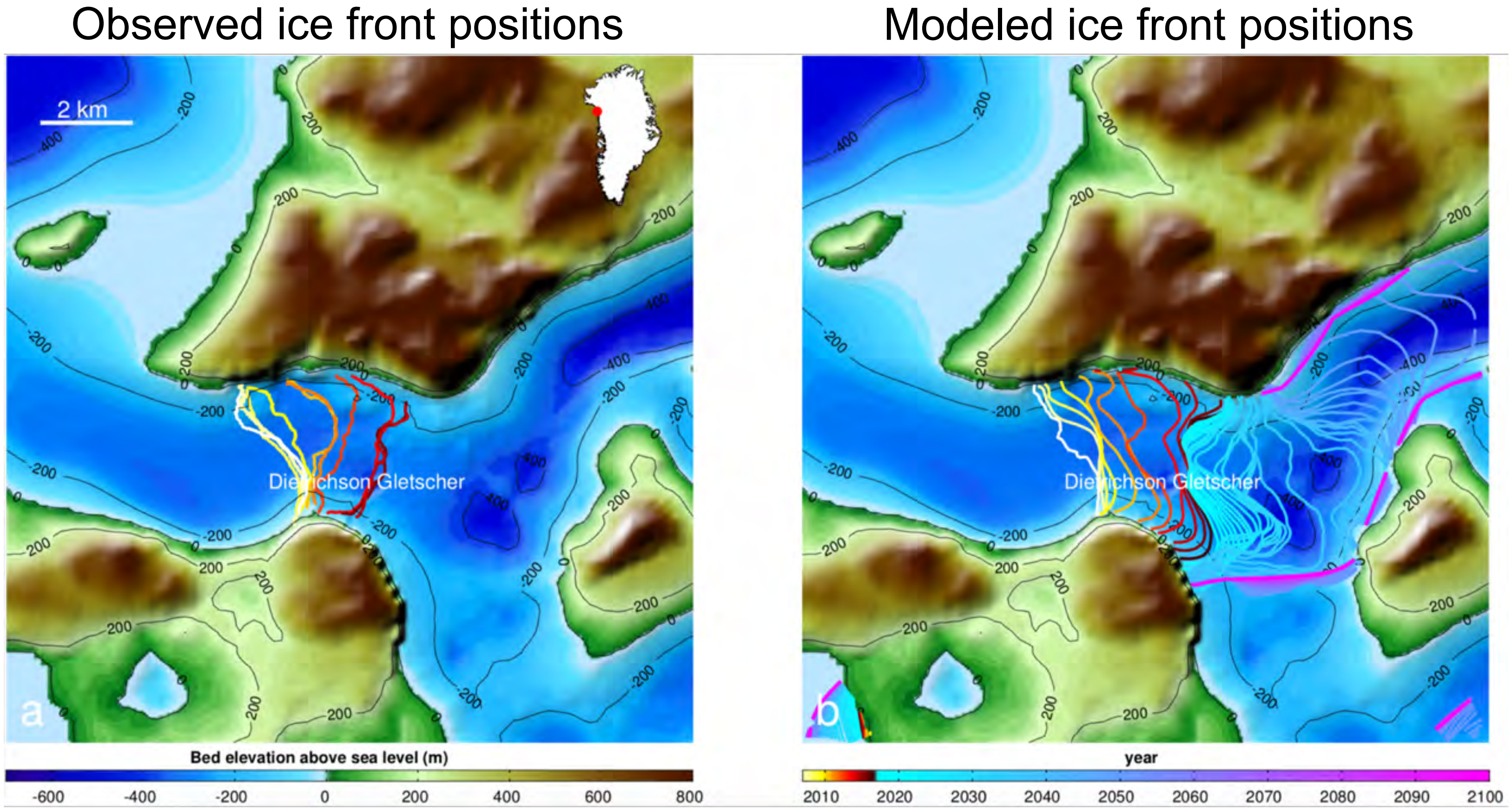
- A: 3e-4 units B: 0.15 units
- α: 0.39 β: 1.18
- h: water depth (BedMachine v3)
- q_{sg}: subglacial water flux (RACMO)
- TF: thermal forcing (ECCO2)

Threshold calibration

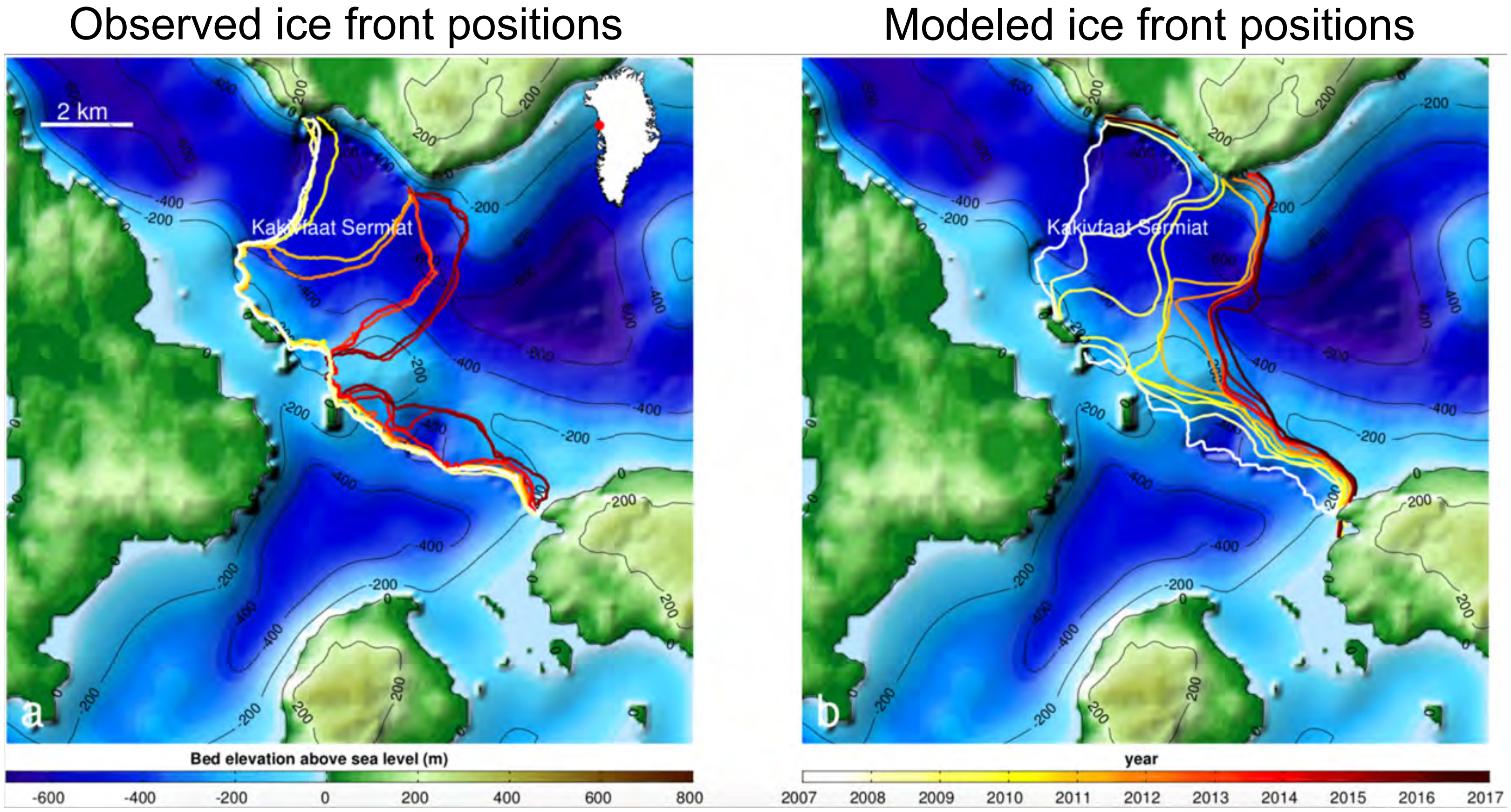
Observed ice front positions | Modeled ice front positions



Threshold calibration

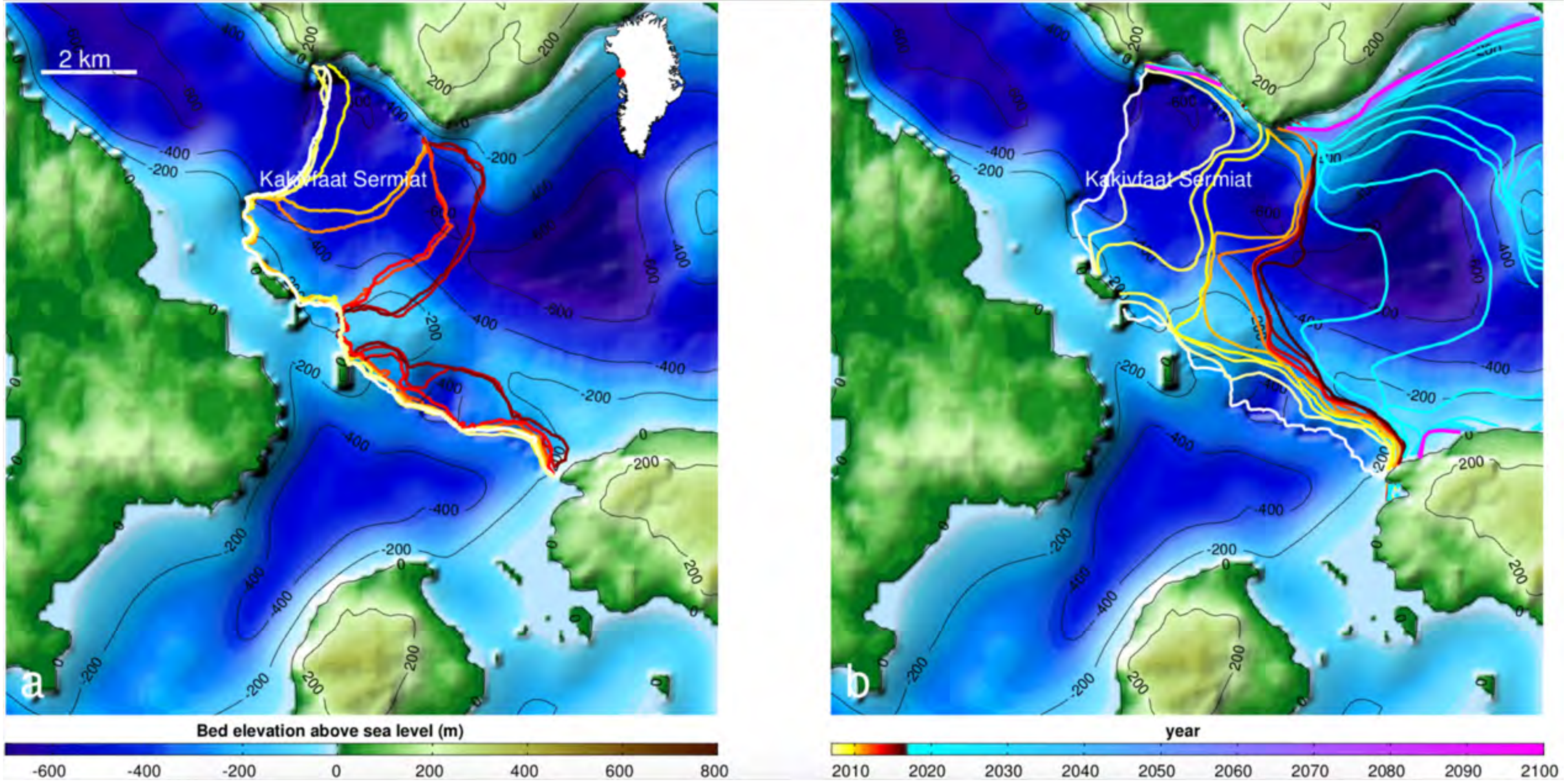


Threshold calibration

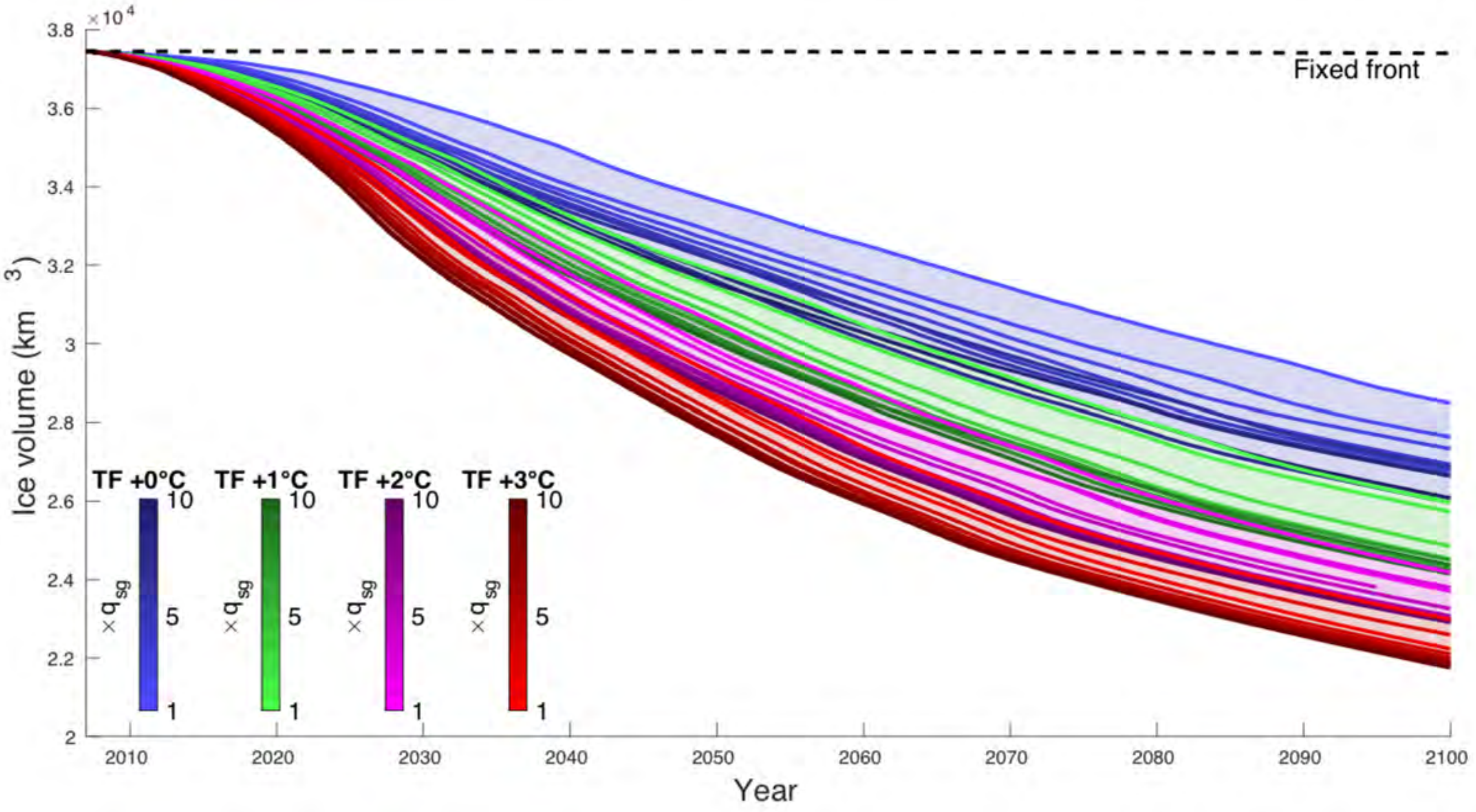


Threshold calibration

Observed ice front positions | Modeled ice front positions



Ice volumes


$\times 10^4$
3.8
3.6
3.4
3.2
3
2.8
2.6
2.4
2.2
2
Ice volume (km^3)
Fixed front
TF +0°C
TF +1°C
TF +2°C
TF +3°C
$\times q_{sg}$
10
5
1
2010
2020
2030
2040
2050
2060
2070
2080
2090
2100
Year

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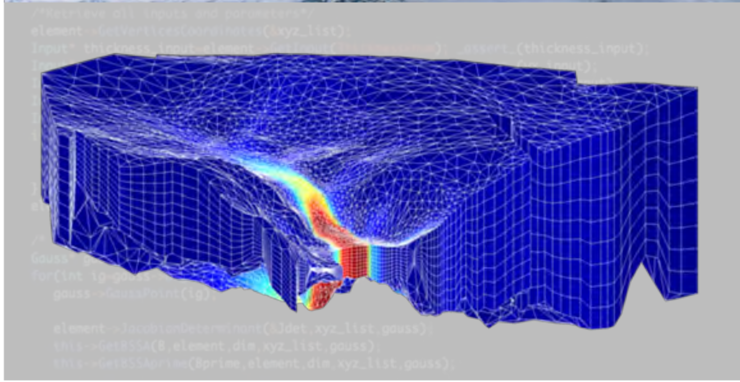


Architecture: some considerations

What Architecture?

- MPI (not just OpenMP)
- Hybrid (MPI/OpenMP)
- As modular as possible

What Language?

- MATLAB/python/… simple user interface
- Fortran: widely used in climate models, supports source-to-source (AD: OpenAD, Tapenade, TAF, etc)
- C: supports pointer, dynamic memory allocation, no (good) AD tool
- C++: same as C but additionally:
    - supports Object-Oriented programing
    - supports Object-Overloading for AD (CoDiPack, Sacado, ADOLC, etc)

Relying on external packages?

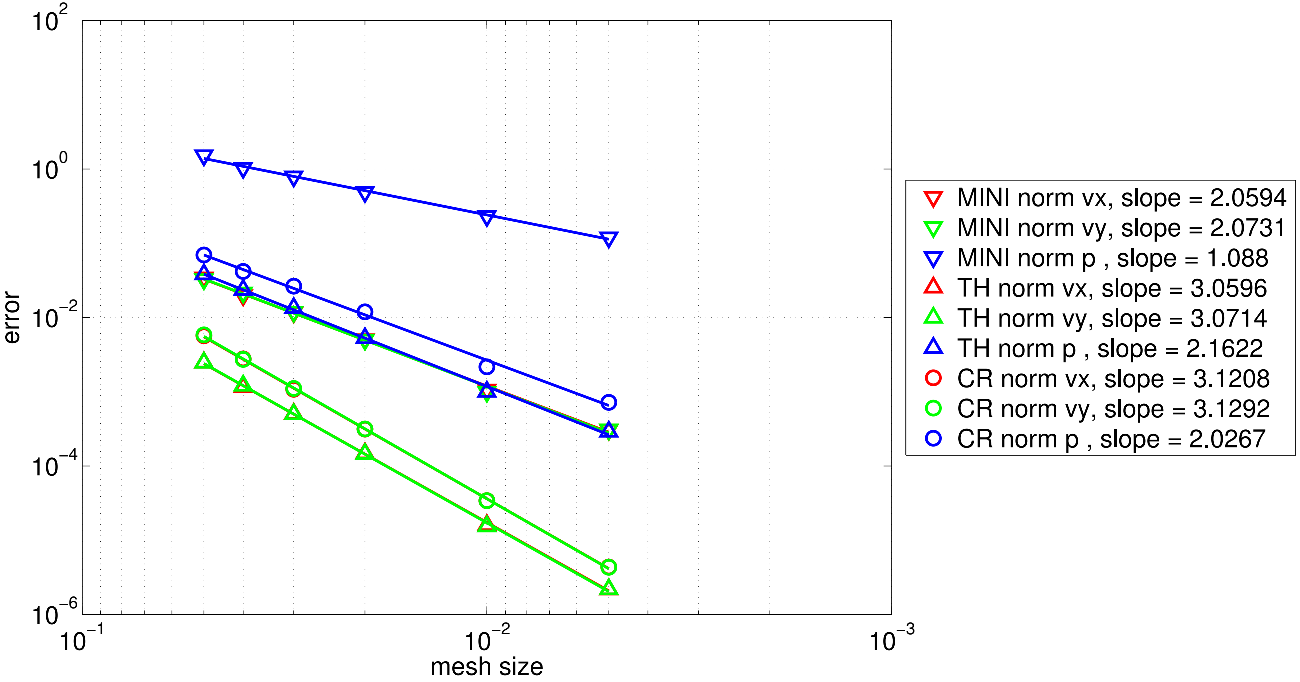
- The fewer the better
- in-house is always better

Team development & Coding:

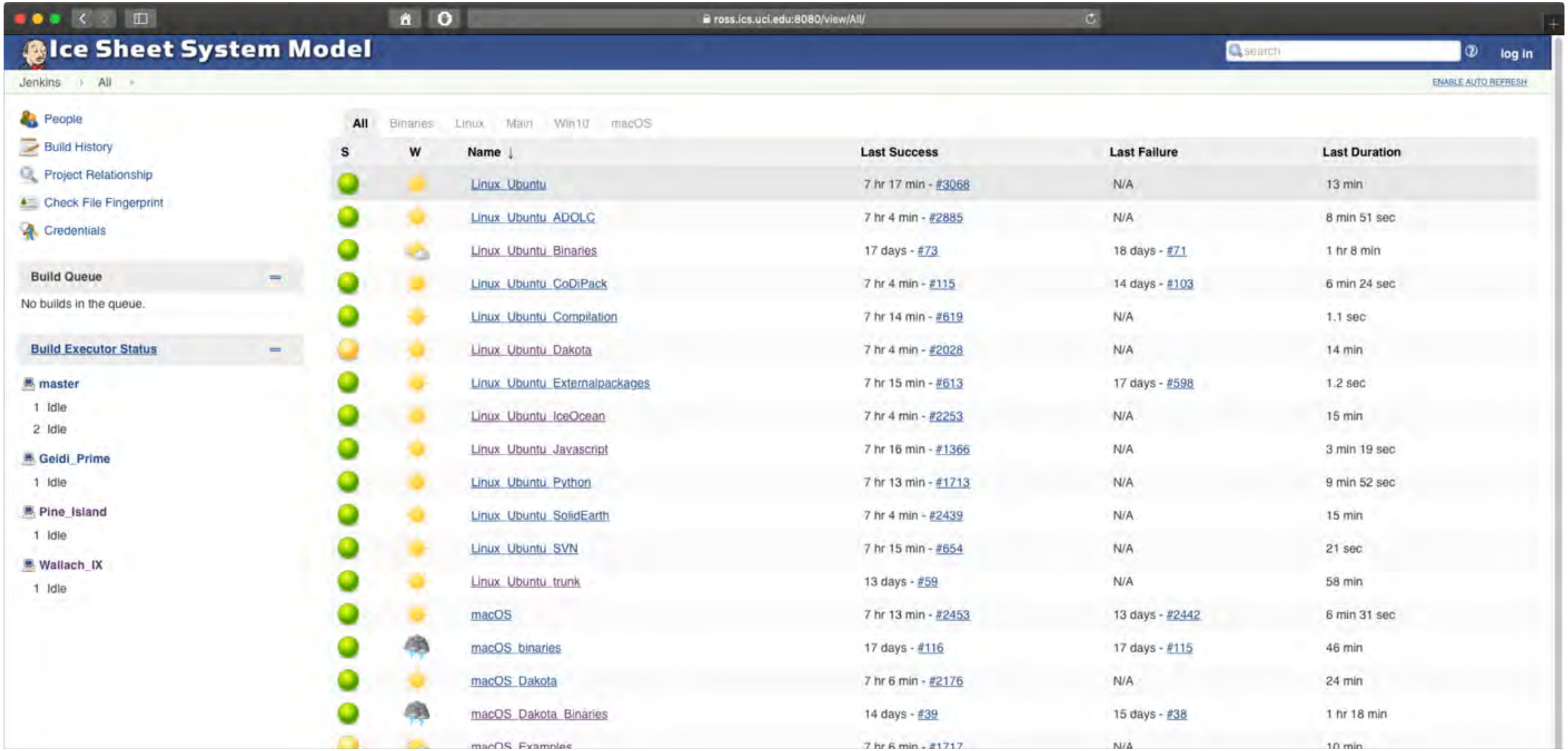
- **Let scientists code!**
- Consistent coding style/clear rules
- **Open source** (svn/git)

Verification & Validation

- Inter-comparison: test against other models (ISMIP-HOM, MISMIP,…)
- Verification: test against analytical solutions
- Validation: test against “reality”

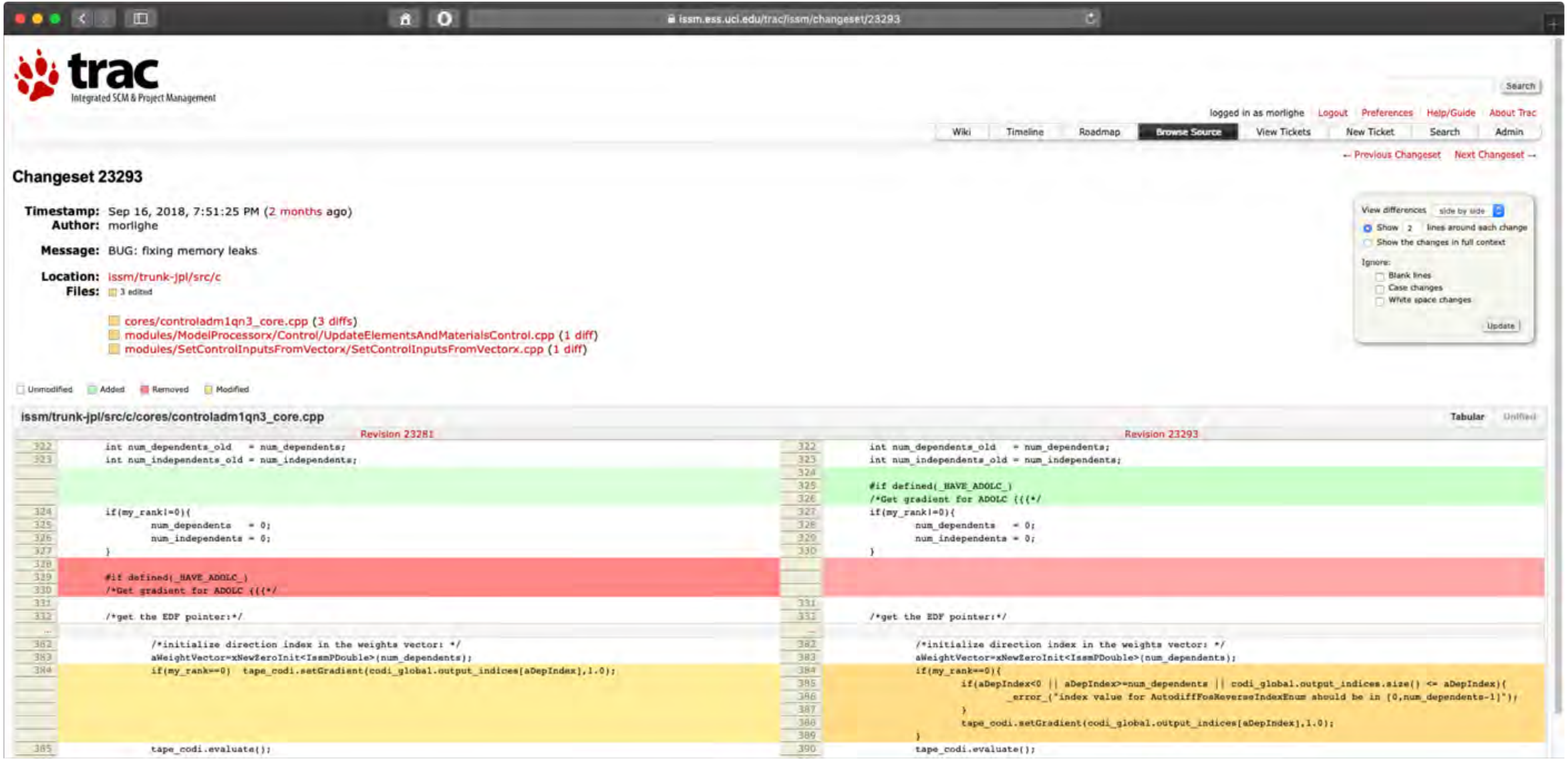


Automatic Regression tests



S	W	Name ↓	Last Success	Last Failure	Last Duration
		Linux_Ubuntu	7 hr 17 min - #3068	N/A	13 min
		Linux_Ubuntu_ADOLC	7 hr 4 min - #2885	N/A	8 min 51 sec
		Linux_Ubuntu_Binaries	17 days - #73	18 days - #71	1 hr 8 min
		Linux_Ubuntu_CoDiPack	7 hr 4 min - #115	14 days - #103	6 min 24 sec
		Linux_Ubuntu_Compilation	7 hr 14 min - #619	N/A	1.1 sec
		Linux_Ubuntu_Dakota	7 hr 4 min - #2028	N/A	14 min
		Linux_Ubuntu_Externalpackages	7 hr 15 min - #613	17 days - #598	1.2 sec
		Linux_Ubuntu_IceOcean	7 hr 4 min - #2253	N/A	15 min
		Linux_Ubuntu_Javascript	7 hr 16 min - #1366	N/A	3 min 19 sec
		Linux_Ubuntu_Python	7 hr 13 min - #1713	N/A	9 min 52 sec
		Linux_Ubuntu_SolidEarth	7 hr 4 min - #2439	N/A	15 min
		Linux_Ubuntu_SVN	7 hr 15 min - #654	N/A	21 sec
		Linux_Ubuntu_trunk	13 days - #59	N/A	58 min
		macOS	7 hr 13 min - #2453	13 days - #2442	6 min 31 sec
		macOS_binaries	17 days - #116	17 days - #115	46 min
		macOS_Dakota	7 hr 6 min - #2176	N/A	24 min
		macOS_Dakota_Binaries	14 days - #39	15 days - #38	1 hr 18 min
		macOS_Examples	7 hr 6 min - #1717	N/A	10 min

Project management website (Trac)



Conclusions

- Ice physics:
  - Do not rely on SIA
  - Should not focus on FS only
  - Model should be capable of capturing Ice front and Grounding line dynamics (at the minimum)

- Numerics:
  - FE or FV, do not use FD
  - Static mesh adaptation, or AMR
  - Handle Moving boundary
  - Check against existing benchmarks

- Code management:
  - Good project management platform
  - Open source
  - Let scientists code!

Thank you !
Questions ?