Ice sheet dynamics: from observations to models

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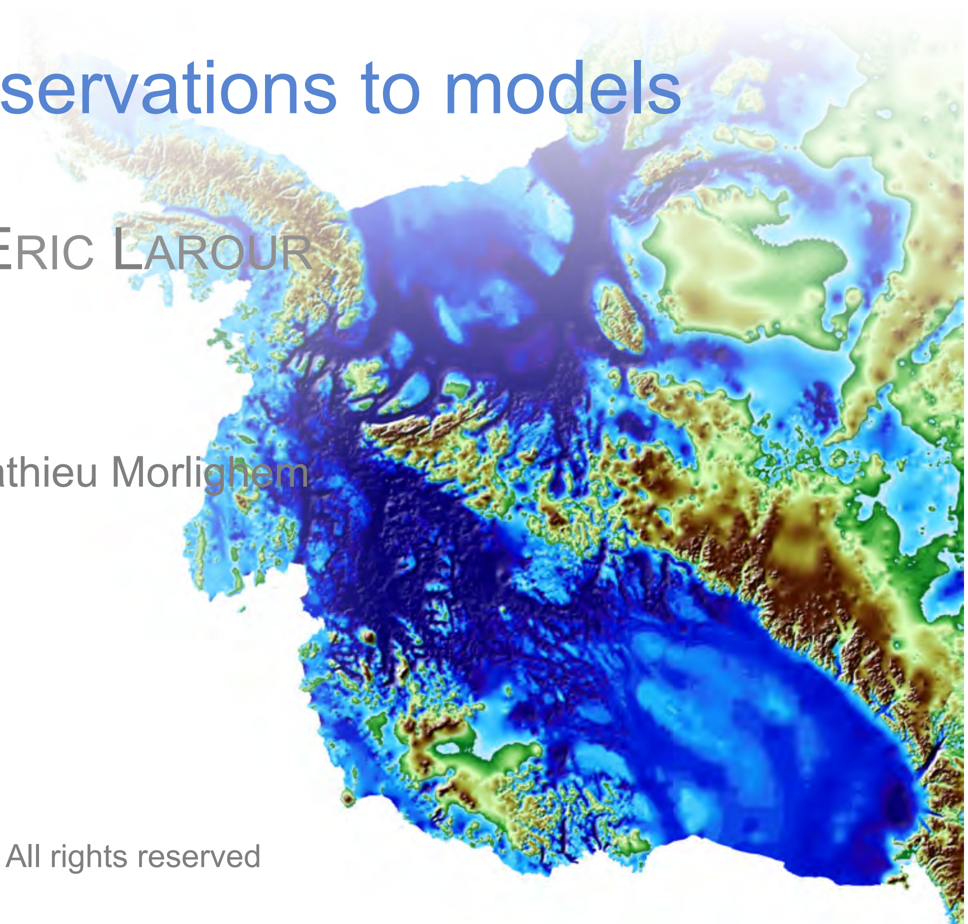
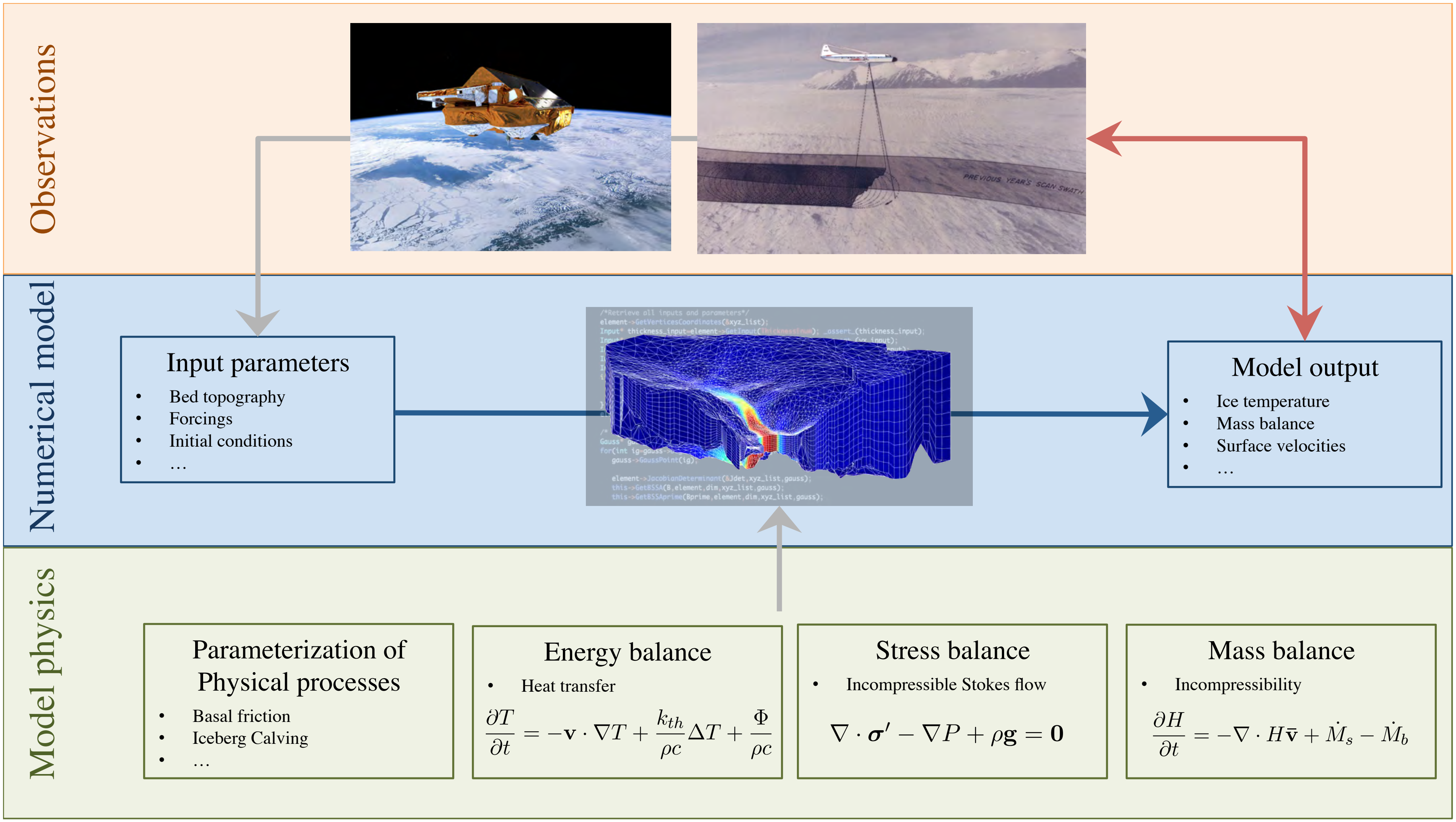
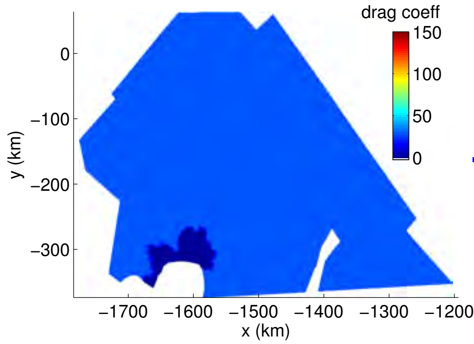
with contributions from Johannes Bondzio and Mathieu Morlighem



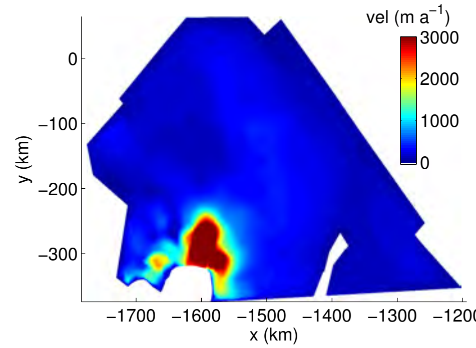

Observations
Numerical model
Model physics
Input parameters
Bed topography
Forcings
Initial conditions
…
Model output
Ice temperature
Mass balance
Surface velocities
…
Parameterization of Physical processes
Basal friction
Iceberg Calving
…
Energy balance
Heat transfer
$\frac{\partial T}{\partial t} = -\mathbf{v} \cdot \nabla T + \frac{k_{th}}{\rho c} \Delta T + \frac{\Phi}{\rho c}$
Stress balance
Incompressible Stokes flow
$\nabla \cdot \boldsymbol{\sigma}' - \nabla P + \rho \mathbf{g} = \mathbf{0}$
Mass balance
Incompressibility
$\frac{\partial H}{\partial t} = -\nabla \cdot H\bar{\mathbf{v}} + \dot{M}_s - \dot{M}_b$

Data Assimilation



$v = F(\alpha)$

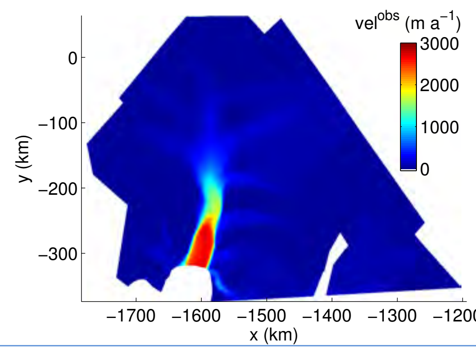
Direct problem



?

$\alpha = F^{-1}(v)$

Inverse problem



Data Assimilation

Minimize the cost function:

$$\mathcal{J}\left(\mathbf{v},\alpha\right)=\frac{1}{2}\int_{\Gamma_s}\left(v_x-v_x^{\mathrm{obs}}\right)^2+\left(v_y-v_y^{\mathrm{obs}}\right)^2 dS+\mathcal{R}\left(\alpha\right)$$

With the constraint:

$$\begin{array}{rcll}
\nabla\cdot\mu\left(\nabla\mathbf{v}+\nabla\mathbf{v}^T\right)-\nabla p+\rho\mathbf{g} & = & \mathbf{0} & \text{in }\Omega\\
\nabla\cdot\mathbf{v} & = & 0 & \text{in }\Omega\\
\boldsymbol{\sigma}\cdot\mathbf{n} & = & \mathbf{f} & \text{on }\Gamma_s\cup\Gamma_w\\
\left(\boldsymbol{\sigma}\cdot\mathbf{n}+\alpha^2\mathbf{v}\right)_{\parallel} & = & \mathbf{0} & \text{on }\Gamma_b\\
\mathbf{v}\cdot\mathbf{n} & = & -\dot{M}_b\,n_z & \text{on }\Gamma_b
\end{array}$$

Data Assimilation

Lagrangian:

$$\mathcal{L}\left(\mathbf{v}, p; \boldsymbol{\lambda}, \lambda_p; \alpha\right) = \frac{1}{2}\int_{\Gamma_s} \left(v_x - v_x^{obs}\right)^2 + \left(v_y - v_y^{obs}\right)^2 d\Gamma_s$$

$$- \int_{\Omega} 2\mu\dot{\boldsymbol{\varepsilon}}_v : \dot{\boldsymbol{\varepsilon}}_\lambda \, d\Omega - \int_{\Gamma_b} \alpha^2 \, \mathbf{v} \cdot \boldsymbol{\lambda} \, d\Gamma_b + \int_{\Gamma_w} \mathbf{f} \cdot \boldsymbol{\lambda} \, d\Gamma_w$$

Model state, $\bar{\mathbf{v}}$, defined by:

$$\forall \delta\boldsymbol{\lambda} \in \mathcal{V} \quad \left\langle \mathcal{D}_{\boldsymbol{\lambda}}\mathcal{L}\left(\bar{\mathbf{v}}\right), \delta\boldsymbol{\lambda}\right\rangle = 0$$

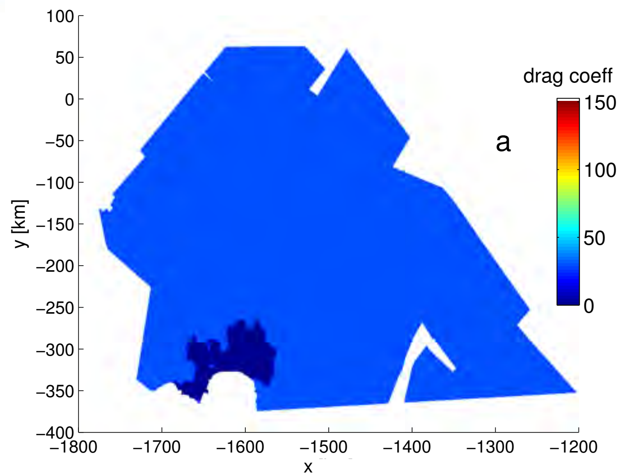
Adjoint state, $\bar{\boldsymbol{\lambda}}$, defined by:

$$\forall \delta\mathbf{v} \in \mathcal{V} \quad \left\langle \mathcal{D}_{\mathbf{v}}\mathcal{L}\left(\bar{\mathbf{v}}, \bar{\boldsymbol{\lambda}}, \alpha\right), \delta\mathbf{v}\right\rangle = 0$$

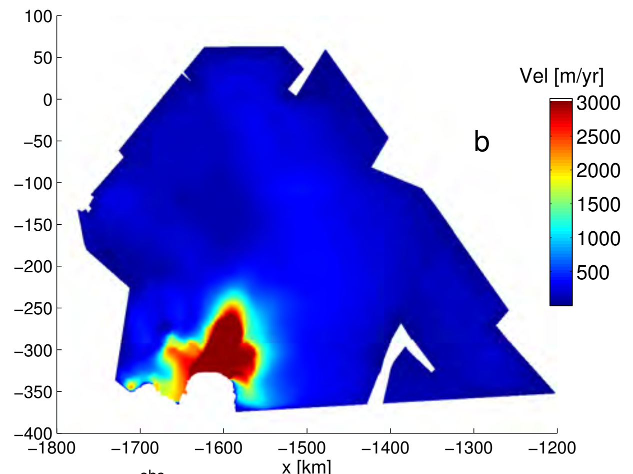
Cost function derivative:

$$\left\langle \mathcal{D}J\left(\alpha\right), \delta\alpha\right\rangle = \left\langle \mathcal{D}_\alpha\mathcal{L}\left(\bar{\mathbf{v}}, \bar{\boldsymbol{\lambda}}\right), \delta\alpha\right\rangle = \int_{\Gamma_b} 2\,\alpha\,\delta\alpha\,\bar{\mathbf{v}} \cdot \bar{\boldsymbol{\lambda}} \, d\Gamma_b$$

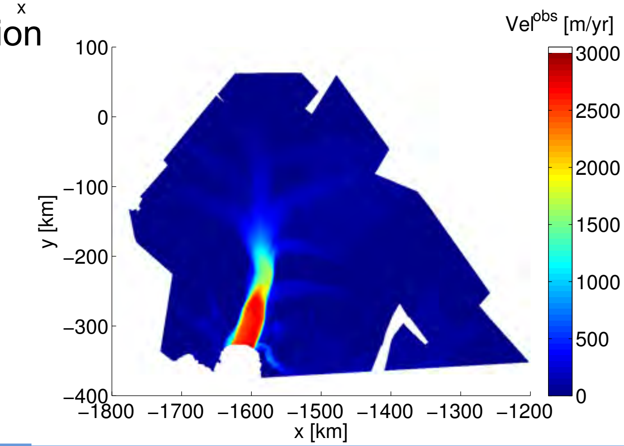
Application to Pine Island Glacier (2/2)



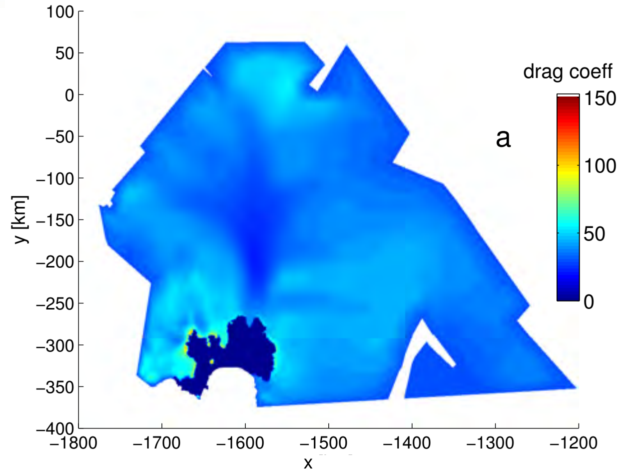
(a) Initial basal friction



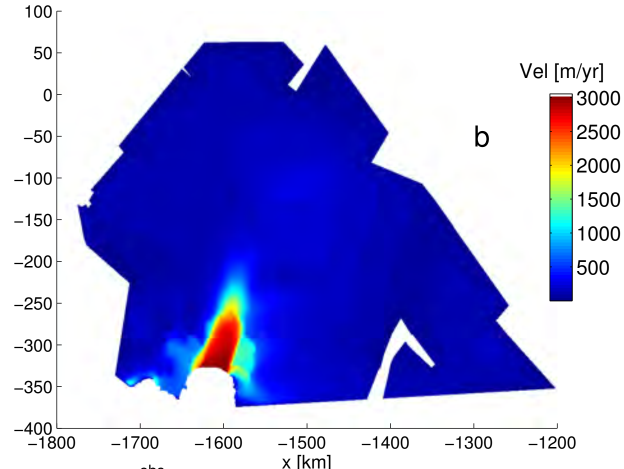
(b) Modeled velocity



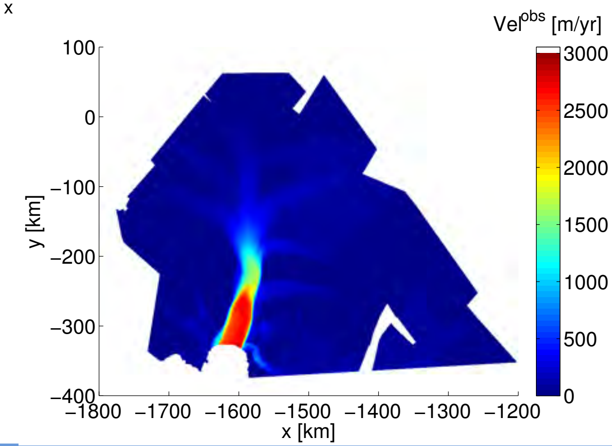
Application to Pine Island Glacier (2/2)



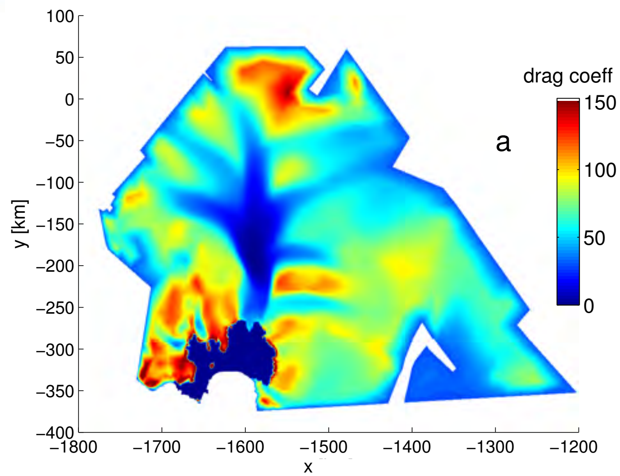
(a) Inferred basal friction
Iteration = 1



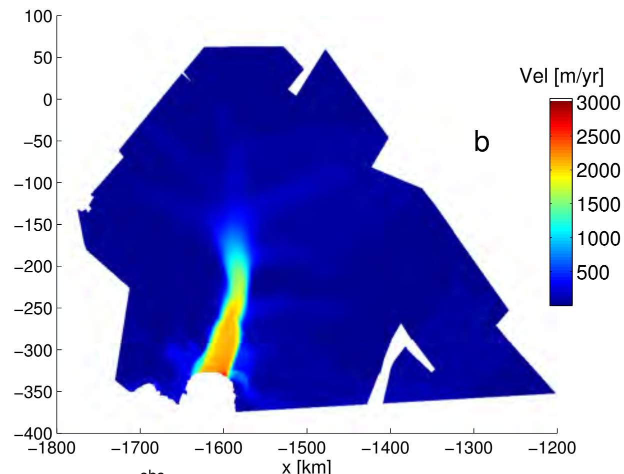
(b) Modeled velocity



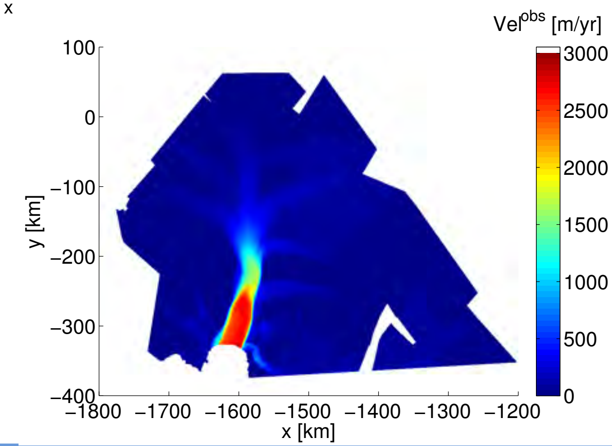
Application to Pine Island Glacier (2/2)



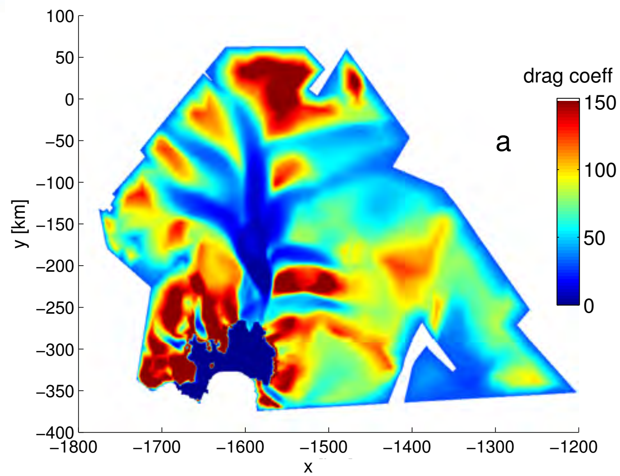
(a) Inferred basal friction
Iteration = 5



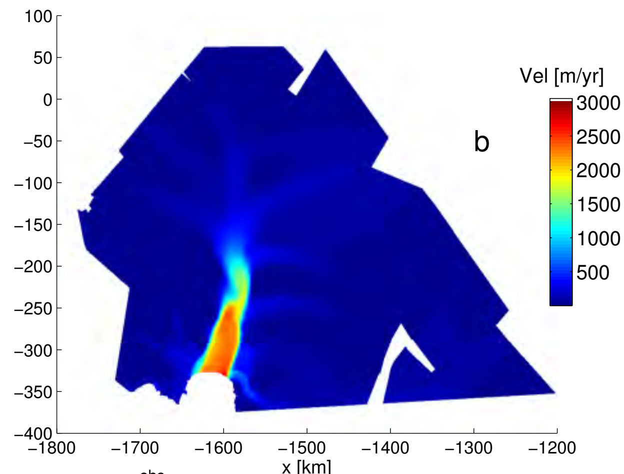
(b) Modeled velocity



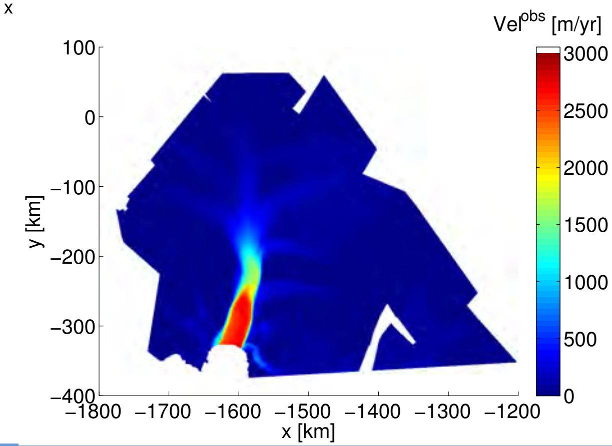
Application to Pine Island Glacier (2/2)



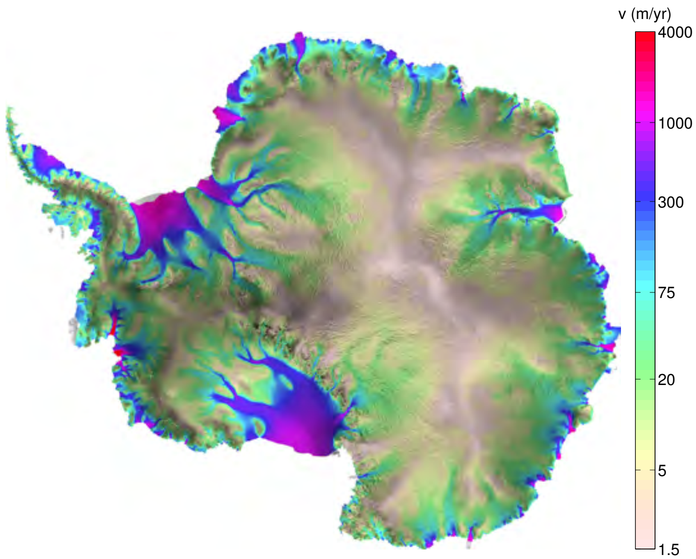
(a) Inferred basal friction
Iteration = 10



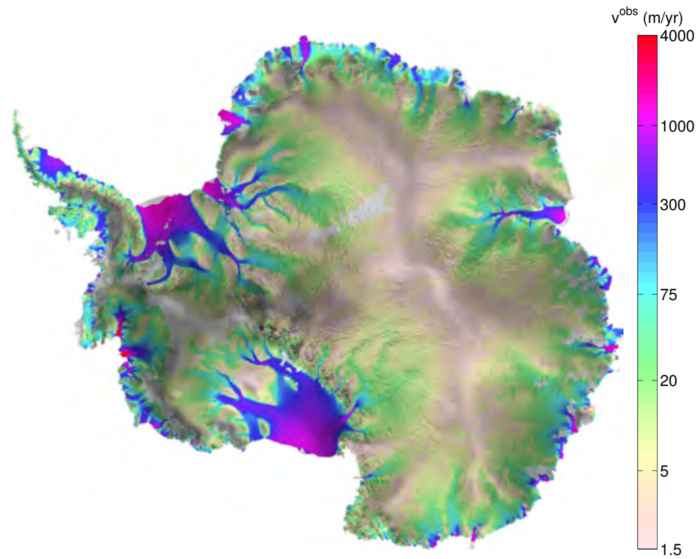
(b) Modeled velocity



Application to the Antarctic Ice Sheet



Modeled surface velocity (Morlighem et al., 2013)



InSAR derived velocity (Rignot et al., 2011)

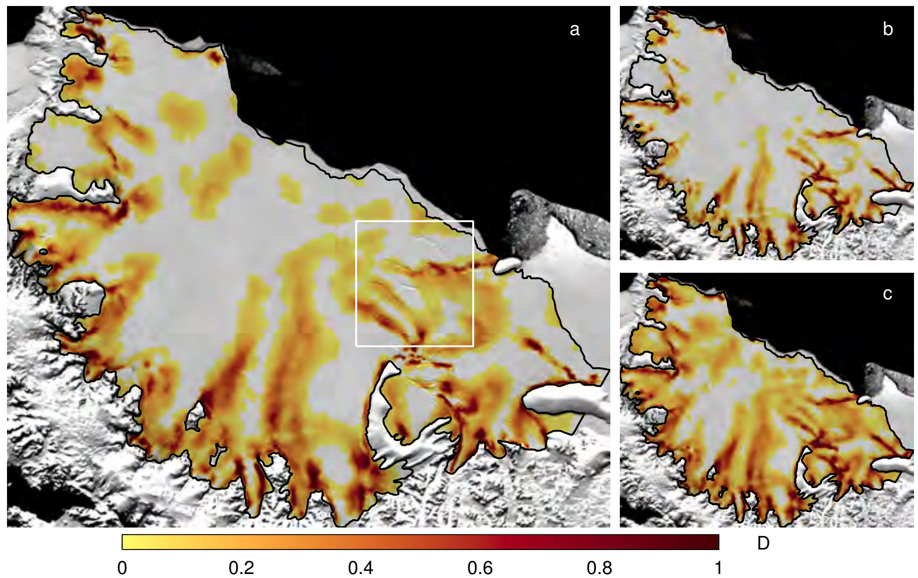
Applications of data assimilation

Observations	Parameter inferred
Surface velocity	Basal friction
Surface velocity	Ice shelf rheology
Surface velocity	Ice shelf damage
Borehole temperature	History of surface temperature
Internal layers	Accumulation rates
Firn temperature	Firn thermal conductivity

Applications of data assimilation

O	red
Su	
Su	gy
Su	e
Boreh	perature
In	es
Fir	ctivity

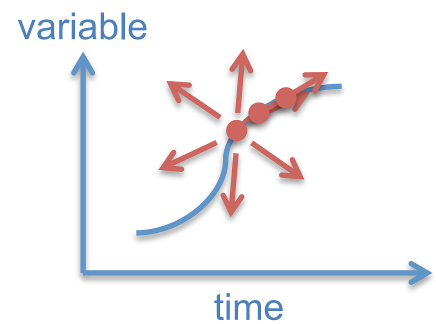
Inferred damage of Larsen C ice shelf



Borstad et al., 2015

Automatic Differentiation

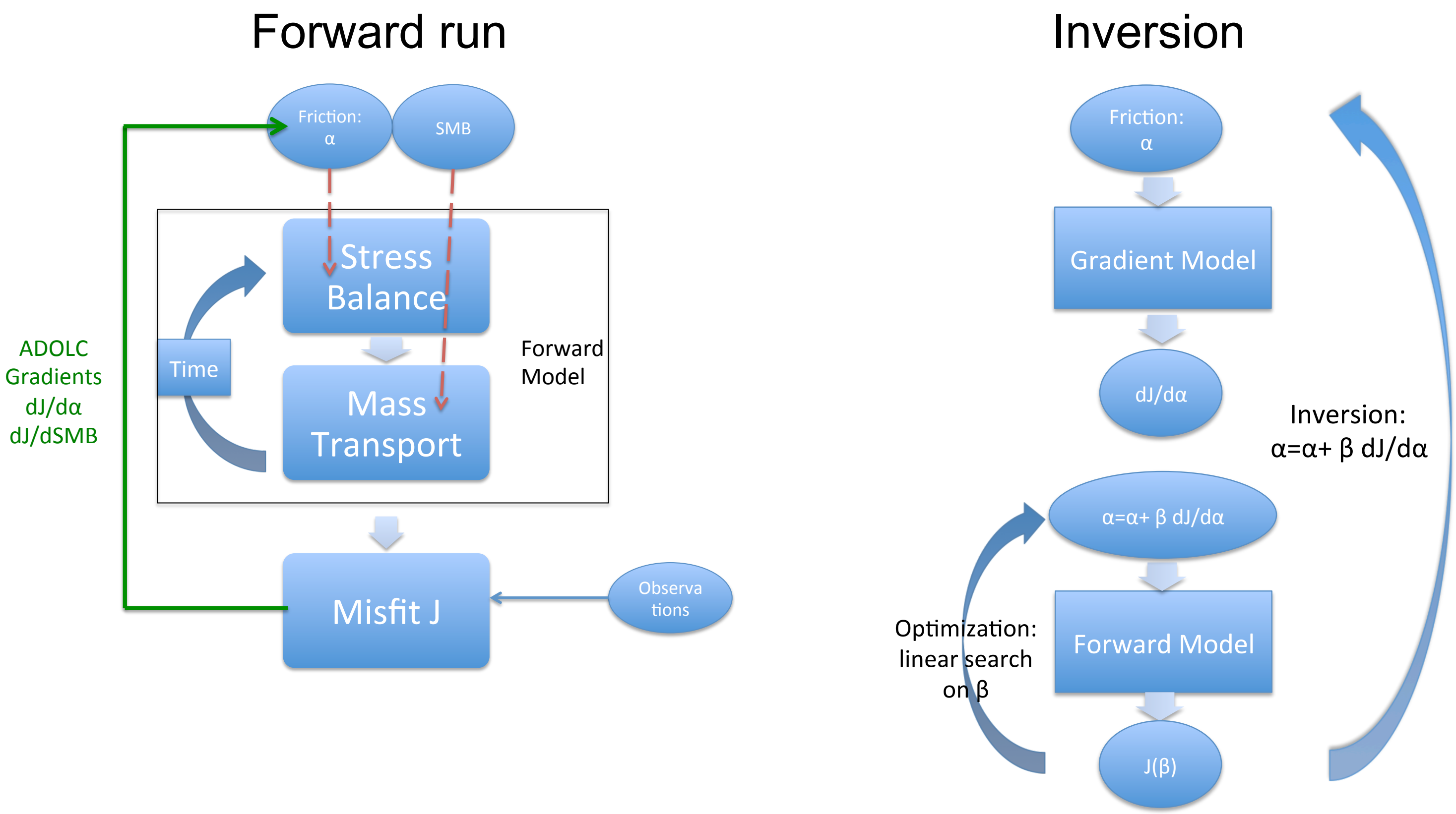
Improvement of ice sheet state estimation and transient sensitivity assessments



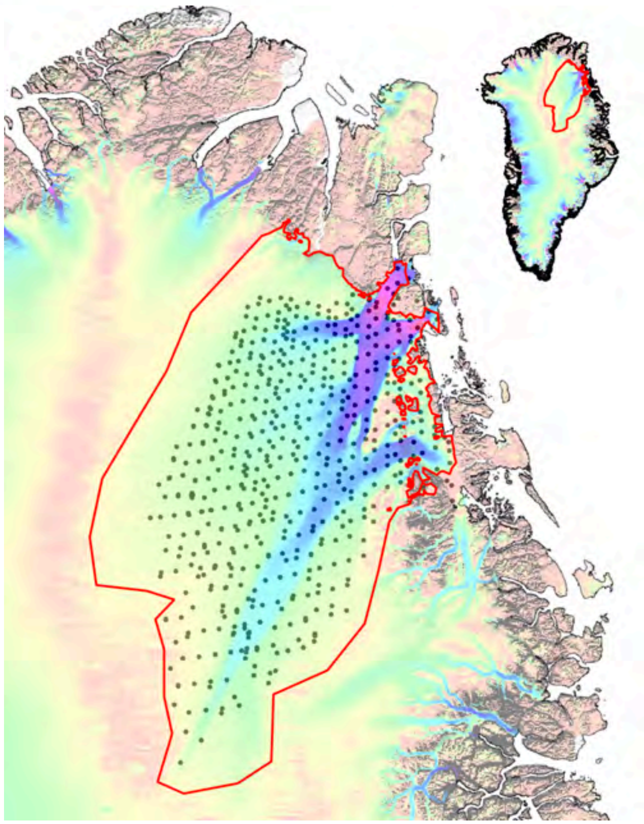
- Most ice sheet models use time-independent inversions
  - Gradients and adjoints manually computed
  - Based on a single observational input
  - Initial states with artificial drift

- Time-dependent inversions starting to emerge
  - Based on a combination of inputs
  - Time dependent problem
  - Need highly efficient model
  - Gradient computation: source to source transformation, object overloading, …
  - Similar to “state and parameter estimation” in oceanography

Algorithm for Automatic Differentiation



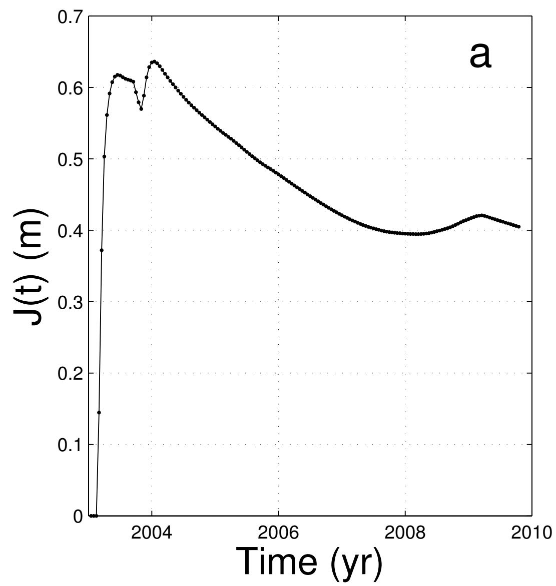
Application to NEGIS, Greenland



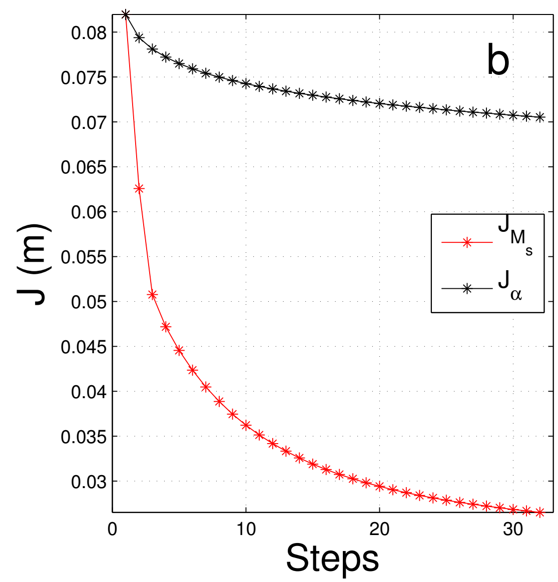
Larour et al., 2014

$$J = \frac{1}{s_\Pi}\frac{1}{T}\int_\Pi \int_{t=0}^{T} \frac{\left(s\left(t\right) - s\left(t\right)_{obs}\right)^2}{2} dx dy dt$$

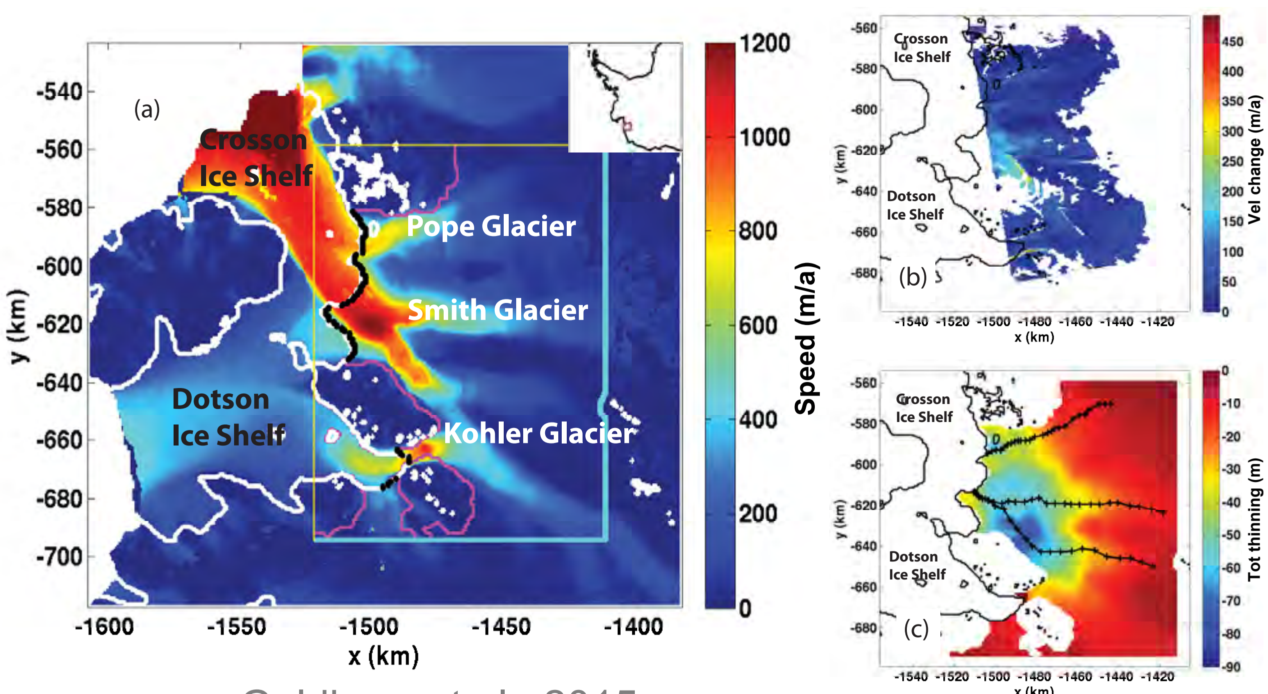
Cost function forward



Cost function inversion



Application to West Antarctic ice streams



Goldberg et al., 2015

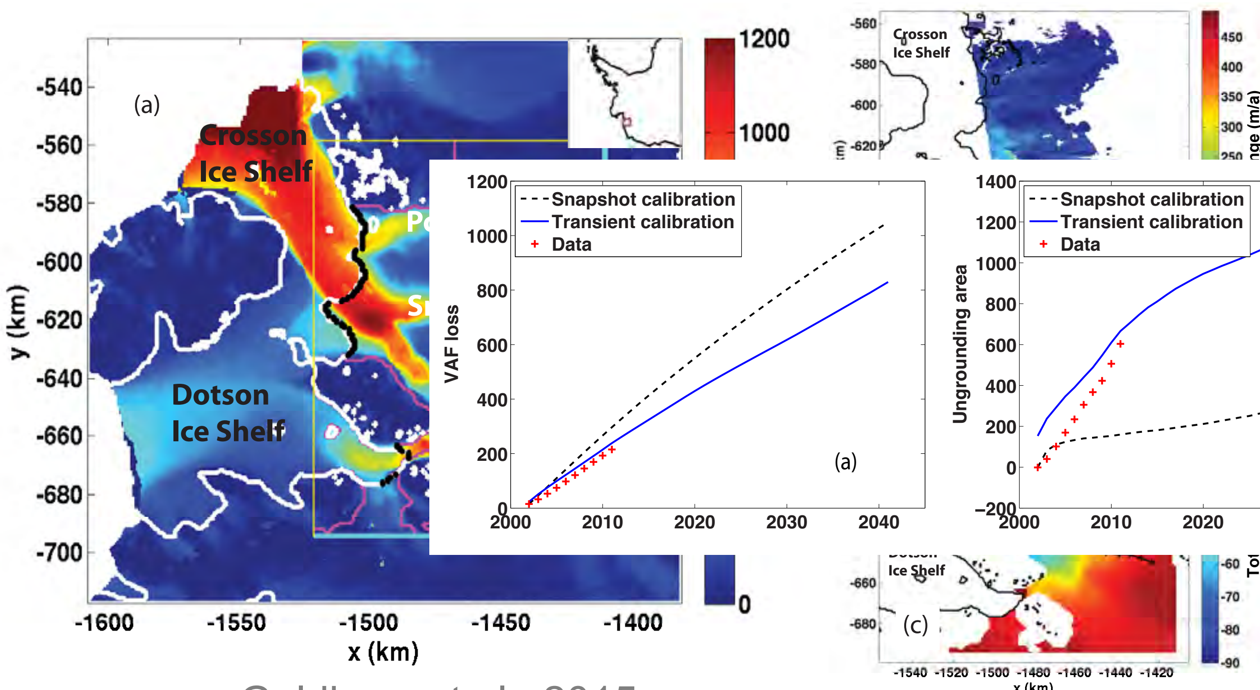
Time-independent cost function

$$J_{\text{snap}} = \sum_{i=1}^{N} \frac{|\boldsymbol{u}_i - \boldsymbol{u}_i^*|^2}{\eta(\boldsymbol{u}_i)^2},$$

Time-dependent cost function

$$J_{\text{trans}} = \omega_u \sum_{k=1}^{T} \sum_{i=1}^{N} \chi_{ki}^{(u)} \frac{|\boldsymbol{u}_i^{(k)} - \boldsymbol{u}_i^{(k)*}|^2}{\eta\left(\boldsymbol{u}_i^{(k)}\right)^2} + \omega_s \sum_{k=1}^{T} \sum_{i=1}^{N} \chi_{ki}^{(s)} \frac{\left(s_i^{(k)} - s_i^{(k)*}\right)^2}{\eta\left(s_i^{(k)}\right)^2},$$

Application to West Antarctic ice streams



Goldberg et al., 2015

Time-independent cost function

$$\sum_{i=1}^{N} \frac{|\boldsymbol{u}_i - \boldsymbol{u}_i^*|^2}{\eta(\boldsymbol{u}_i)^2}$$

endent cost function

$$\sum^{T} \sum_{i=1}^{N} \chi_{ki}^{(u)} \frac{|\boldsymbol{u}_i^{(k)} - \boldsymbol{u}_i^{(k)*}|^2}{\eta\left(\boldsymbol{u}_i^{(k)}\right)^2}$$

$$\sum_{k=1}^{T} \sum_{i=1}^{N} \chi_{ki}^{(s)} \frac{\left(s_i^{(k)} - s_i^{(k)*}\right)^2}{\eta\left(s_i^{(k)}\right)^2},$$

Calibration of parameters

Jakobshavn Isbrae, West Greenland

- Significant front retreat since 1980's
- Dynamics dominated by ice front evolution
- Calibrate parameters against observations
- Weight simulations

Bondzio et al., 2017; 2018

Ice front evolution

$$\frac{\partial \phi}{\partial t} + (\mathbf{v} - (c + m_{\mathrm{fr}})\,\mathbf{n}) \cdot \nabla \phi = 0 \qquad (1)$$

where

$$c = \|\mathbf{v}\| \frac{\tilde{\sigma}}{\sigma_{\max}} \qquad (2)$$

$$m_{\mathrm{fr}} = \left(A h q_{\mathrm{sg}}^{\alpha} + B\right) TF^{\beta} \qquad (3)$$

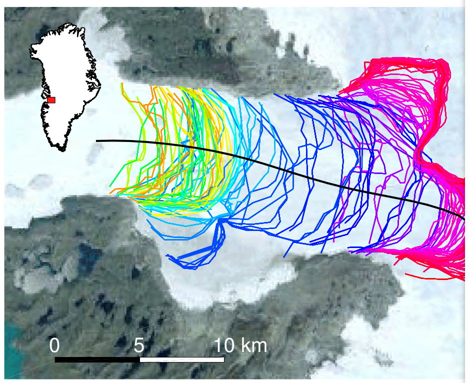
$$m_{\mathrm{sub}} = -\rho_M c_{\mathrm{pM}} \gamma_T \left(TF - T_{\mathrm{pmp}}\right) \qquad (4)$$

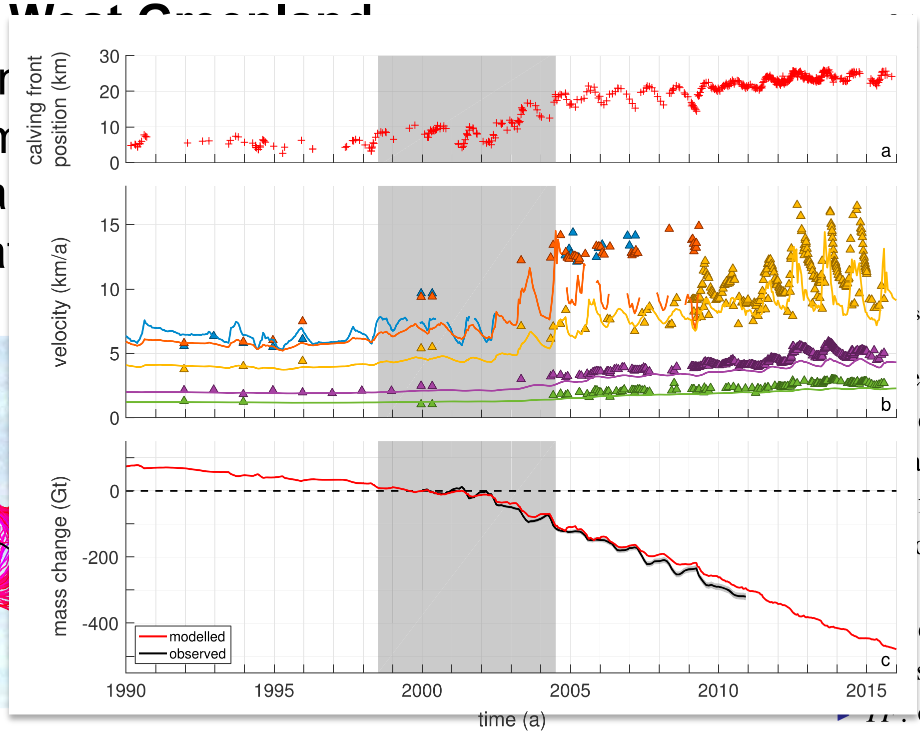
- ϕ: level-set function
- $\mathbf{v}$: ice velocity at the ice front
- c: calving rate
- m_{fr}: frontal melting rate ("undercutting")
- $\tilde{\sigma}$: von-Mises tensile stress
- $\sigma_{\max}$: stress threshold parameter
- h: ice thickness
- q_{sg}: subglacial discharge
- TF: ocean thermal forcing
- m_{sub}: subglacial melt
- T_{pmp}: ice pressure melting point

Calibration of parameters

Jakobshavn Isbrae, W...

- Significant fro...
- Dynamics dom...
- Calibrate para...
- Weight simula...





Bondzio et al., 2017; 2018

Ice front evolution

$$+ (\mathbf{v} - (c + m_{\mathrm{fr}})\,\mathbf{n}) \cdot \nabla\phi = 0 \qquad (1)$$

$$c = \|\mathbf{v}\| \frac{\tilde{\sigma}}{\sigma_{\max}} \qquad (2)$$

$$m_{\mathrm{fr}} = \left(A h q_{\mathrm{sg}}^{\alpha} + B\right) TF^{\beta} \qquad (3)$$

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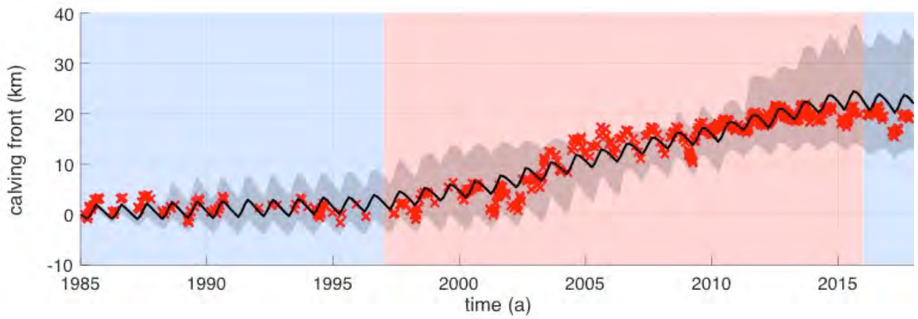
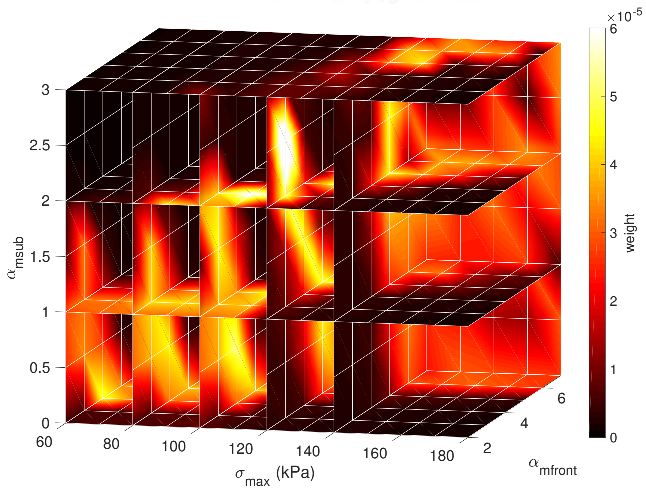
- ...vel-set function
- ...e velocity at the ice front
- ...lving rate
- ...frontal melting rate (“undercutting”)
- ...on-Mises tensile stress
- ...: stress threshold parameter
- ...e thickness
- ...subglacial discharge
- ...: ocean thermal forcing
- m_{sub}: subglacial melt
- T_{pmp}: ice pressure melting point

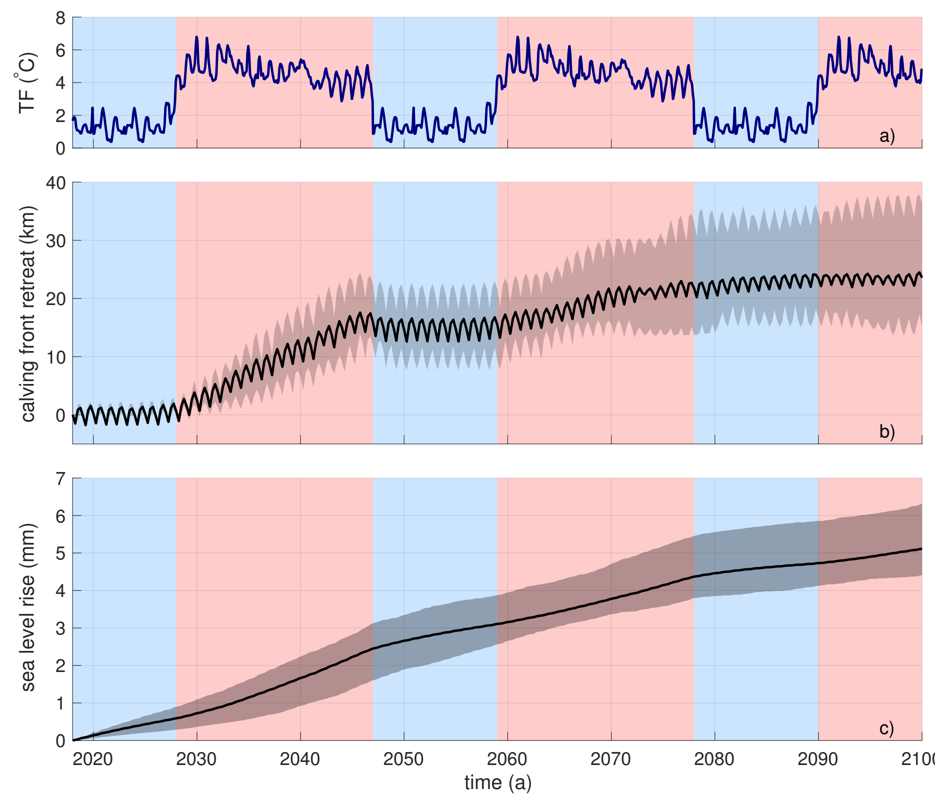
Calibration of parameters

Hindcast



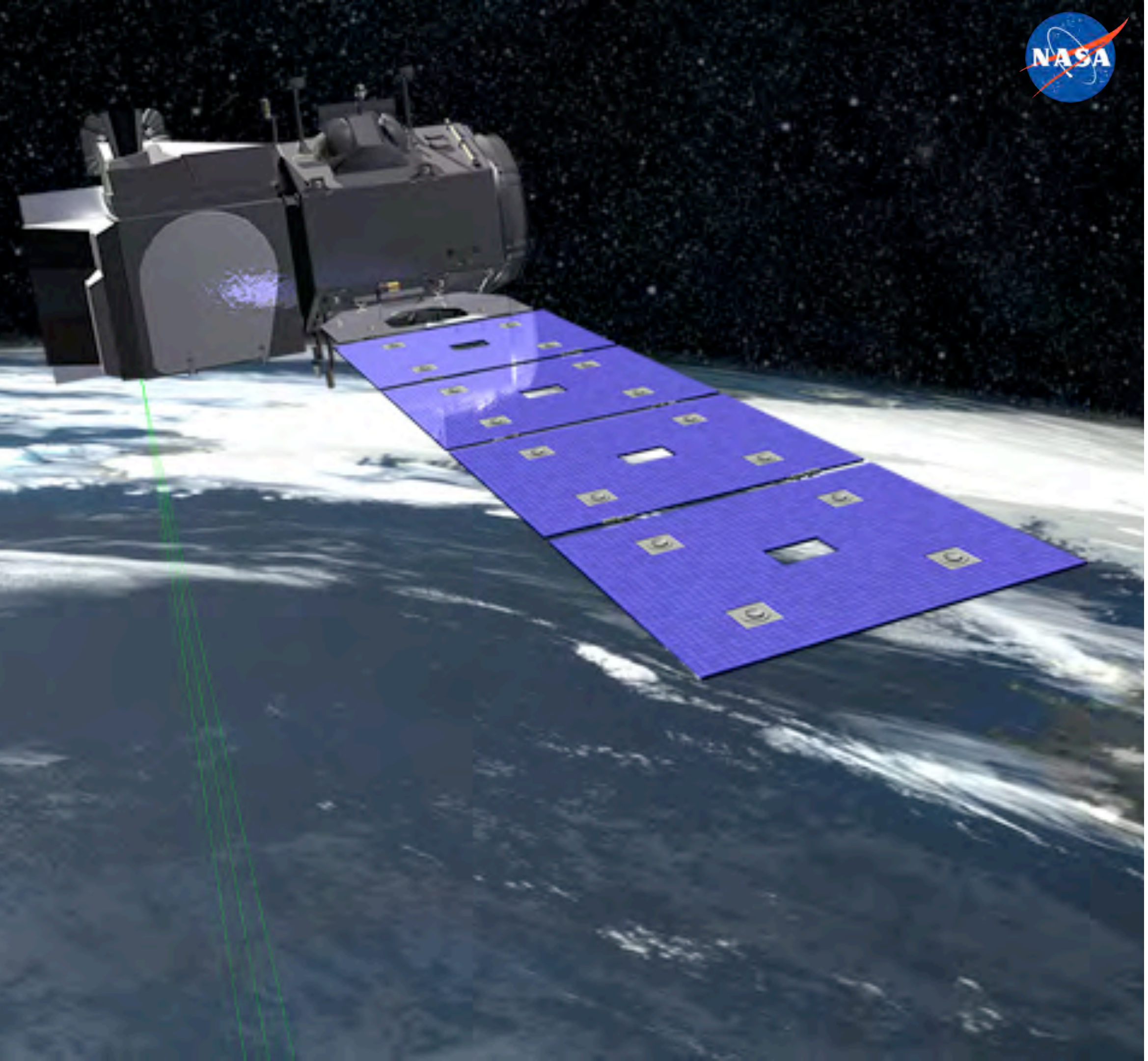


and forecast



Conclusions

- Rapid shift from almost no observations to observations acquired at high temporal resolution
- Observations used for:
    - Data assimilation (unknown parameters, initial conditions)
    - Calibration of parameters (unknown parameters)
- So far, time-independent assimilation mostly, but lead to significant and prolonged model drifts
- **New models should be designed around time-dependent data assimilation** to improve initial conditions and model drifts



The Future of Earth System Modeling:
Polar Climates

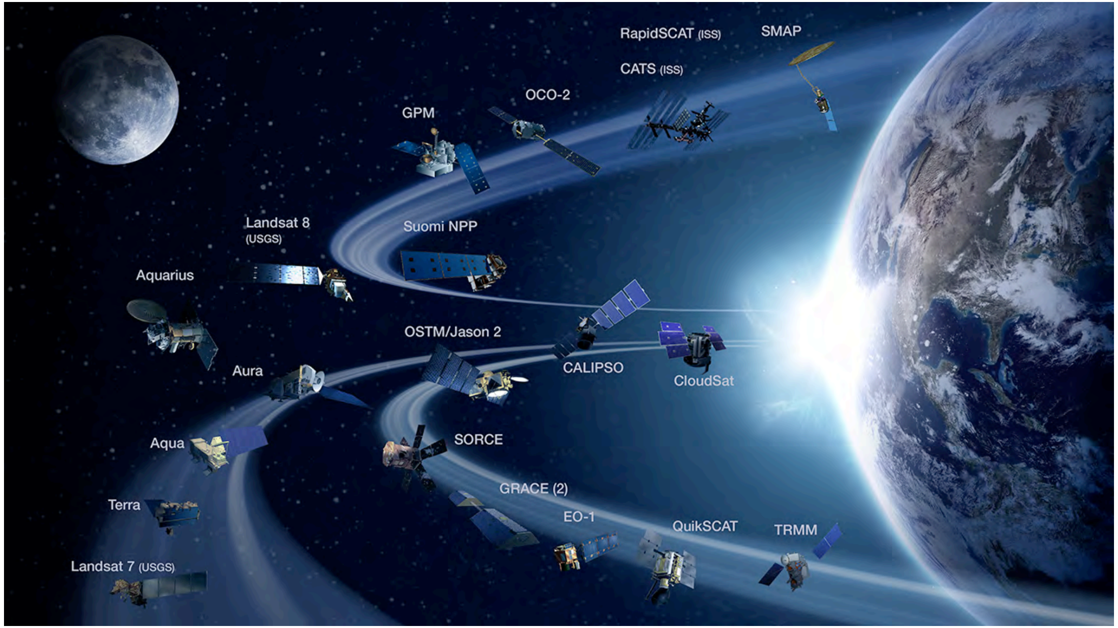
Ice Sheet Response: From Observations to Models

Alex Gardner
Helene Seroussi
Eric Larour

Jet Propulsion Laboratory
California Institute of Technology

Key properties that can be directly measured from space

- Ice Sheet Elevation (since-1992)
- Ice Sheet Mass Change (2002)
- Surface melting – binary (1979)
- Surface velocity (patchy)
- Surface reflectance (2000)
- Ice sheet extent (patchy)
- Grounding lines (patchy)



Challenging to measure

- Precipitation (CloudSat?)
- Melt magnitude
- Sublimation / blowing snow
- Runoff
- Ice thickness
- Basal properties
- Ice viscosity

Summary of current state

- Numerous measurements **but** …
- Data are generally difficult to work with and require specialized knowledge of the measurement and instrument
- It’s going to get easier… and soon

What's coming

- We are entering an era where availability of observations will not be the limiting factor
- Sentinel-1A/B SAR data is pouring in
- Landsat 8 and Sentinel-2 A/B data provide nearly continuous coverage
- GRACE-FO launched in May
- ICESat-2 launched in September
- NISAR will launch in late-2021/early-2022
- New NASA MEaSUREs initiative: ITS_LIVE

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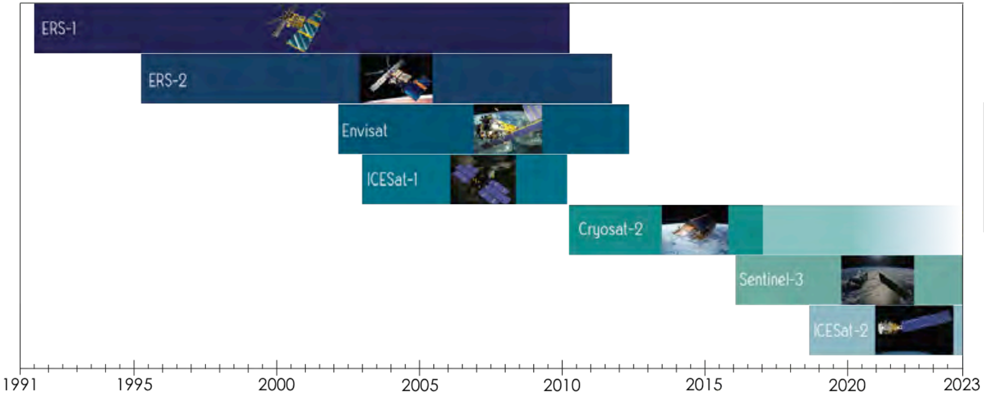
ITS_LIVE: Inter-mission Time Series of Land Ice Velocity and Elevation

- Global coverage
- Standardized nested grid
- Open-source code
- Synthesized products
- Low-latency
- HDF5, netCDF-4 compliant



Surface velocity

- image-pair results
- monthly/quarterly mosaics
- 240m resolution
- 2013-present
- Sparse data back to 1985

Elevation change

- Multi-mission synthesis
- 1992-present
- Monthly
- 960 m resolution
- Ice shelf melt rates

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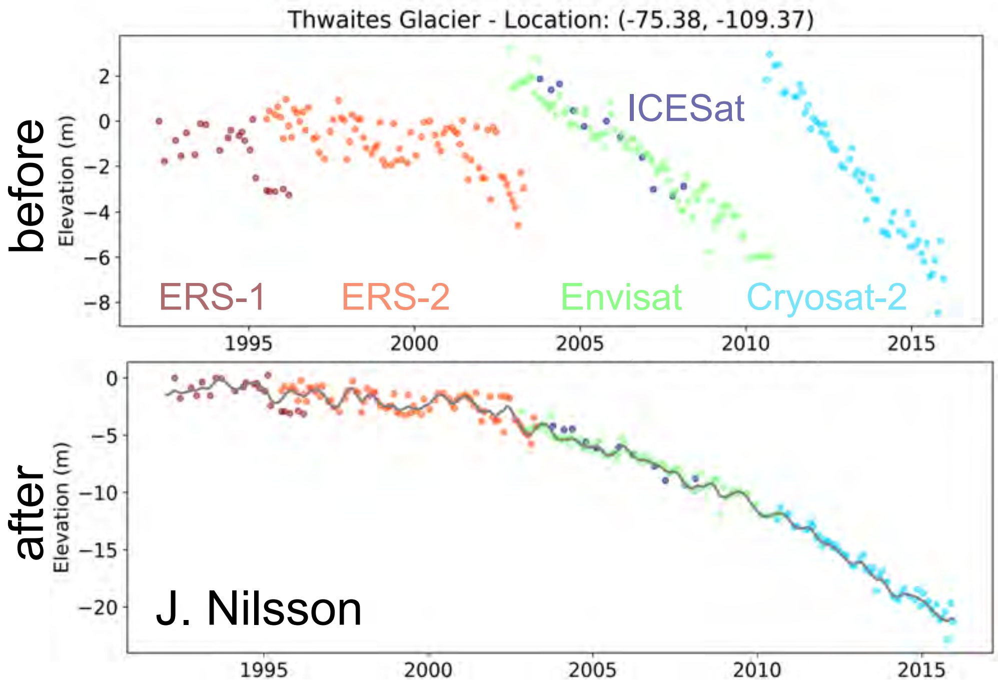
ITS_LIVE: Example of elevation processing

- Remove mean topography
- Cross-calibration
- Scattering correction
- Sensor synthesis using Kalman-Smoother algorithm

Each monthly spatial field is interpolated to 10 km resolution to create 3D data cube for further analysis



Single pixel time series over Thwaites Glacier cross-calibrated and integrated to produce a continuous record of elevation change.

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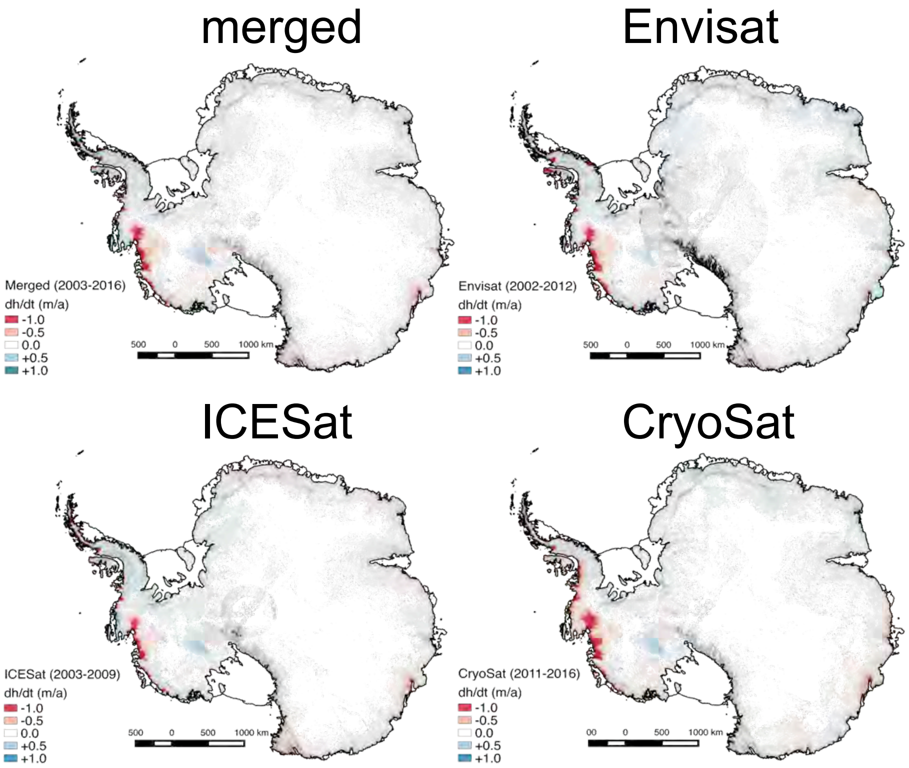
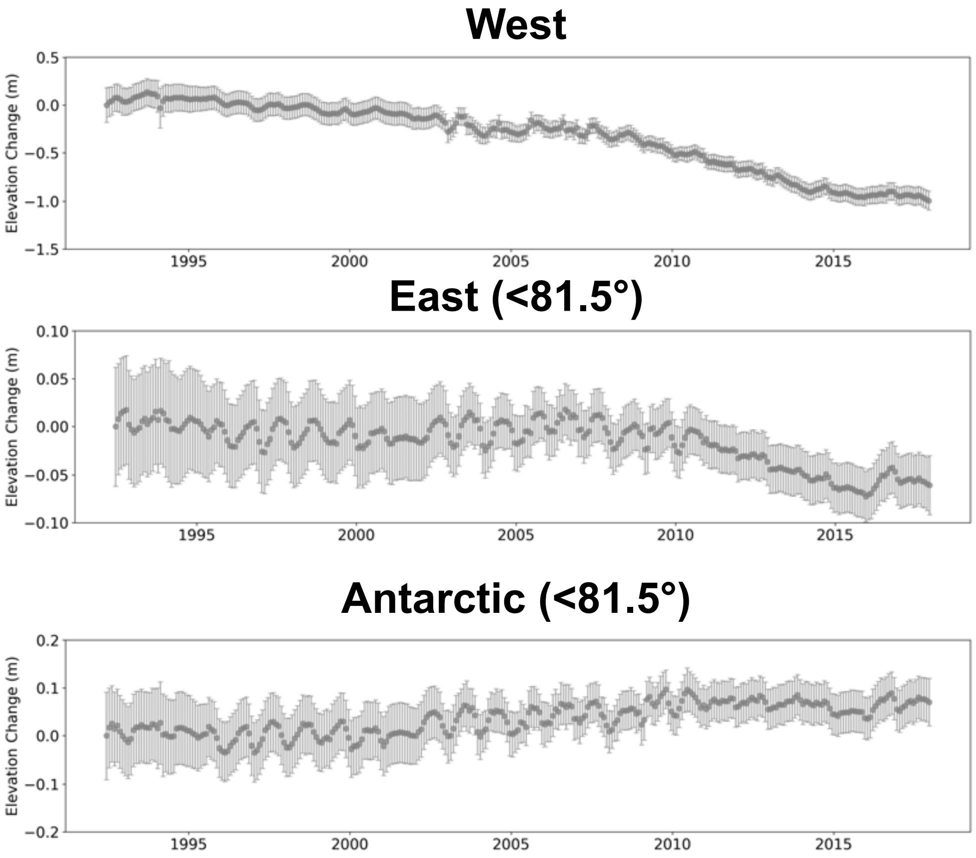
ITS_LIVE: 27 year record of grounded ice change





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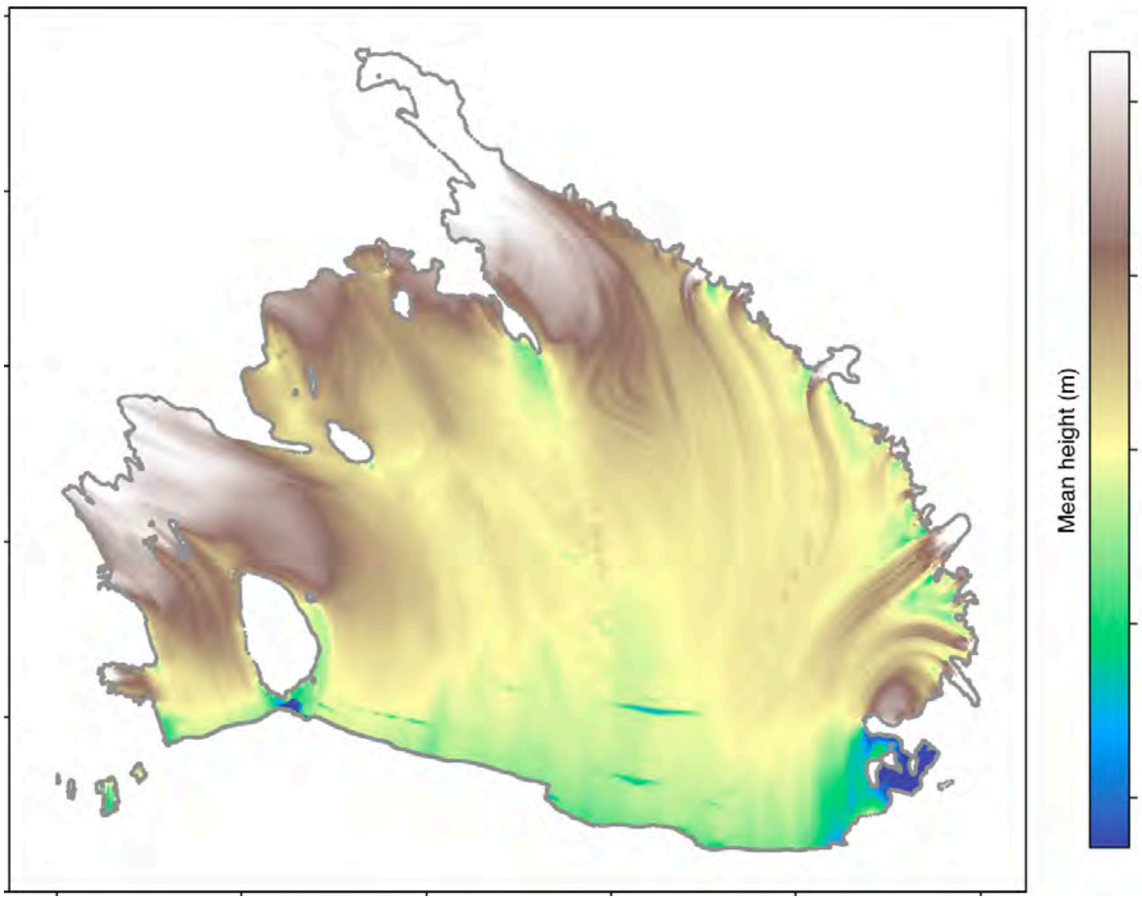
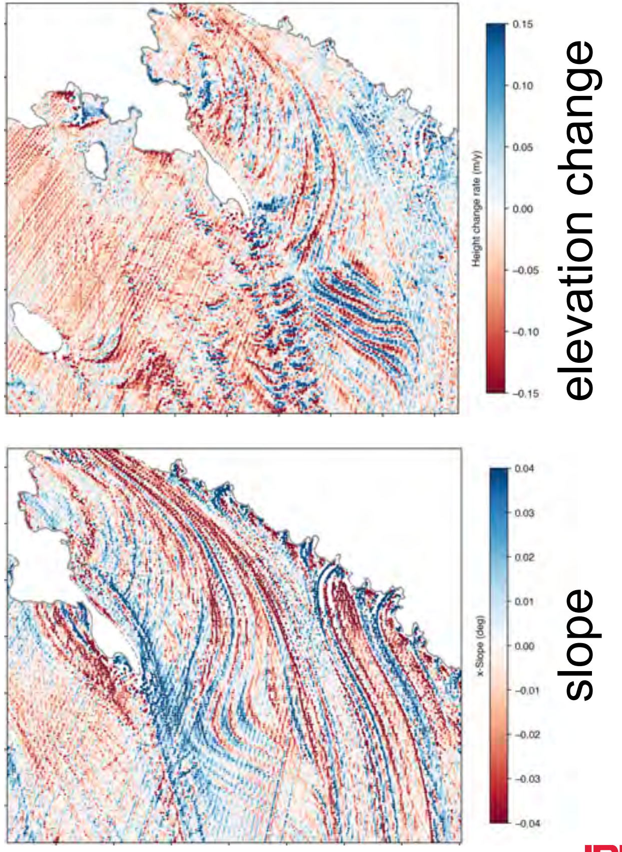
ITS_LIVE: Ice shelf melt





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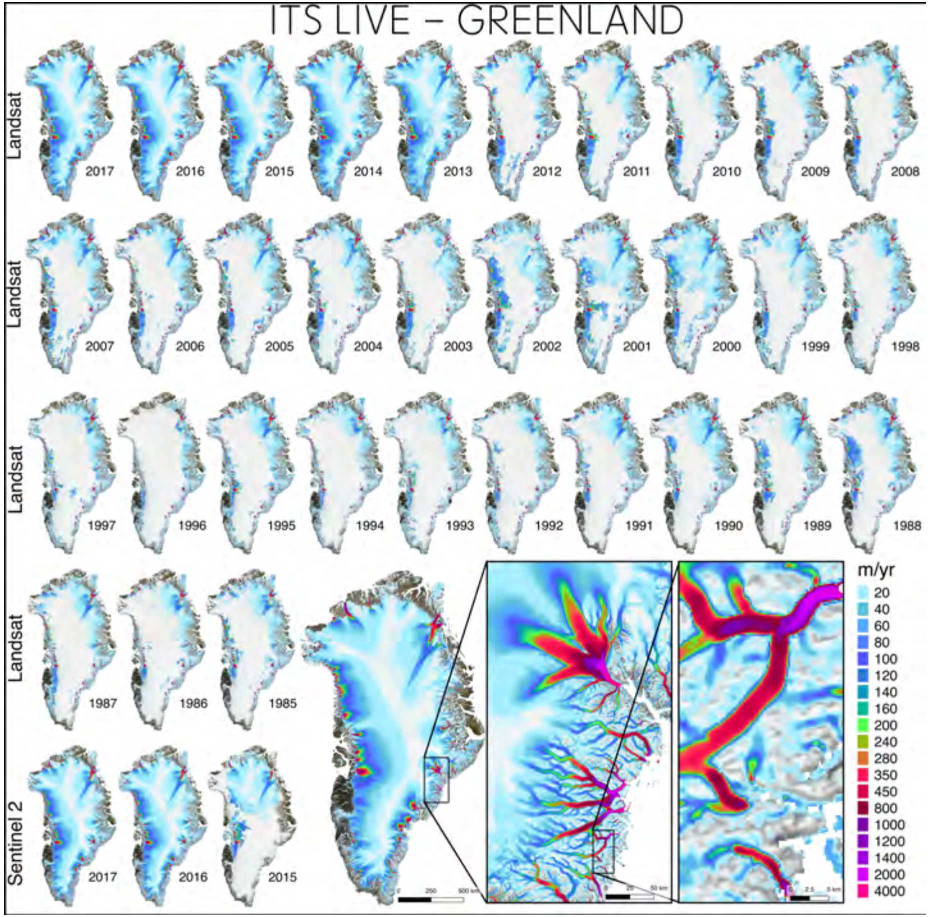
ITS_LIVE: 33 year record of Greenland ice discharge

- Extracting surface velocities from historic Landsat data… it's messy but the record of flow is there

- Requires heavy post processing: robust least squares and Kalman methods



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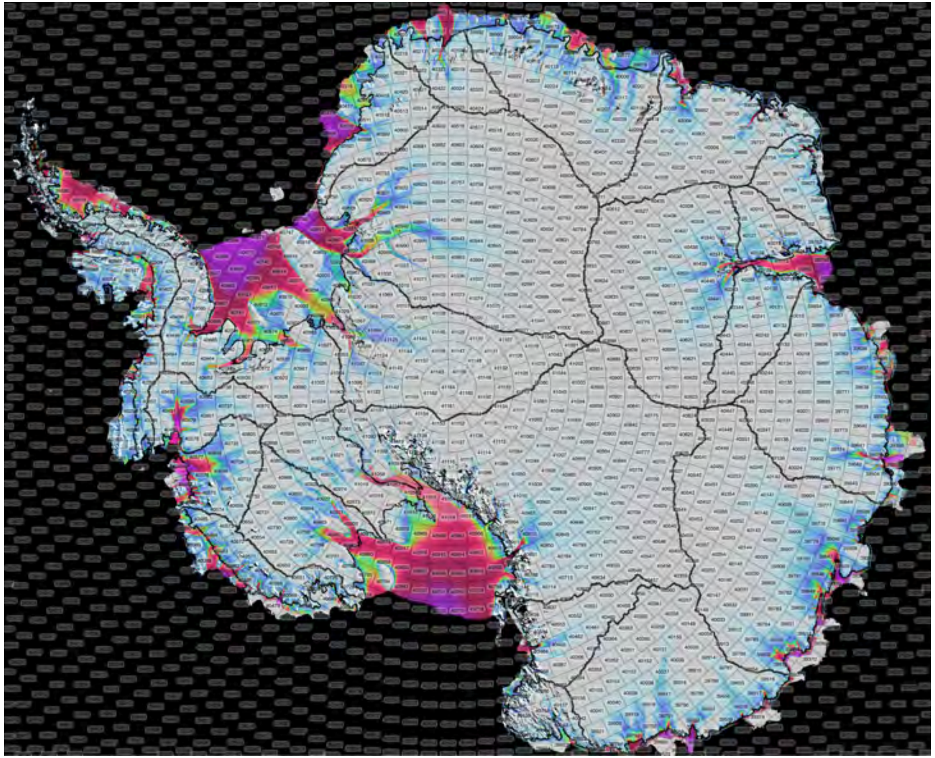
Joint Inversion of Satellite Gravimetry + Altimetry

- Data combination at the level of the GRACE normal equations
- 1° by 1° solution (41,164 disks on an ellipse)
- Corrections
  - Glacial Isostatic Adjustment
  - Elastic Loading
  - Firn Air Content



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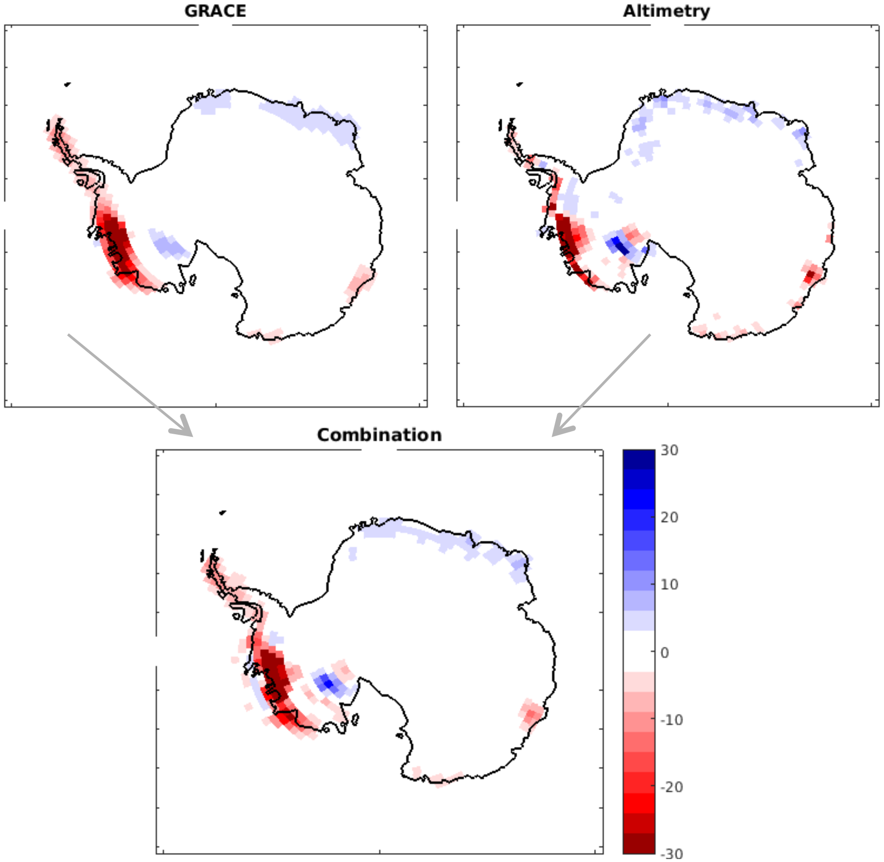
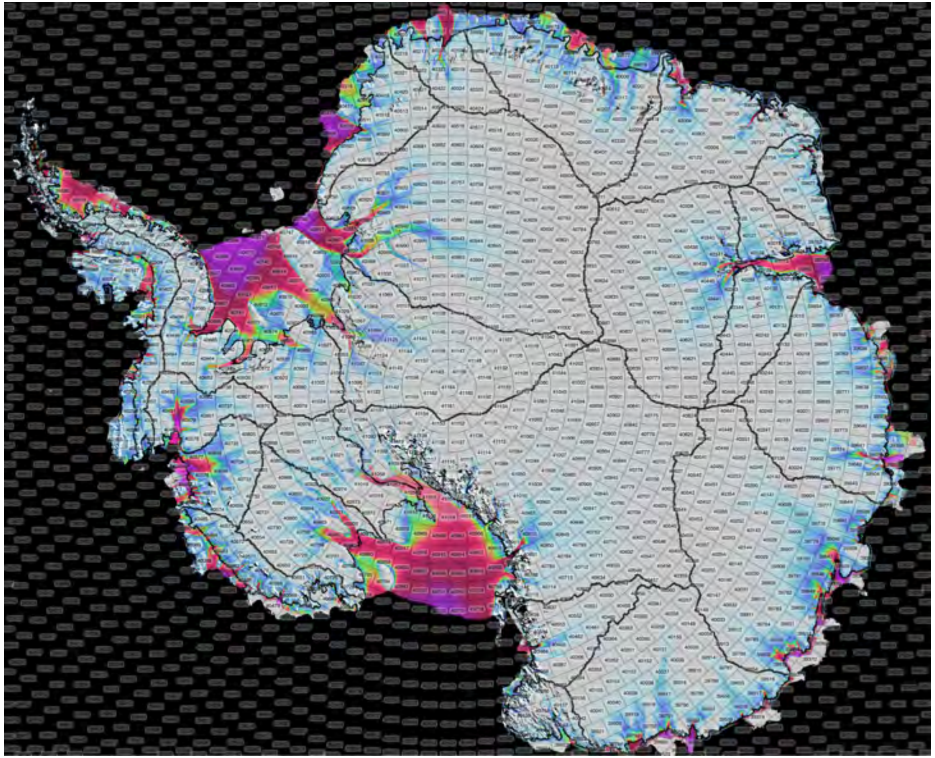
Joint Inversion of Satellite Gravimetry + Altimetry





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Summary of future state

In the near future measurements of ice sheet surface properties and their change through time will be bountiful and all will be merry

- Records of elevation change will be continuous from 1992
- Records of ice shelf melt will be continuous from 1992
- Records of ice velocity will be opportunistic prior starting in 1985 and continuous from 2014
- Get ready for a flood of observationally driven discoveries

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