Beyond $g(h)$
Modeling and Evaluating
Sea Ice Thickness in Earth System Models

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Outline

1 How should we design sea ice models to obtain better predictions of polar climate?

2 How can we better integrate observations with models?

3 What additional observations would help improve models?
Outline

1. How should we design coupled sea ice models to obtain better predictions of sea ice thickness, concentration and drift?

2. How can we better integrate thickness observations with models?

3. What additional thickness observations would help improve models?
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3 How should we design coupled sea ice models to obtain better predictions of sea ice thickness, concentration and drift?
Brief Introduction to the Thickness Distribution $g(h)$

model grid cell

$g(h)$

USGS Global Fiducials Library
Sea Ice Thickness Distribution

\[
m = \rho \int_{0}^{\infty} g(h) \cdot h \, dh
\]

\( g(h) \) is used to describe mass conservation in sea ice models:

\[
\frac{dg}{dt} = \Psi + \Theta - g(\nabla \cdot \dot{x})
\]

\( \Psi \) Dynamic Redistribution, 
\( \Theta \) Thermodynamic Redistribution
Introduction to $g(h)$

Beaufort Sea 2007
Introduction to $g(h)$

Ridge Cross Section

Introduction to \( g(h) \)

Sea Ice Thickness Distribution

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\( \Psi \) Dynamic Redistribution, \( \Theta \) Thermodynamic Redistribution
1. How can we better integrate thickness observations with models?

- Thickness ($h$) is an important variable to evaluate well, because it carries the dynamic and thermodynamic history of the pack.

- A robust method for evaluating modeled thickness has been elusive, partly due to uncertainty in observed snow thickness ($h_s$) and sea ice density ($\rho$).

- The method used opposite is not the answer to our problems.

1. How can we better integrate thickness observations with models?

Answer: Evaluate and train against freeboard, not thickness

Calculate $\bar{h}_{fm}$ in Earth System Models

$$\bar{h}_{fm} = \int_0^\infty \left[ h \left( \frac{\rho_w - \rho}{\rho_w} \right) + h_s \left( \frac{\rho_w - \rho_s}{\rho_w} \right) \right] g(h) \, dh$$

Instead of comparing $g(h)$ in a model with $\bar{h}$ from the satellite, we compare $\bar{h}_{fm}$ with $h_f$
1. How can we better integrate thickness observations with models?

We can evaluate: \( \text{Bias}[\bar{h}_{fm}(t)] \approx \bar{h}_{fm}(t) - \frac{1}{n} \sum_{i=1}^{n} \hat{h}_f(t_i) \)
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Evaluation of the Regional Arctic System Model (RASM) Freeboard

Bias $[\bar{h}_{fm}]$

Bias $[\bar{h}_{fm}, h_s \rightarrow 0]$
Details of the skill score in:
2. What additional **freeboard** observations would help improve models?

AR(1) series: \( h_i = 0.994 h_{i-1} + \varepsilon_i \)

Due to the high degree of autocorrelation in sea ice thickness time series, we need regular seasonal measurements of sea ice freeboard taken at the top of the snow layer, just as provided by ICESat and to be provided by ICESat-2 with a 91-day repeat orbit.
2. What additional **freeboard** observations would help improve models?

**Answer:** Seasonal *absolute* freeboard for as long as possible

ICESat, ICESat-2, ..., ICESat-N
3. How should we design models to better predict sea ice thickness?

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<table>
<thead>
<tr>
<th>Model Element</th>
<th>Variable or Component</th>
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<td>Compressive Strength</td>
<td>Internal Stress ( \sigma )</td>
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<tr>
<td>Scale Aware Density</td>
<td>Mass per unit area ( m )</td>
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<tr>
<td>Form Drag</td>
<td>External Stress ( \tau_a, \tau_w )</td>
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<td>Landfast Ice</td>
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<td>Melt Pond Drainage</td>
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<tr>
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<td>Carbon Cycle</td>
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</table>

The ‘Barrow List’ of parts of a modern sea ice model that need more than \( g(h) \)
‘We haven’t got any money, so we’ve got to think’
- Ernest Rutherford
The Variational Method:
Building up from a ridge, not down from a grid cell

\[ \int_{t_i}^{t_f} \int_A \left( \nabla \cdot \bar{\sigma} - m \frac{d\mathbf{x}}{dt} \right) \cdot \delta \mathbf{x} \, dA \, dt = 0 \]

Step 1: Coarse-grain ridging using simple polygons

Define a relationship between horizontal stress $\bar{\sigma}$ and potential energy density $\mathcal{V}$

$$0 = \frac{1}{3} \frac{d}{dt} \frac{\partial \mathcal{T}}{\partial \dot{x}} - \frac{\partial \mathcal{V}}{\partial \dot{x}}$$

Imposing Coulombic failure, we isolate the conservative part of the system ($\mathcal{T}$ and $\mathcal{V}$ are kinetic and potential energy density, respectively)
Demonstration of the influence of macroporosity

Comparative effect of strain $\epsilon_{R_I}$ and macroporosity $\phi_R$
Step 2: Apply the variational constraint to the ridge, giving $\nabla_R \cdot \mathbf{d} = 0$

We now have a solution to the initial value problem, with a state space trajectory (red).
The bivariate thickness distribution $g(h, \phi)$

Redistribution of an individual ridge:

$$m = \rho \int_0^\infty \int_0^1 g(h, \phi) (1-\phi) \ h \ d\phi \ dh$$

Ice density $\rho$ includes microporosity. Ridges defined by strain $\epsilon_R$, porosity $\phi_R$, and angle of repose $\alpha_R$. 
Step 3: Derive the resulting ridge frequency statistics

The $\hat{\zeta}$-plane, along the initial value state space trajectory on the previous slide.
Consequently, stationarity applies to collections of ridges

\[ 0 = \delta \iint_A \mathcal{V}_R \, dA \]
Which defines redistribution of $g(h)$ analytically

$$g(h) = \int g(h, \phi) \, d\phi$$

$$\frac{dg(h, \phi)}{dt} = \Psi(h, \phi) - g(h, \phi)(\nabla \cdot \dot{x})$$
Combined variational ridging and ICESat-2 emulator

\[ \bar{h}_{fm} = \int_0^\infty \int_0^1 \left[ h \left( \frac{\rho_w - (1 - \phi)\rho}{\rho_w} \right) + h_s \left( \frac{\rho_w - (1 - \phi)\rho_s}{\rho_w} \right) \right] g(h, \phi) \, d\phi \, dh \]

Calculate \( \bar{h}_{fm} \) in Earth System Models
Conclusions

1. How can we better integrate thickness observations with models?
   - Use satellite freeboard measurements with an emulator.

2. What additional freeboard observations would help improve models?
   - Ongoing laser altimetry using a seasonal repeat orbit: ICESat-N

3. How should we design sea ice models to obtain better predictions of
   sea ice thickness, concentration and drift?
   - Expand the model state space to accommodate the local
     organization of ridges and floes, e.g. $g(h, \phi)$