# Some Thoughts on Using Machine Learning for Cumulus Parameterization

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# Scope

- Construct cumulus parameterizations from cloud resolving model simulations through machine learning.
- This is not a substitute for global cloud resolving models
- But it is still an interesting and useful problem given the success of super-parameterization.

# Potential difficulties

- Intrinsic noise
- Memory
- Coupling with large-scale
- High dimensionality

# Noise

- There are intrinsic noises in convective tendencies from CRM simulations.

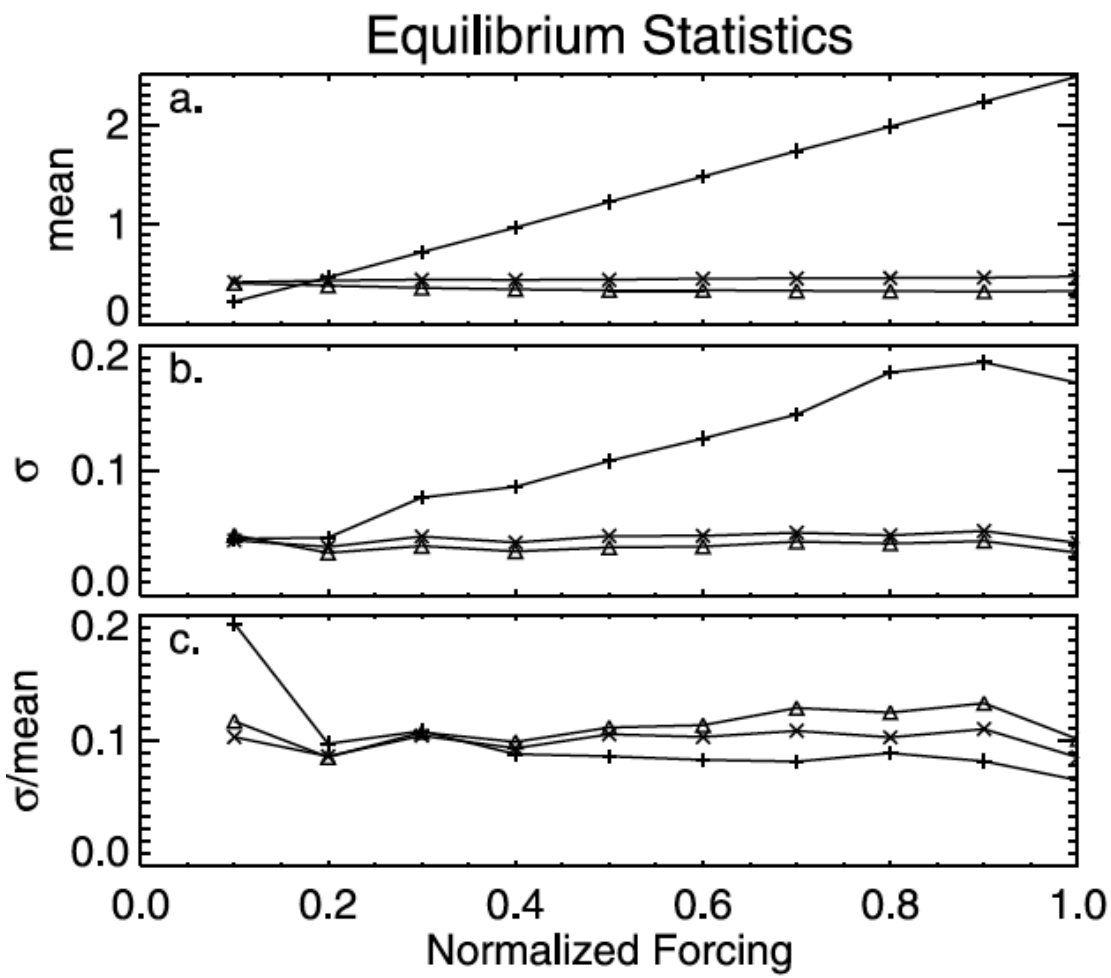



Figure from Jones and Randall (2011)

For a 256kmx256km domain (dx=dy=2km) subject to constant forcing

# Noise

- Care must be taken to ensure correct direction of causation.

x is the state variable

y is the convective tendency

n indicates the nth time step

$$y_n = -\lambda x_n + \sigma_n$$

$$\lambda = 1$$
$$\Delta t = 1$$

$$x_{n+1} = x_n + y_n \Delta t$$

$$\frac{\langle x, y \rangle}{\langle x, x \rangle} = -1$$

x is uncorrelated with the noise

Do a two-point average (sum)

$$X_n \equiv x_{2n-1} + x_{2n}$$
$$Y_n \equiv y_{2n-1} + y_{2n}$$

$$\frac{\langle X, Y \rangle}{\langle X, X \rangle} = -\frac{1}{2}$$

X is no longer uncorrelated with the noise

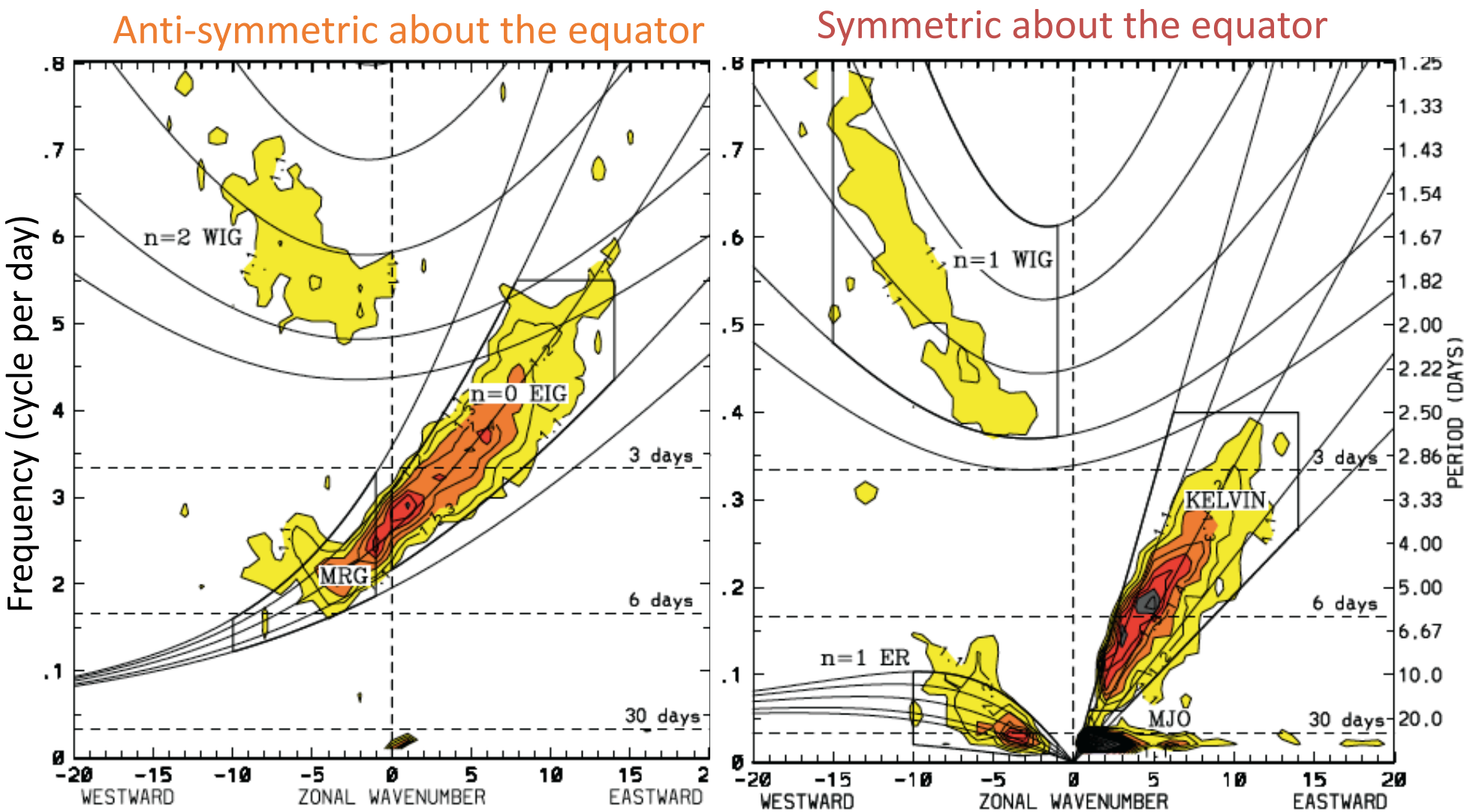
# Noise

- Care must be taken to ensure correct direction of causation.
- For example, using hourly averages as in Krasnopolsy et al. (2013) will confuse the direction of causation, as noise can now influence the state vector. This influence will be interpreted incorrectly as the dependence of convective tendencies on the state vector.
- The use of instantaneous values can also be complicated by the presence of memory (in the noises).

# Coupling with large-scale dynamics

- In a typical machine learning problem, minimizing the prediction error for the process in question is the end goal.
- Here, convection is a sub-component. Errors in convective tendencies along different directions can be amplified or damped by coupling.
- **This makes the problem distinct from a typical machine learning problem**.

# Convectively coupled waves as an example




Space-time spectra (averaged over 15N-15S) (with background red noise removed)

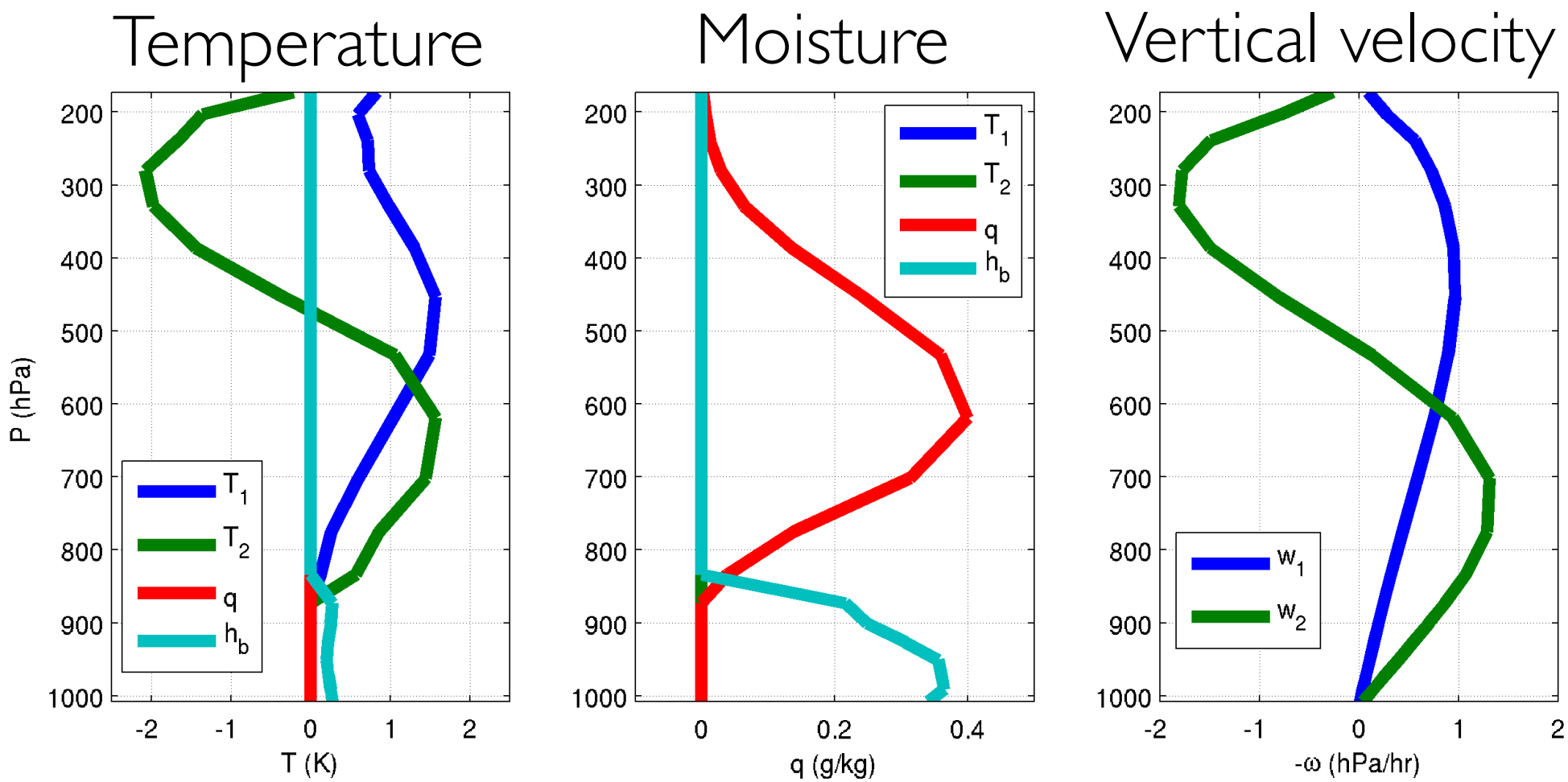
After Wheeler and Kiladis, J. Atmos. Sci., 1999

# A linear model of convectively coupled waves

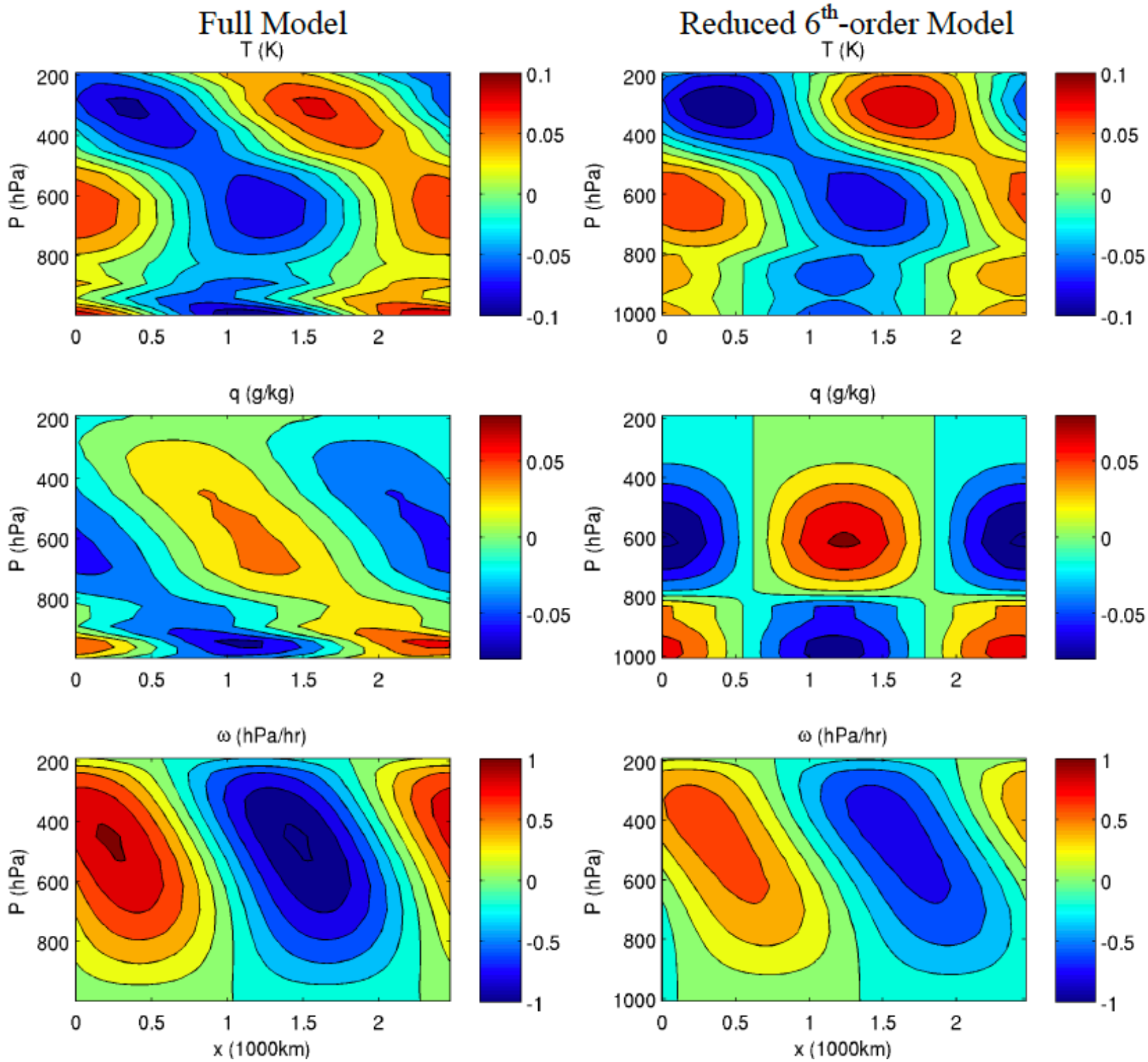
$$\frac{d}{dt}\begin{pmatrix} \vec{x} \\ \vec{w} \end{pmatrix} = \begin{pmatrix} \mathbf{M} & \mathbf{A} \\ k^2\mathbf{C} & \mathbf{D} \end{pmatrix}\begin{pmatrix} \vec{x} \\ \vec{w} \end{pmatrix} + \vec{f}$$

x: thermodynamic profiles (temperature and humidity)
w: vertical velocity profiles
f: forcing
k: horizontal wavenumber
M: thermodynamic linear response function for convection
A: effect of vertical advection on temperature and humidity
C: effect of temperature and humidity on vertical velocity
D: momentum damping

# Reduce convection to 4 degrees of freedom




These are not set *a priori*, and come from the order reduction procedure.

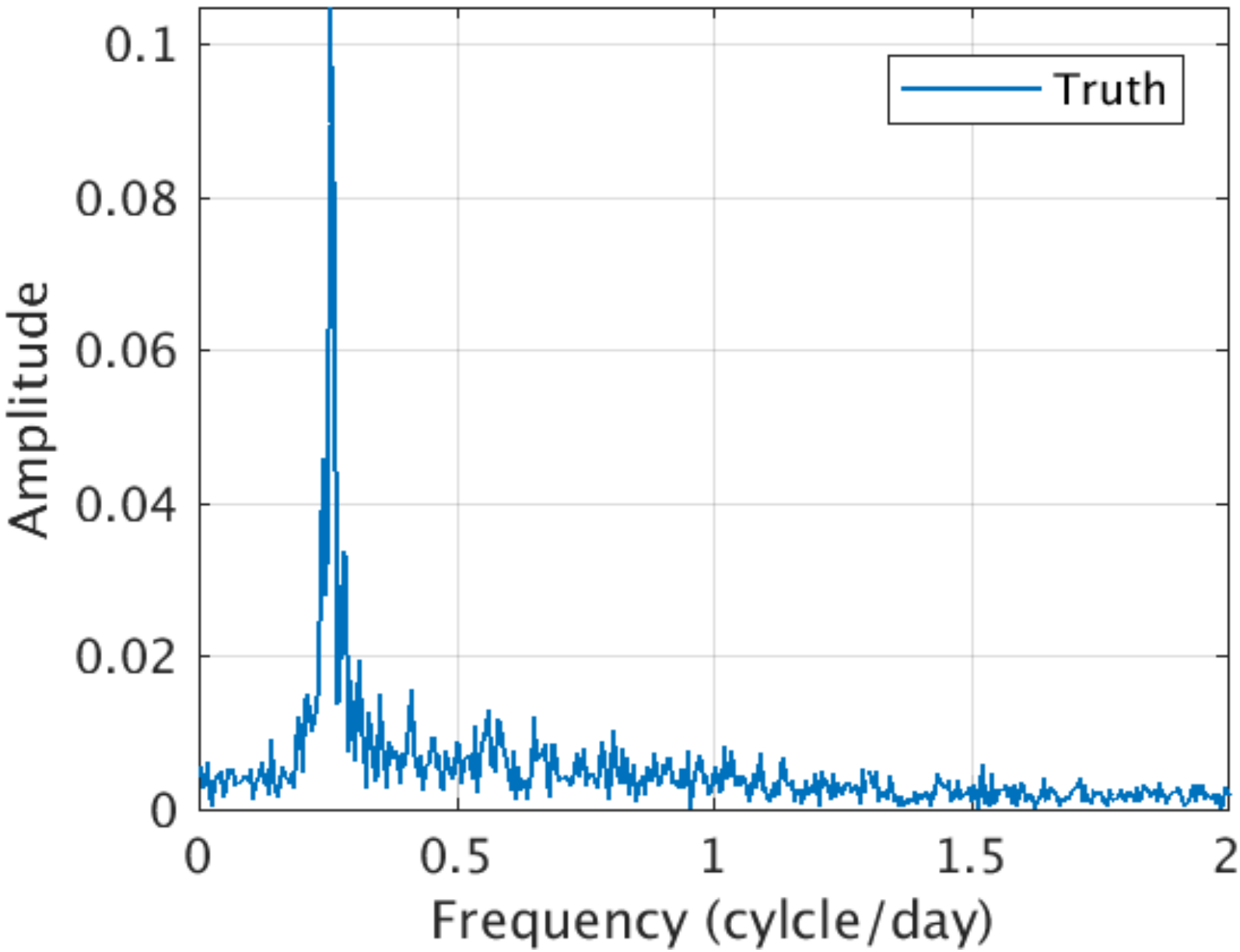

Full Model
Reduced 6th-order Model
T (K)
q (g/kg)
ω (hPa/hr)
P (hPa)
x (1000km)
200
400
600
800
1000
0
0.5
1
1.5
2
0.1
0.05
-0.05
-0.1
-0.5
-1

# Learn this small linear system

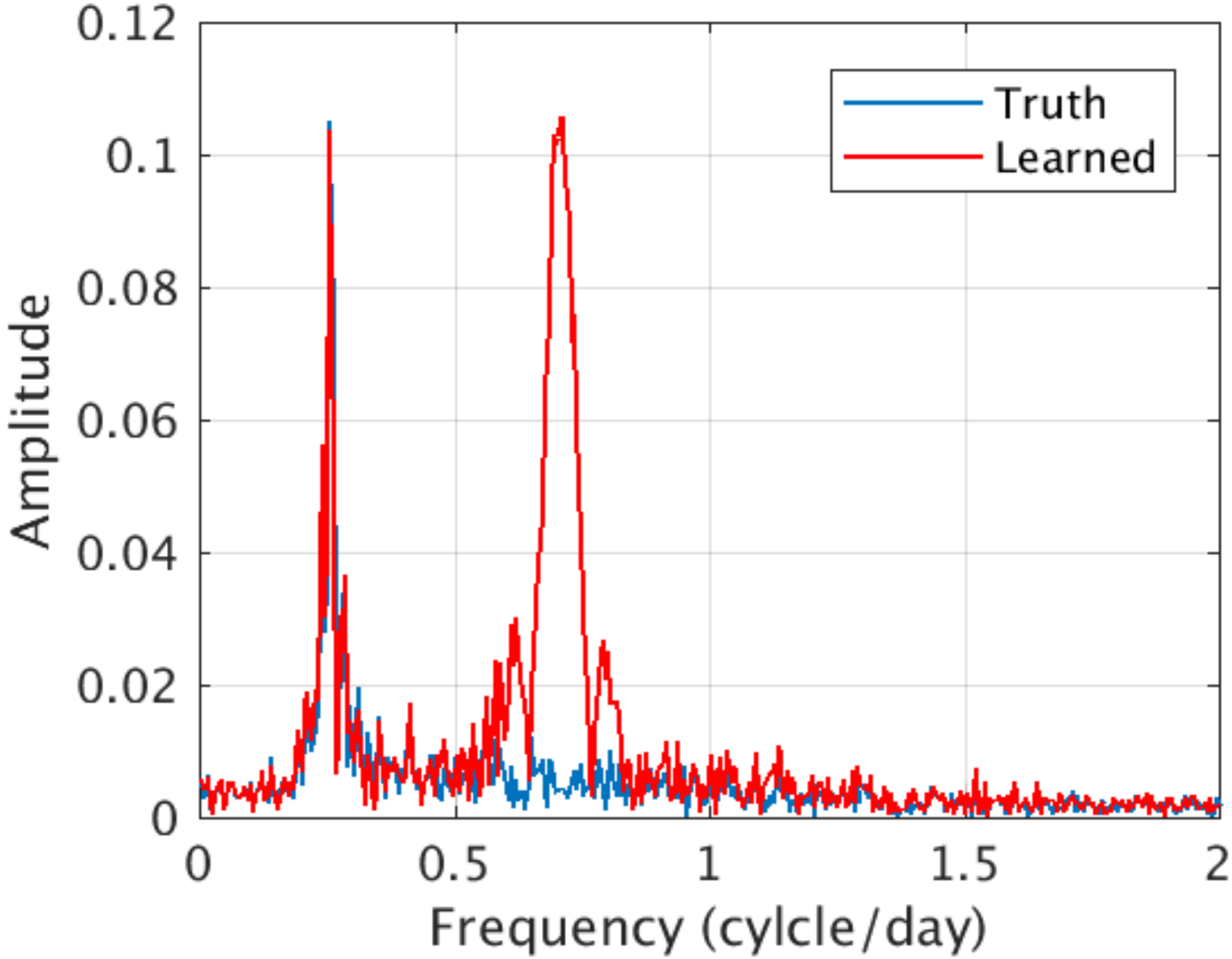
- Stable linear system driven by random noise
- No noise in convective tendencies
- 4 degrees of freedom for convection (6 overall)
- A random forest approach (10 trees, 5 leaves)
- 1.2e5 training samples

- **Test set $R^2$:**
  - Lower troposphere temperature: 0.97
  - Upper troposphere temperature: 0.97
  - Free troposphere temperature: 0.99
  - Boundary layer moist static energy: 0.96

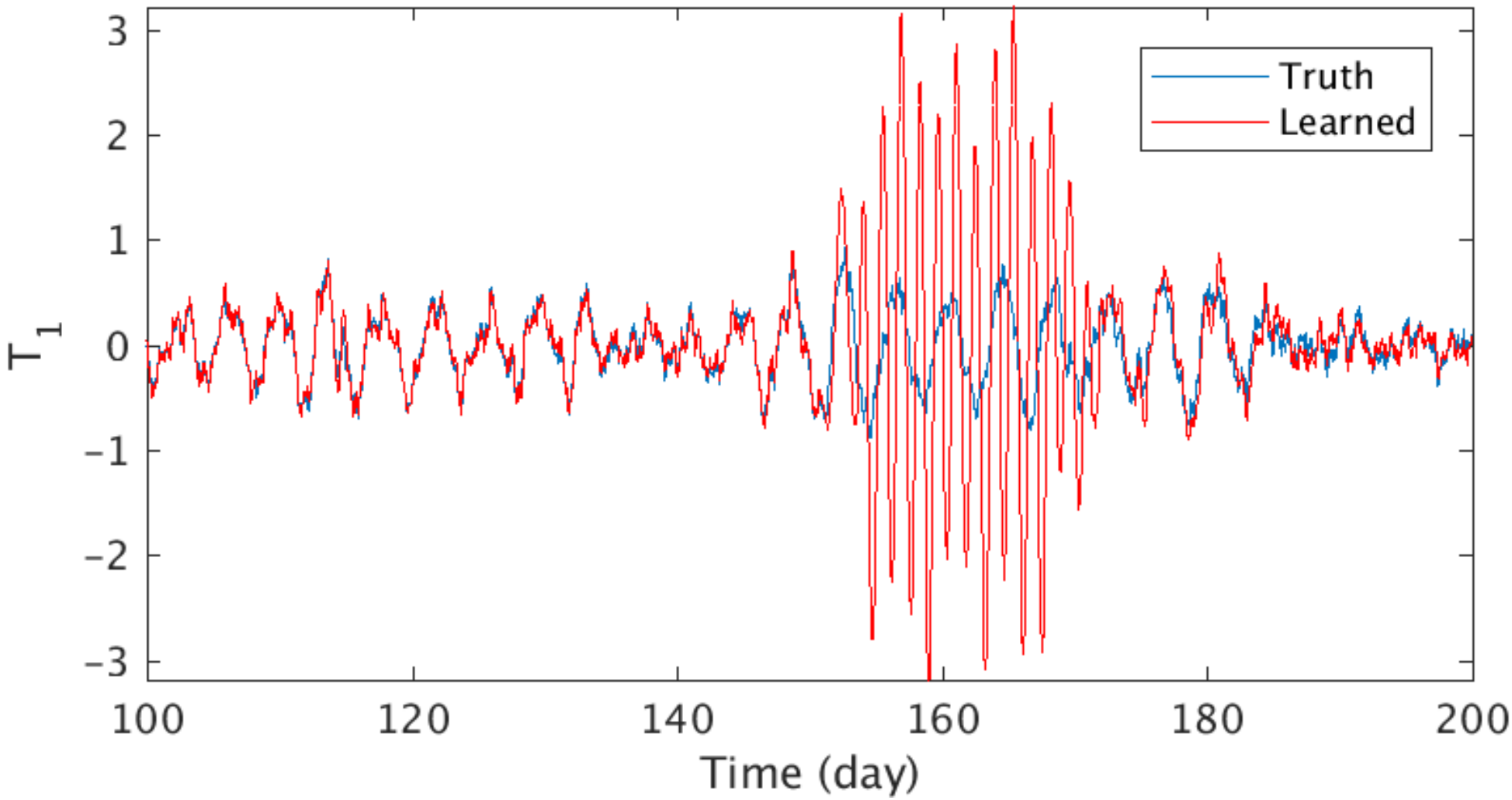
# Spectrum from coupled run with the true matrix



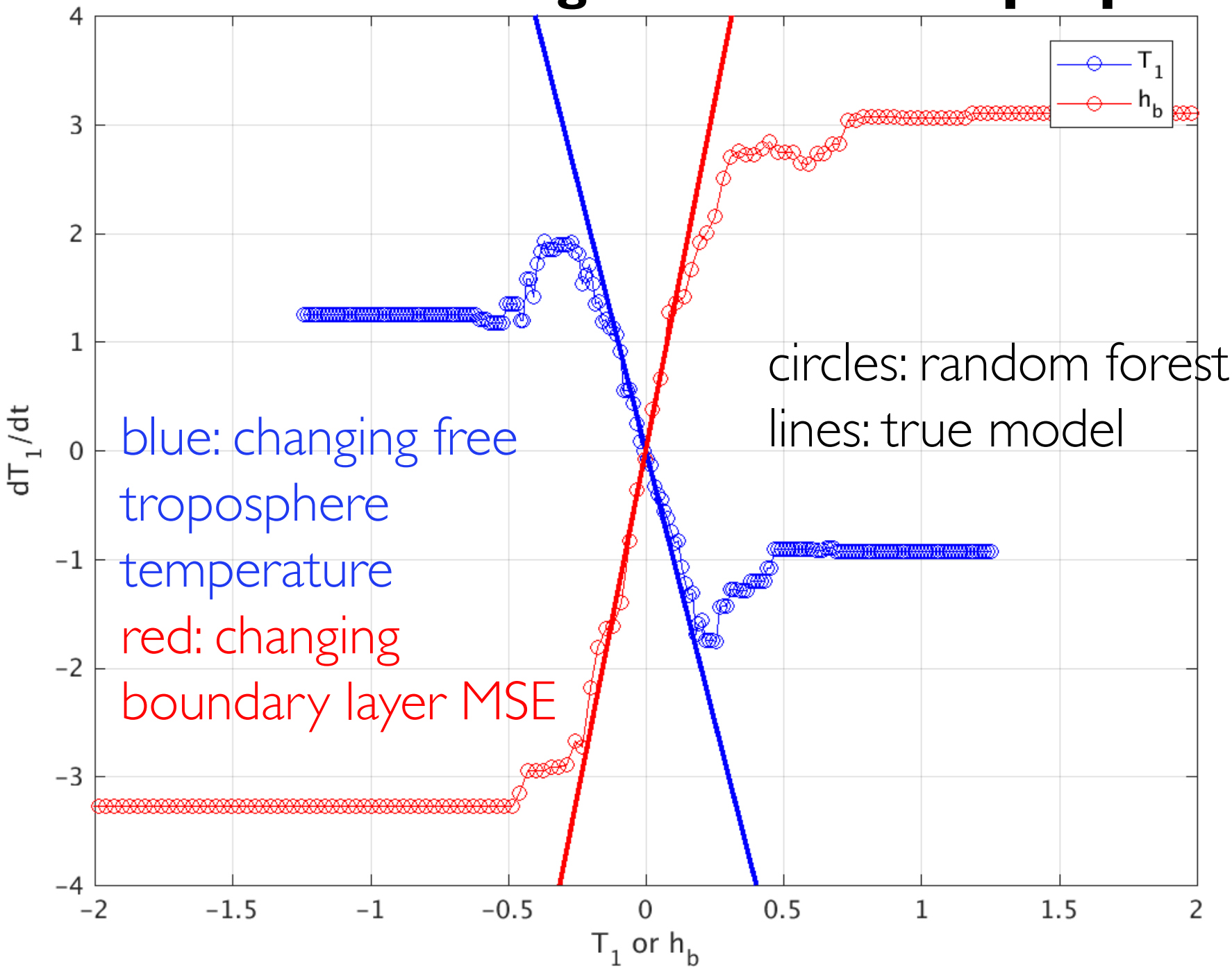
# True matrix replaced by random forest



# Time series



# Convective heating of the free troposphere



# Binary decision tree

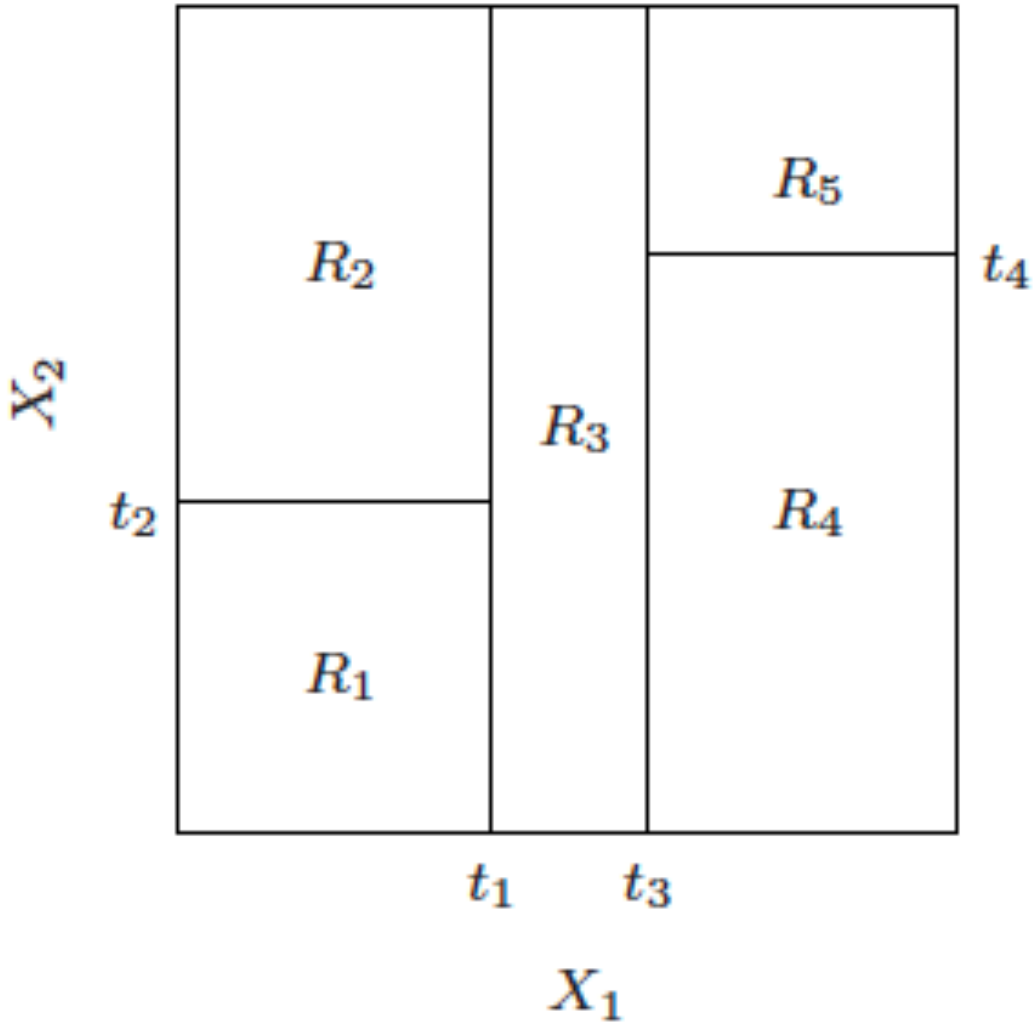
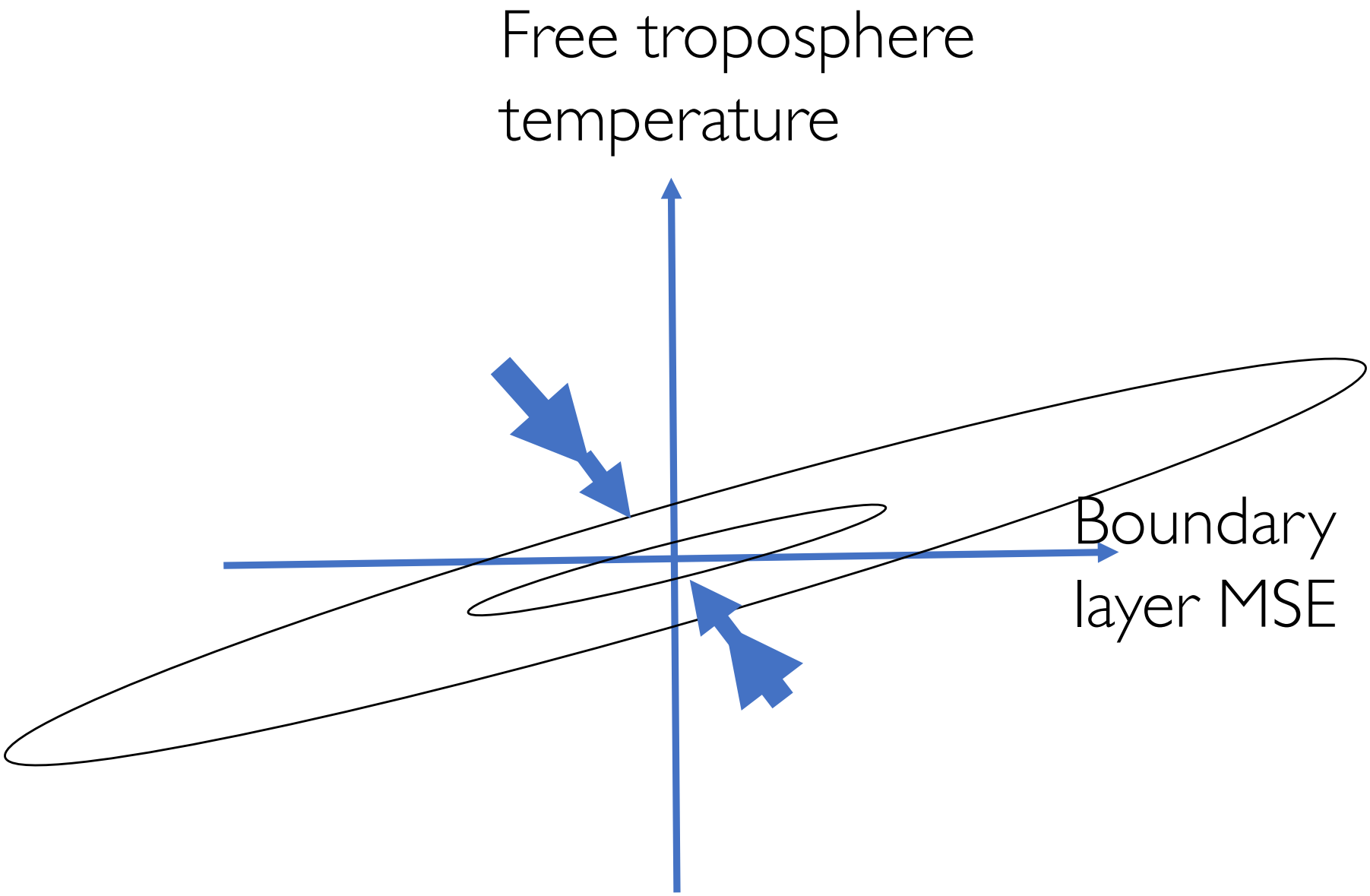
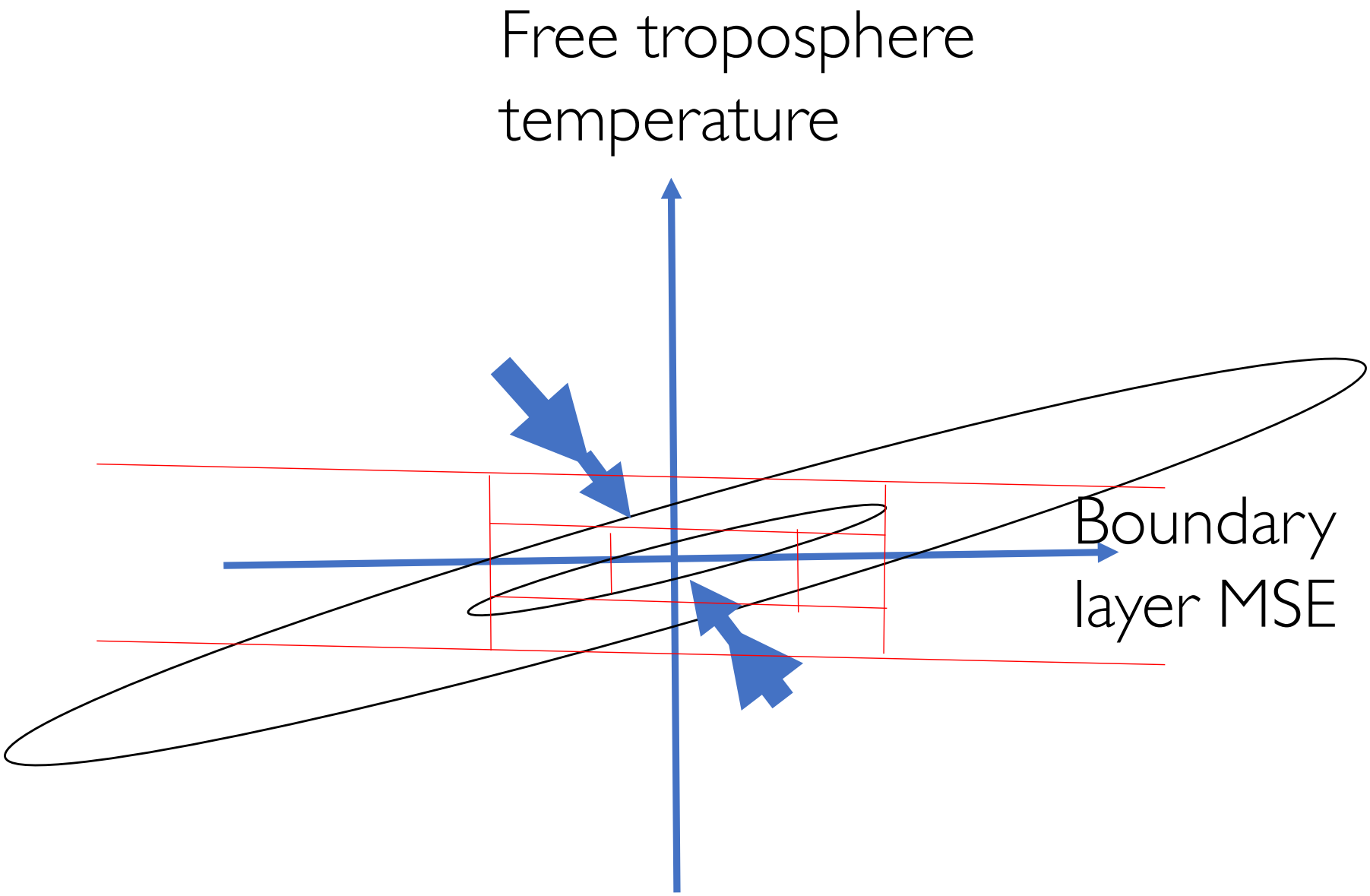
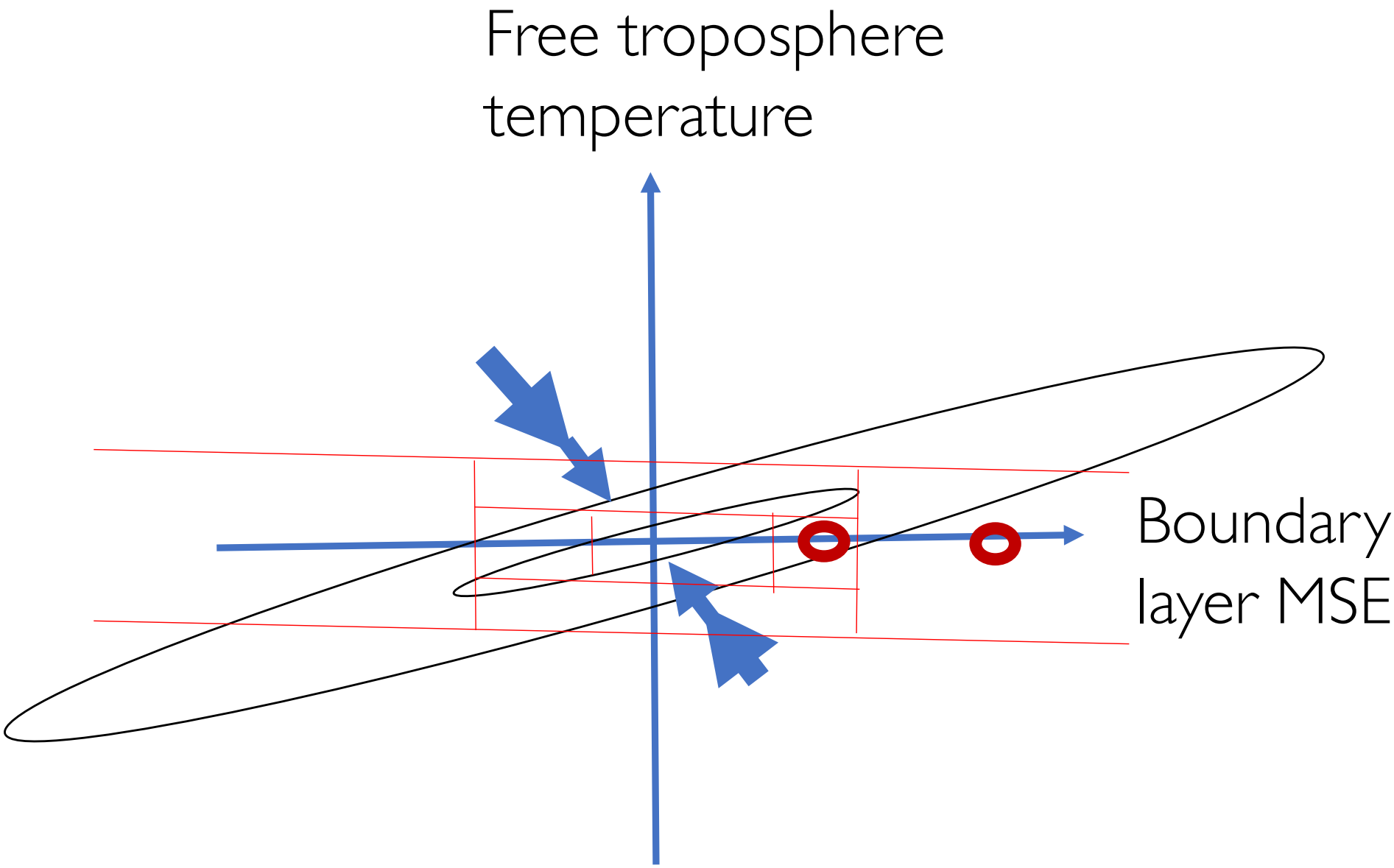
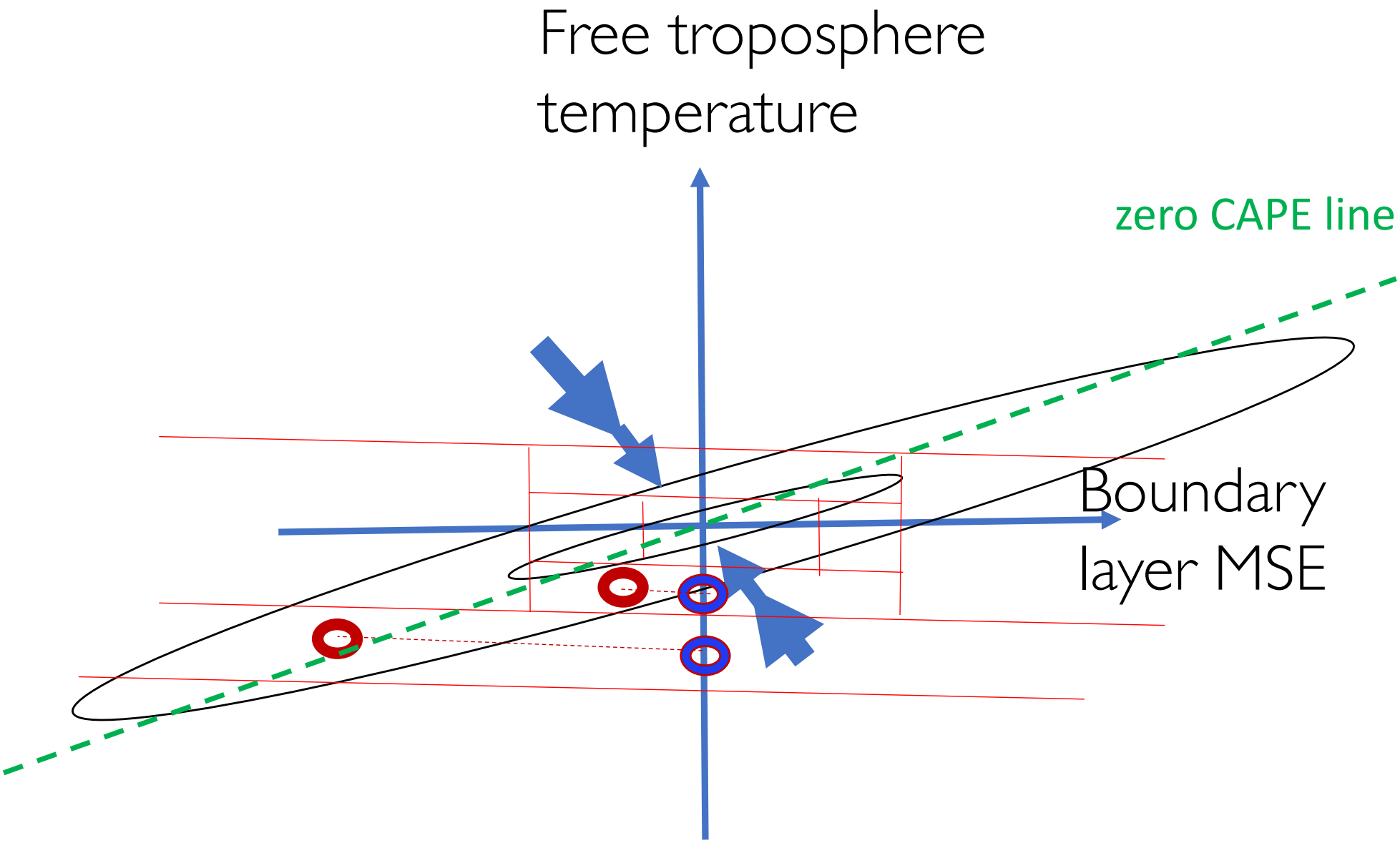



Figure from Hastie, Tibshirani and Friedman

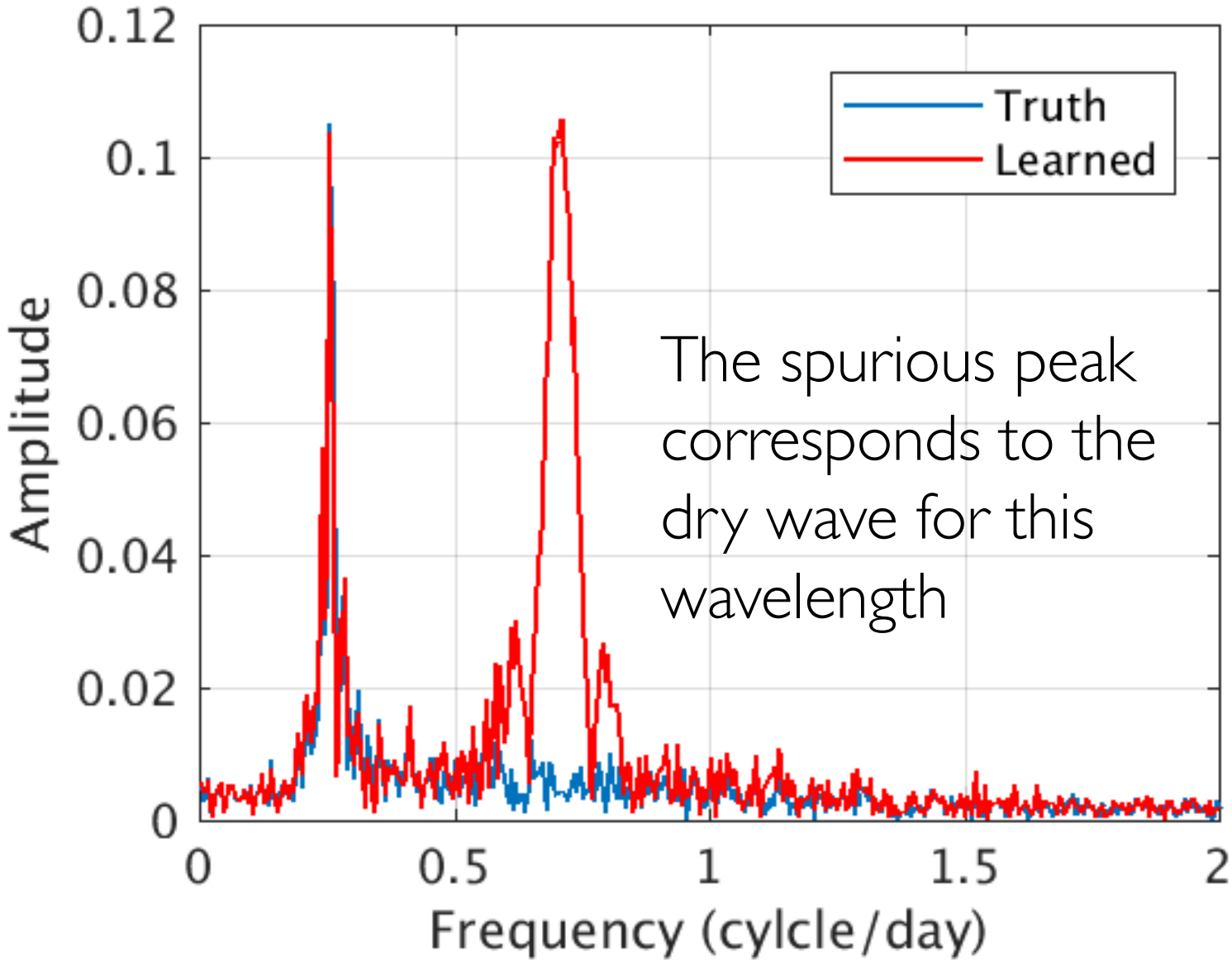

Free troposphere
temperature
Boundary
layer MSE


Free troposphere
temperature
Boundary
layer MSE


Free troposphere
temperature
Boundary
layer MSE

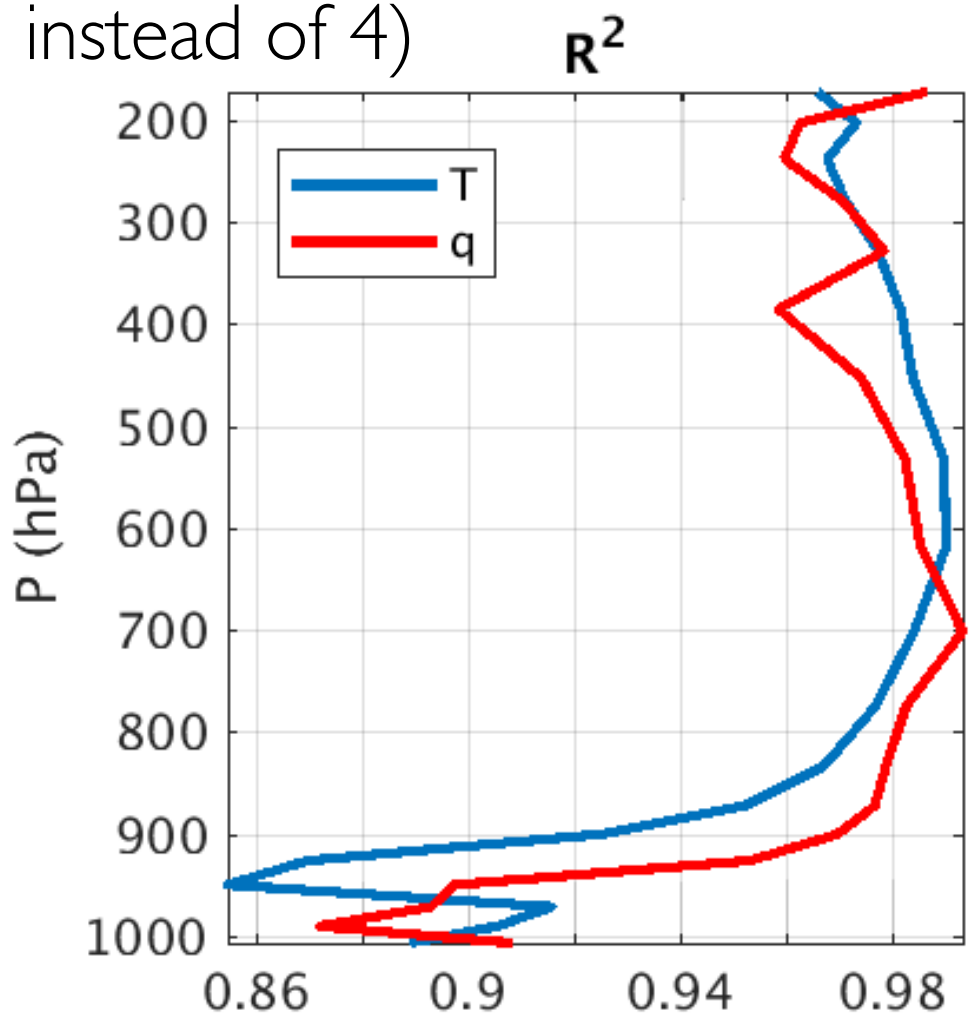

Free troposphere temperature
zero CAPE line
Boundary layer MSE

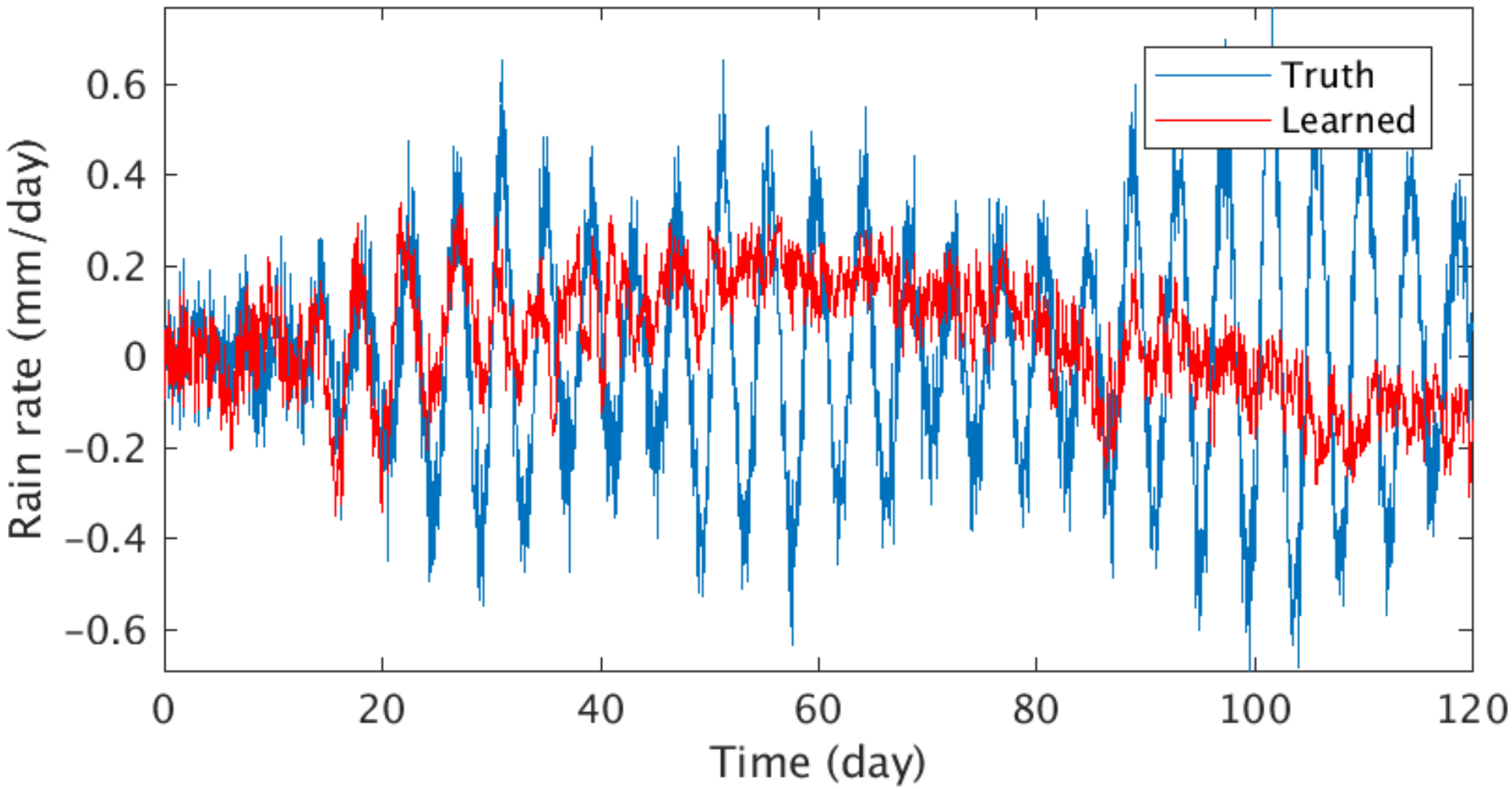
# True matrix replaced by random forest

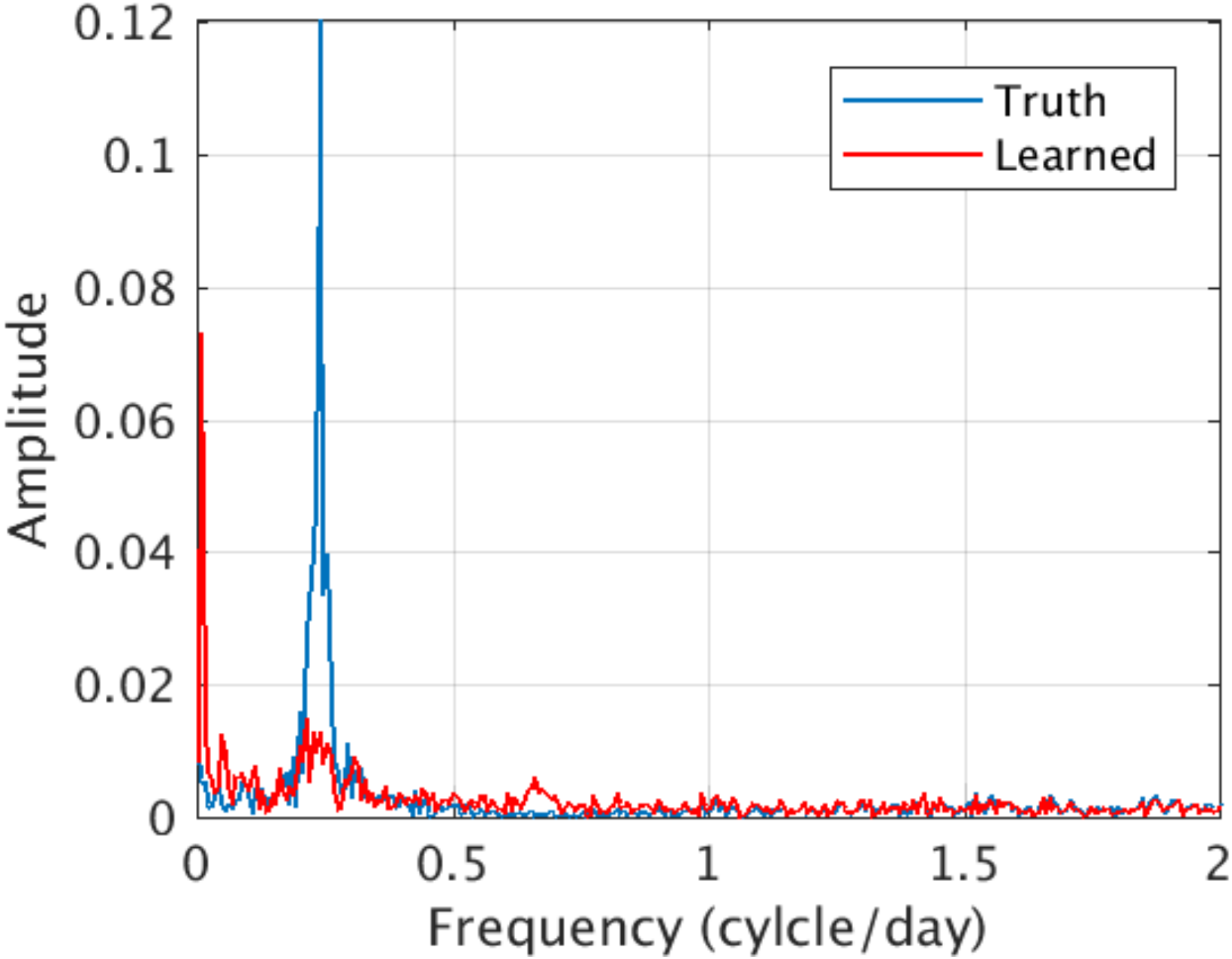


# Higher dimensionality

- Use the full linear response function for convection (46 degrees of freedom instead of 4)
- 2.4e6 training samples

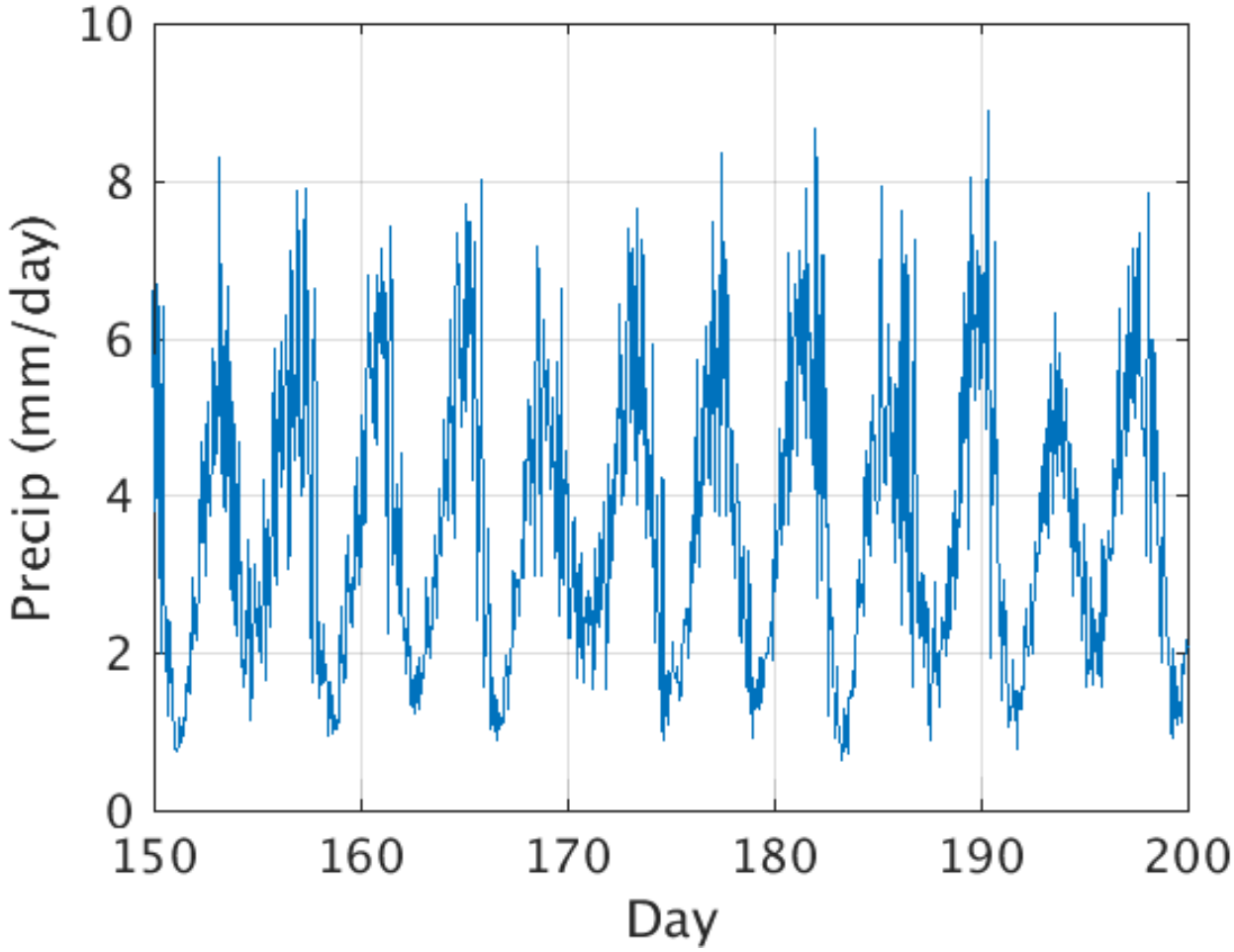



Truth
Learned
Rain rate (mm/day)
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
20
40
60
80
100
120
Time (day)


0.12
0.1
0.08
0.06
0.04
0.02
0
Amplitude
0
0.5
1
1.5
2
Frequency (cylcle/day)
Truth
Learned

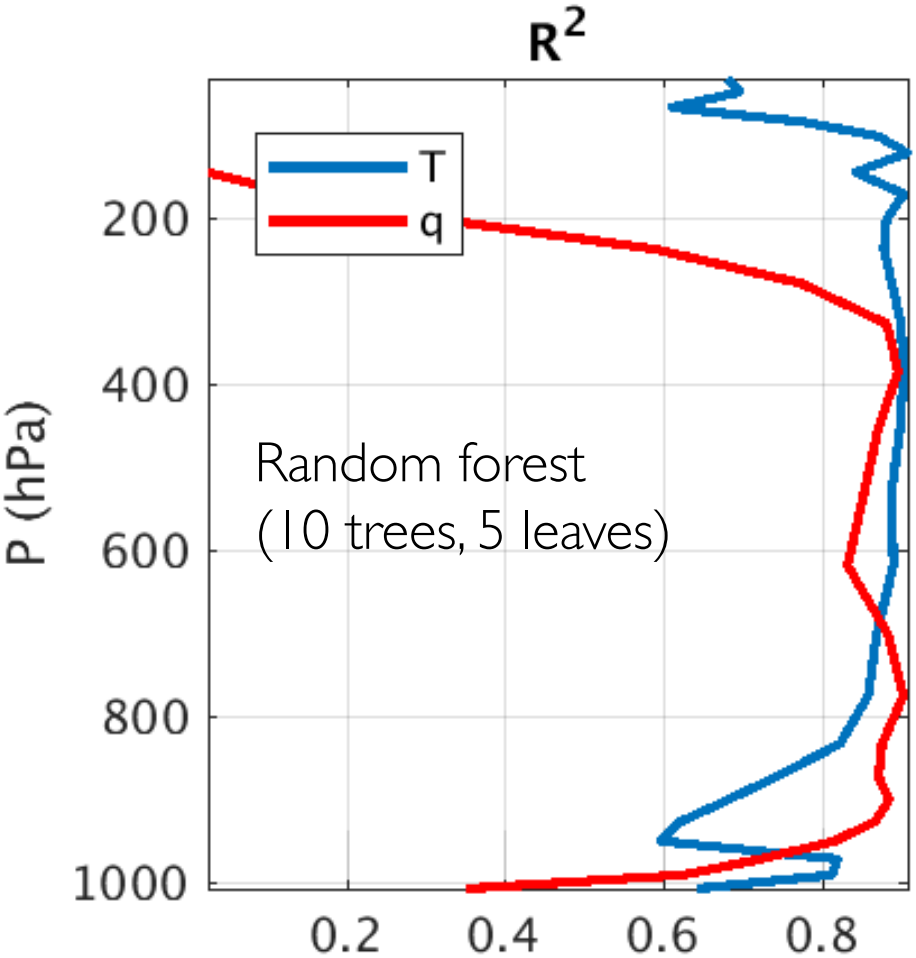
# A more realistic example

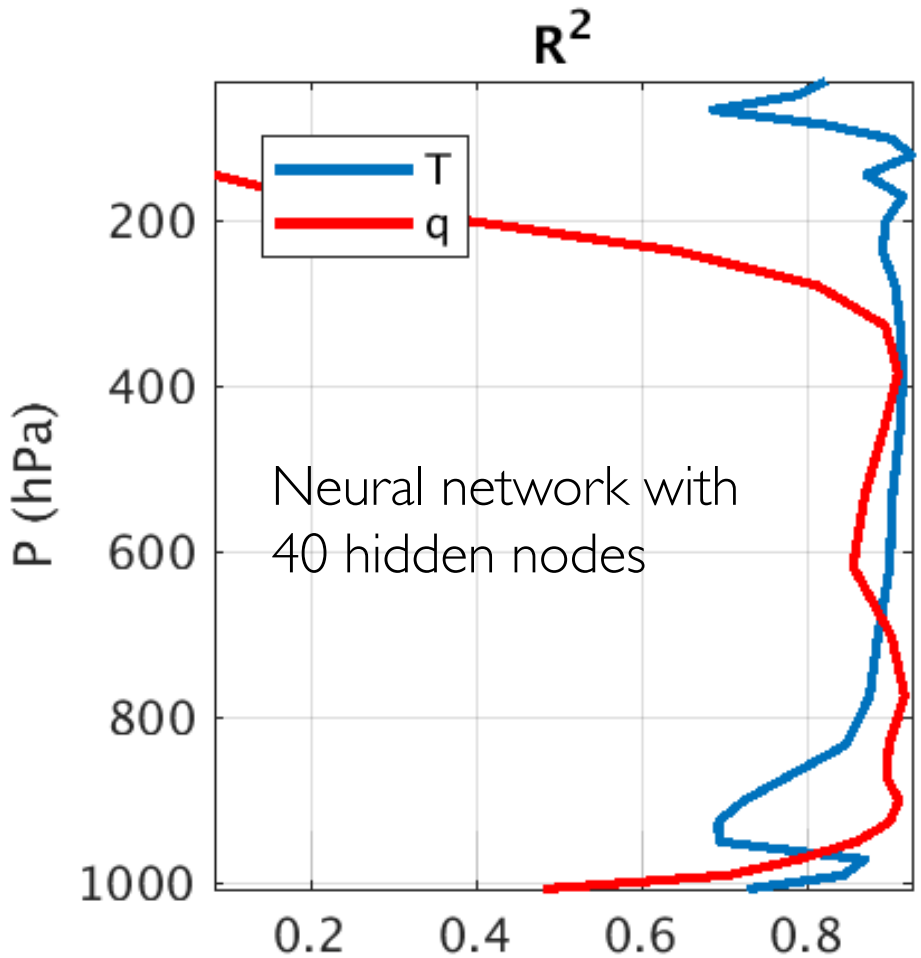
- A cyclic CRM (512kmx512km) coupled with 2D linear gravity wave (horizontal wavelength of 5000km).
- Long period (~4 days) makes the memory issue less acute



# Reasonable test error

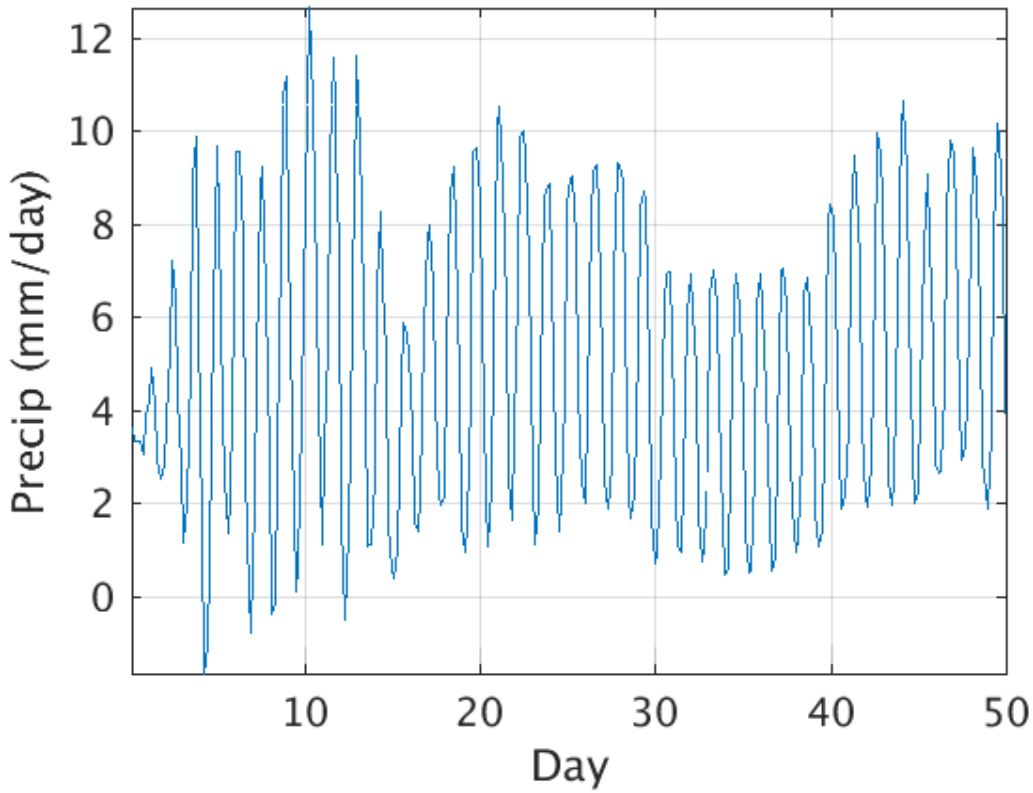
- 70000 training samples



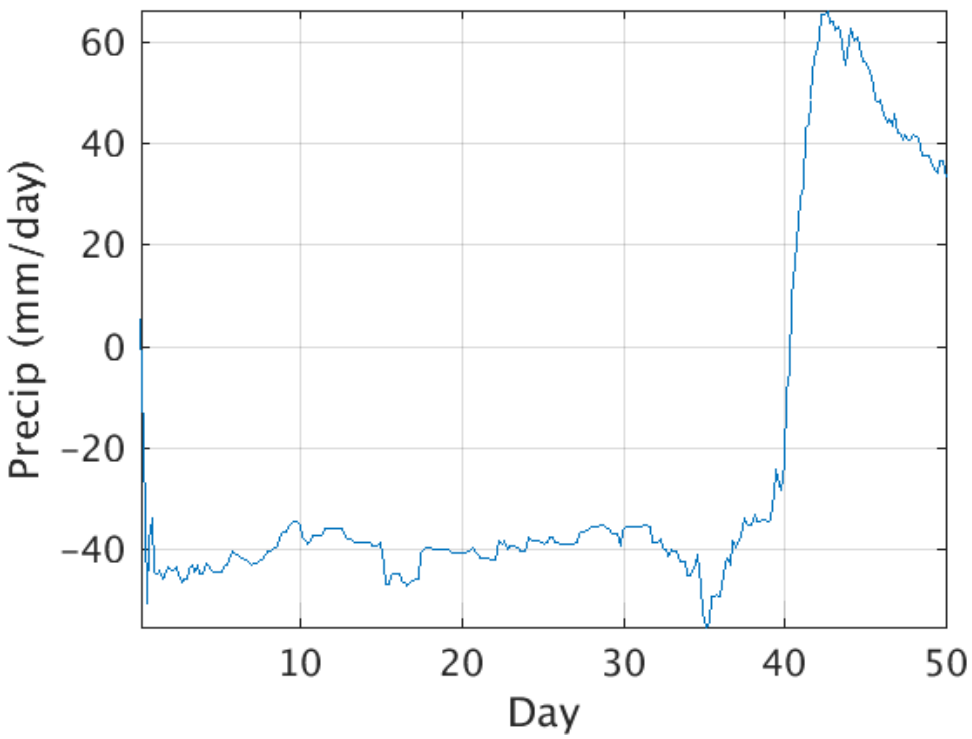
# Big errors when coupled

Random forest
(10 trees, 5 leaves)




Period 1.3 days, phase speed
45m/s

Neural network with
40 hidden nodes



- The coupled system can behave differently even when test set errors are small for two reasons:
- 1. Coupling can selectively amplify or damp errors, which makes learning based on minimizing those errors less effective.
- 2. Erroneous behaviors can arise as the system travels to the boundaries of the training set.
- **The error function needs to take coupling into consideration.**

# “Active” learning?

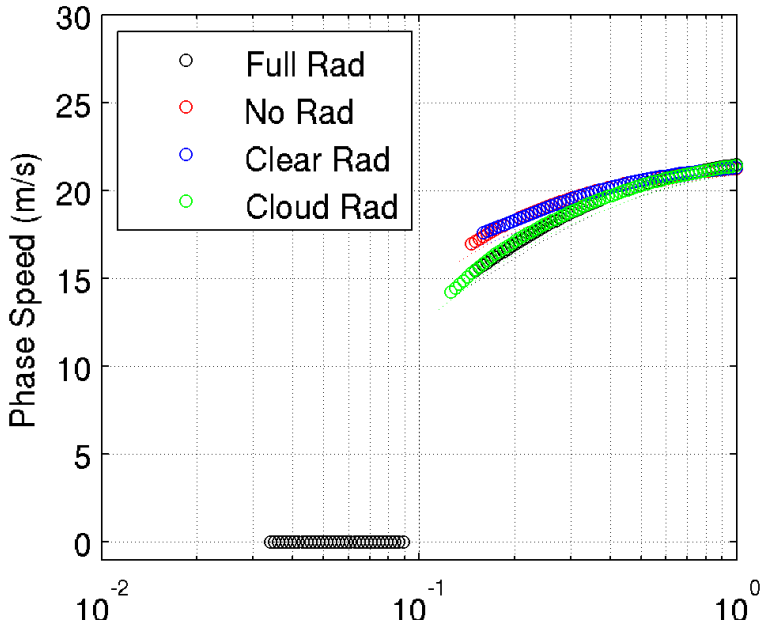
- Unlike in typical machine learning problem, we can make new CRM runs as needed.
- We can build linear response functions for different mean states and stitch them together using e.g. a decision tree, and also build new ones on demand
  - This should be more efficient than a brute-force database (look-up table) of the state vectors and the tendencies.

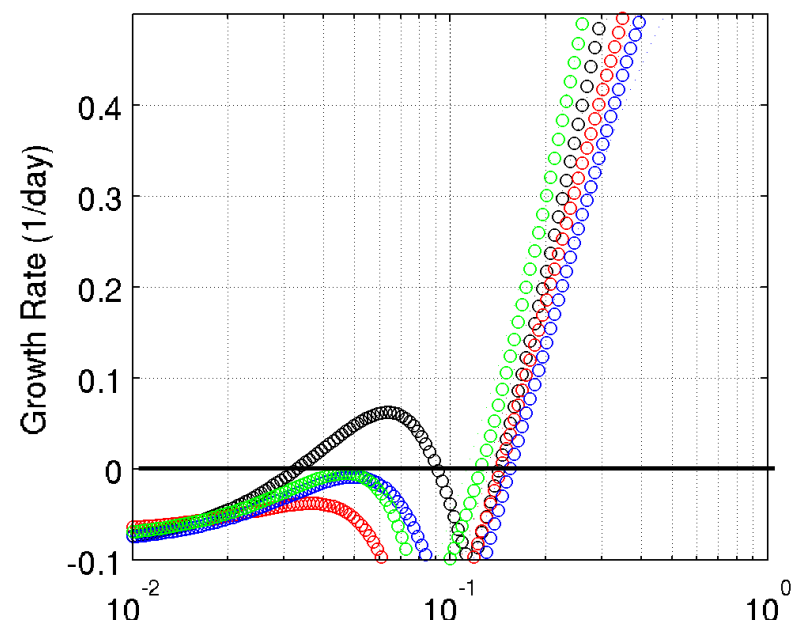
# Concluding remarks

- There are a number of issues to work through in order to learn a cumulus parameterization from CRM simulations:
  - Intrinsic noise
  - Memory (not explored here)
  - **Coupling with large-scale**
  - High dimensionality
- Some "active" learning is likely needed.

# Model order reduction from control theory

Full linear response function

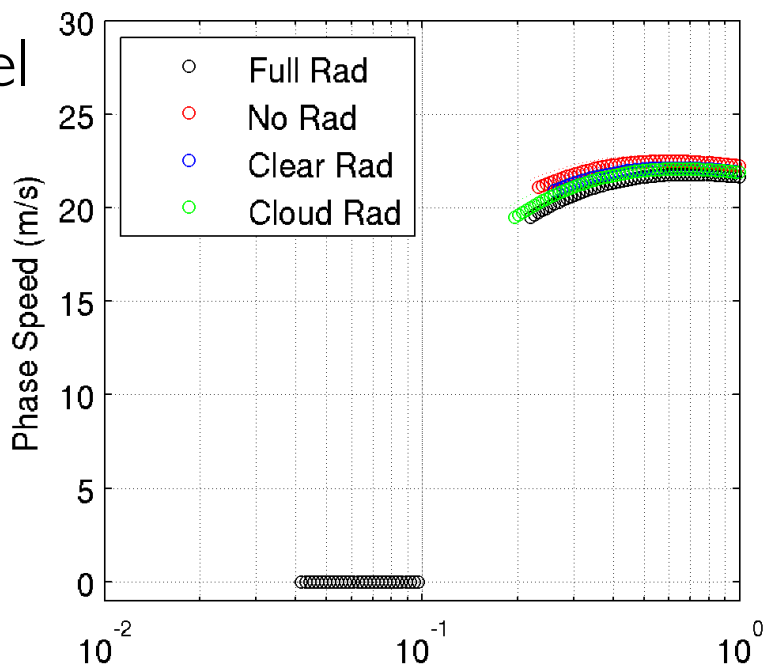


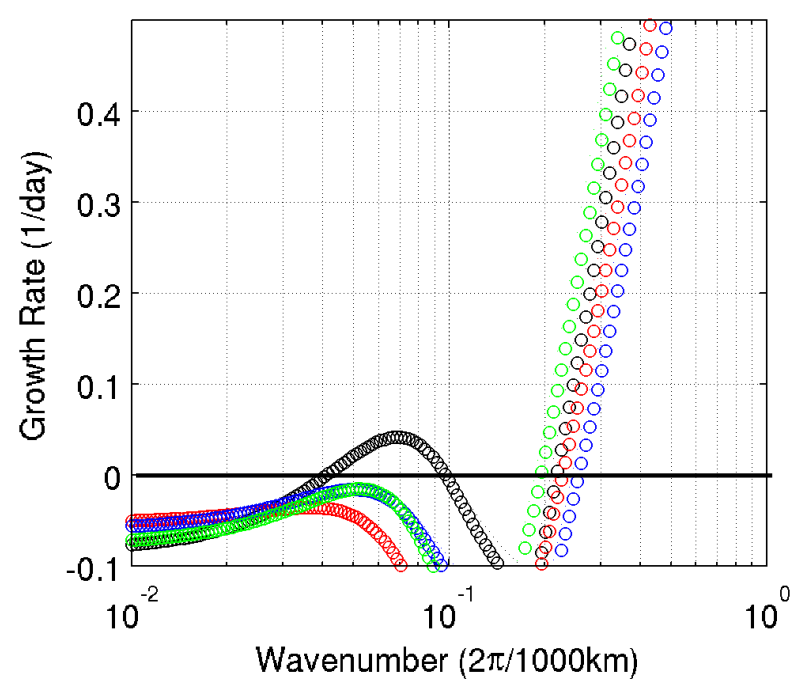




Reduced model

4 T&q modes with structural constraints

2 w modes



$$X_n \equiv x_{2n-1} + x_{2n}$$
$$Y_n \equiv y_{2n-1} + y_{2n}$$

$$X_{n+1} = (2 - \lambda \Delta t\,) x_{2n-1} + \sigma_{2n-1} \Delta t$$

$$\boxed{Y_{n+1} = -\lambda X_{n+1} + \sigma_{2n-1} + \sigma_{2n}}$$

$$\frac{\langle X, Y \rangle}{\langle X, X \rangle} = -\frac{1}{2}$$

X is no longer uncorrelated with the noise